

Some Results on Fractional Integro-Differential Equations with Nonlocal Integral Boundary Conditions of Riemann-Liouville Type

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Abstract

In this paper, we discuss about the existence and uniqueness of solutions for the fractional integro-differential equations with nonlocal integral boundary conditions of Riemann- Liouville Type. Existence result is based on Krasnoselskii's fixed point theorem and the uniqueness result is based on the contraction mapping principle.

Keywords: Fractional Differential Equation, Nonlocal integral boundary conditions, Existence, Banach Contraction Principle, Fixed point.

I. INTRODUCTION

Fractional Differential equation defined as fractional derivatives involving an equation. Fractional differential equations has recently used by several researchers such as aerodynamics, polymer rheology, biophysics and etc. In recent years, many authors interest in the study of fractional – order differential equations with boundary conditions. [see 1-6, 10].

In this paper, we consider the following fractional integro-differential equations with nonlocal integral boundary conditions of Riemann-Liouville Type :

$${}^c D^\alpha u(t) = g\left(t, u(t), \int_0^t h(t, s, u(s)) ds\right), \quad t \in J = [0,1], \quad 1 < \alpha \leq 2 \quad (1)$$

$$u(0) = a I^p u(\zeta) = a \int_0^\zeta \frac{(\zeta - s)^{p-1}}{\Gamma(p)} u(s) ds, \quad 0 < p \leq 1,$$

$$u(1) = b I^q u(\xi) = b \int_0^\xi \frac{(\xi - s)^{q-1}}{\Gamma(q)} u(s) ds, \quad 0 < q \leq 1 \quad (2)$$

where the function $f: J \times \mathbb{R} \rightarrow \mathbb{R}$ is a continuous function. Here ${}^c D^\alpha$ is the caputo fractional derivative of order $1 < \alpha \leq 2$, I^α is the Riemann-Liouville fractional integral of order $\alpha > 0$ and a, b, ζ, ξ are real constants with $0 < \zeta, \xi < 1$.

In section 2 is devoted to preliminaries related to the existence of solution. The proof of main results of the paper discussed in section 3. Finally, an example is illustrated in the section 4.

II. PRELIMINARIES

In this section, we introduce definitions [7, 8, 9] and preliminary facts which are used throughout this paper. Denote by $Y = C(J, \mathbb{R})$, the Banach space of all continuous functions from J into \mathbb{R} with the norm

$$\|u\| = \sup_{t \in J} \{|u(t)|\}$$

Definition 2.1. The Riemann-Liouville fractional integral of order $\alpha > 0$ for a function $f(t)$ is defined as

$$I^\alpha f(t) = \frac{1}{\Gamma(\alpha)} \int_0^t (t-s)^{\alpha-1} f(s) ds, \quad t > 0,$$

provided that the right hand side is point wise defined on $[0, \infty)$, where Γ is the gamma function.

Definition 2.2. The Caputo fractional derivative of order $\alpha > 0$ of a function $f: [0, \infty) \rightarrow \mathbb{R}$ is defined as

$${}^c D^\alpha f(t) = \frac{1}{\Gamma(n-\alpha)} \int_0^t (t-s)^{n-\alpha-1} f^{(n)}(s) ds, \quad n-1 < \alpha < n,$$

where $n = [\alpha] + 1$ and $[\alpha]$ denotes the integral part of the real number.

Lemma 2.3. For any given $w \in Y, u \in Y^2$ is a solution of the fractional differential equation

$${}^c D^\alpha u(t) = w(t), \quad 1 < \alpha \leq 2 \tag{3}$$

with the boundary conditions

$$u(0) = aI^p u(\zeta), \quad u(1) = bI^q u(\xi), \quad 0 < p \leq 1, q \leq 1 \tag{4}$$

if and only if $u(t)$ is a solution of the fractional integral equation

$$u(t) = I^\alpha w(t) + (z_1 - z_4 t)I^{\alpha+p} w(\zeta) + (z_2 + z_3 t)[bI^{\alpha+q} w(\xi) - I^\alpha w(1)] \tag{5}$$

Where

$$\begin{aligned} z_1 &= \frac{a}{z} \left(1 - \frac{b\xi^{q+1}}{\Gamma(q+2)} \right), & z_2 &= \frac{a}{z} \left(\frac{\zeta^{p+1}}{\Gamma(p+2)} \right), \\ z_3 &= \frac{1}{z} \left(1 - \frac{a\zeta^p}{\Gamma(p+1)} \right), & z_4 &= \frac{a}{z} \left(1 - \frac{b\xi^q}{\Gamma(q+1)} \right) \\ z &= \left(1 - \frac{a\zeta^p}{\Gamma(p+1)} \right) \left(1 - \frac{b\xi^{q+1}}{\Gamma(q+2)} \right) + \left(\frac{\zeta^{p+1}}{\Gamma(p+2)} \right) \left(1 - \frac{b\xi^q}{\Gamma(q+1)} \right) \end{aligned} \tag{6}$$

Proof. We know that, the general solution of the fractional differential equation (3) can be written as

$$u(t) = I^\alpha w(t) + c_0 + c_1 t \tag{7}$$

where c_0, c_1 are arbitrary constants.

Applying the boundary conditions (4) in (7), we have

$$\left(1 - \frac{a\zeta^p}{\Gamma(p+1)} \right) c_0 - \left(\frac{a\zeta^{p+1}}{\Gamma(p+2)} \right) c_1 = aI^{\alpha+p} w(\zeta) \tag{8}$$

$$\left(1 - \frac{b\xi^q}{\Gamma(q+1)} \right) c_0 + \left(1 - \frac{b\xi^{q+1}}{\Gamma(q+2)} \right) c_1 = bI^{\alpha+q} w(\xi) - I^\alpha w(1) \tag{9}$$

Solving the system of equations (8) and (9), we get

$$\begin{aligned} c_0 &= \frac{1}{z} \left\{ a \left(1 - \frac{b\xi^{q+1}}{\Gamma(q+2)} \right) I^{\alpha+p} w(\zeta) + \left(\frac{a\zeta^{p+1}}{\Gamma(p+2)} \right) [bI^{\alpha+q} w(\xi) - I^\alpha w(1)] \right\} \text{ and} \\ c_1 &= \frac{1}{z} \left\{ -a \left(1 - \frac{b\xi^q}{\Gamma(q+1)} \right) I^{\alpha+p} w(\zeta) + \left(1 - \frac{a\zeta^p}{\Gamma(p+1)} \right) [bI^{\alpha+q} w(\xi) - I^\alpha w(1)] \right\}. \end{aligned}$$

Substituting the values of c_0, c_1 in (7), we get (5).

Conversely, it is clear that the integral solution (5) satisfies the equation (3) and boundary conditions (4). This completes the proof.

III. MAIN RESULTS

In view of lemma 2.3, we define the operator $\mathcal{G}: Y \rightarrow Y$ by

$$(\mathcal{G}u)(t) = I^\alpha g(s, u(s), Hu(s))(t) + (z_1 - z_4 t)I^{\alpha+p} g(s, u(s), Hu(s))(\zeta) + (z_2 + z_3 t) [bI^{\alpha+q} g(s, u(s), Hu(s))(\xi) - I^\alpha g(s, u(s), Hu(s))(1)], t \in J$$

For the forthcoming analysis, we need the following assumptions:

(H1) There exists constants L_g and L_h such that

- (i) $|g(t, u_1, v_1) - g(t, u_2, v_2)| \leq L_g (|u_1 - u_2| + |v_1 - v_2|), t \in J, u_1, u_2, v_1, v_2 \in Y$
- (ii) $|h(t, s, u_1) - h(t, s, u_2)| \leq L_h (|u_1 - u_2|)$

(H2) $|f(t, u, v)| \leq l(t)\phi(\|u\|), (t, u, v) \in J \times \mathbb{R}^2$, where $l \in L^1(J, \mathbb{R}^+)$ and $\phi: [0, \infty) \rightarrow [0, \infty)$ is a continuous non-decreasing function.

For our convenience, we can take

$$\Psi_1 = \frac{1}{\Gamma(\alpha+1)} + (|z_1| + |z_4|) \frac{\zeta^{\alpha+p}}{\Gamma(\alpha+p+1)} + (|z_2| + |z_3|) \left[\frac{|b|\xi^{\alpha+q}}{\Gamma(\alpha+q+1)} + \frac{1}{\Gamma(\alpha+1)} \right]$$

and

$$\Psi_2 = L_g \left\{ \left(\frac{1}{\Gamma(\alpha + 1)} + (|z_1| + |z_4|) \frac{\zeta^{\alpha+p}}{\Gamma(\alpha + p + 1)} + (|z_2| + |z_3|) \left[\frac{|b|\xi^{\alpha+q}}{\Gamma(\alpha + q + 1)} + \frac{1}{\Gamma(\alpha + 1)} \right] \right) \right. \\ \left. + L_h \left(\frac{1}{\Gamma(\alpha + 1)} + (|z_1| + |z_4|) \frac{\zeta^{\alpha+p}}{\Gamma(\alpha + p + 1)} + (|z_2| + |z_3|) \left[\frac{|b|\xi^{\alpha+q}}{\Gamma(\alpha + q + 1)} + \frac{1}{\Gamma(\alpha + 1)} \right] \right) \right\}$$

A. Existence result via Krasnoselskii’s fixed point theorem

Lemma 3.1. [Krasnoselskii’s Theorem]. Let S be a closed, bounded, convex and nonempty subset of a Banach space X.

Let P, Q be two operators such that

- $Px + Qy \in S$, whenever $x, y \in S$,
- P is compact and continuous,
- Q is a contraction mapping.

Then there exists $z \in S$ such that $z = Pz + Qz$.

Theorem 3.2. Assume that the hypotheses (H1) and (H2) are satisfied. Then the boundary value problem (1)-(2) has

at least a solution in $C(J, \mathbb{R})$, provided that

$$L_g \left\{ \left((|z_1| + |z_4|) \frac{\zeta^{\alpha+p}}{\Gamma(\alpha + p + 1)} + (|z_2| + |z_3|) \left[\frac{|b|\xi^{\alpha+q}}{\Gamma(\alpha + q + 1)} + \frac{1}{\Gamma(\alpha + 1)} \right] \right) \right. \\ \left. + L_h \left((|z_1| + |z_4|) \frac{\zeta^{\alpha+p}}{\Gamma(\alpha + p + 1)} + (|z_2| + |z_3|) \left[\frac{|b|\xi^{\alpha+q}}{\Gamma(\alpha + q + 1)} + \frac{1}{\Gamma(\alpha + 1)} \right] \right) \right\} < 1.$$

Proof. We consider the set $B_r = \{u \in \mathbb{R} : \|u\| \leq r\}$, for $u \in B_r$.

We define the two operators \mathcal{G}_1 and \mathcal{G}_2 by

$$\mathcal{G}_1 = I^\alpha g(s, u(s), Hu(s))(t), \quad t \in J$$

$$\mathcal{G}_2 = (z_1 - z_4 t) I^{\alpha+p} g(s, u(s), Hu(s))(\zeta) + (z_2 + z_3 t) [b I^{\alpha+q} g(s, u(s), Hu(s))(\xi) - I^\alpha g(s, u(s), Hu(s))(1)]$$

Choosing $r \geq \|l\| \phi(r) \Psi_1$. For $u, v \in B_r$, we have

$$|\mathcal{G}_1 u(t) + \mathcal{G}_2 v(t)| \\ \leq \sup_{t \in J} \{ I^\alpha |g(s, u(s), Hu(s))|(t) + (|z_1| + |z_4|) I^{\alpha+p} |g(s, v(s), Hv(s))|(\zeta) \\ + (|z_2| + |z_3|) [|b| I^{\alpha+q} |g(s, v(s), Hv(s))|(\xi) + I^\alpha |g(s, v(s), Hv(s))|(1)] \} \\ \leq I^\alpha |g(s, u(s), Hu(s))|(1) + (|z_1| + |z_4|) I^{\alpha+p} |g(s, v(s), Hv(s))|(\zeta) + (|z_2| + |z_3|) \\ [|b| I^{\alpha+q} |g(s, v(s), Hv(s))|(\xi) + I^\alpha |g(s, v(s), Hv(s))|(1)] \\ \leq \|l\|_{L^1} \phi(r) \left\{ \frac{1}{\Gamma(\alpha + 1)} + (|z_1| + |z_4|) \frac{\zeta^{\alpha+p}}{\Gamma(\alpha + p + 1)} + (|z_2| + |z_3|) \left[\frac{|b|\xi^{\alpha+q}}{\Gamma(\alpha + q + 1)} + \frac{1}{\Gamma(\alpha + 1)} \right] \right\} \\ \leq \|l\| \phi(r) \Psi_1 \\ \leq r$$

Hence, $\mathcal{G}_1 u + \mathcal{G}_2 v \in B_r$.

Next we prove that \mathcal{G}_2 is a contraction.

$$|\mathcal{G}_2 u(t) - \mathcal{G}_2 v(t)| \\ \leq \sup_{t \in J} \{ (|z_1| + |z_4|) I^{\alpha+p} |g(s, u(s), Hu(s)) - g(s, v(s), Hv(s))|(\zeta) \\ + (|z_2| + |z_3|) [|b| I^{\alpha+q} |g(s, u(s), Hu(s)) - g(s, v(s), Hv(s))|(\xi) \\ + I^\alpha |g(s, u(s), Hu(s)) - g(s, v(s), Hv(s))|(1)] \} \\ \leq (|z_1| + |z_4|) I^{\alpha+p} [L_g \|u - v\| + \|Hu(s) - Hv(s)\|](\zeta) \\ + (|z_2| + |z_3|) [|b| I^{\alpha+q} [L_g \|u - v\| + \|Hu(s) - Hv(s)\|](\xi) \\ + I^\alpha [L_g \|u - v\| + \|Hu(s) - Hv(s)\|](1)] \\ \leq L_g \left\{ \left((|z_1| + |z_4|) \frac{\zeta^{\alpha+p}}{\Gamma(\alpha + p + 1)} + (|z_2| + |z_3|) \left[\frac{|b|\xi^{\alpha+q}}{\Gamma(\alpha + q + 1)} + \frac{1}{\Gamma(\alpha + 1)} \right] \right) \right. \\ \left. + L_h \left((|z_1| + |z_4|) \frac{\zeta^{\alpha+p}}{\Gamma(\alpha + p + 1)} + (|z_2| + |z_3|) \left[\frac{|b|\xi^{\alpha+q}}{\Gamma(\alpha + q + 1)} + \frac{1}{\Gamma(\alpha + 1)} \right] \right) \right\} \|u - v\|$$

Thus \mathcal{G}_2 is a contraction mapping.

Moreover, continuity of g and h implies that the operator \mathcal{G}_1 is continuous. Also, \mathcal{G}_1 is uniformly bounded on B_r as

$$\begin{aligned} |\mathcal{G}_1 u(t)| &\leq \sup_{t \in J} \{I^\alpha |g(s, u(s), Hu(s))|(t)\} \\ &\leq I^\alpha |g(s, u(s), Hu(s))|(1) \\ &\leq \frac{\|l\| \phi(r)}{\Gamma(\alpha + 1)} \end{aligned}$$

Next we prove the compactness of the operator \mathcal{G}_1 .

Now, for any $t_1, t_2 \in J$, $t_1 < t_2$, and $u \in B_r$

$$\begin{aligned} |(\mathcal{G}_1 x)(t_2) - (\mathcal{G}_1 x)(t_1)| &= |I^\alpha g(s, u(s), Hu(s))(t_2) - I^\alpha g(s, u(s), Hu(s))(t_1)| \\ &\leq \frac{\|l\| \phi(r)}{\Gamma(\alpha)} \left| \int_0^{t_1} [(t_2 - s)^{\alpha-1} - (t_1 - s)^{\alpha-1}] ds + \int_{t_1}^{t_2} (t_2 - s)^{\alpha-1} ds \right| \\ &\leq \frac{\|l\| \phi(r)}{\Gamma(\alpha + 1)} [2|t_2 - t_1|^\alpha + |t_2^\alpha - t_1^\alpha|] \end{aligned}$$

which is independent of u and tends to zero as $t_2 - t_1 \rightarrow 0$. Thus \mathcal{G}_1 is equicontinuous. Hence, by the Arzelà-Ascoli Theorem, \mathcal{G}_1 is compact on B_r . Hence, by the Krasnoselkii fixed point theorem, there exists a fixed point $u \in Y$ such that $\mathcal{G}x = x$ which is a solution to the boundary value problem (1)-(2) has at least one solution on J . This completes the proof.

B. Uniqueness result via Banach’s fixed point Theorem

Theorem 3.3: Assume that the hypotheses (H1) and (H2) hold and $\Psi_2 < 1$. Then the boundary value problem (1)-(2) has a unique solution on J .

Proof:

Let $M_1 = \sup_{t \in J} |g(s, 0, 0)|$ and $M_2 = \sup_{t \in J} |h(t, s, 0)|$

and choose

$$\frac{M_1 \Delta_1 + L_g M_2 \Delta_2}{1 - L_g(\Delta_1 - L_h \Delta_2)} \leq r.$$

We take

$$\begin{aligned} \Delta_1 &= \frac{1}{\Gamma(\alpha + 1)} + (|z_1| + |z_4|) \frac{\zeta^{\alpha+p}}{\Gamma(\alpha + p + 1)} + (|z_2| + |z_3|) \left[\frac{|b| \xi^{\alpha+q}}{\Gamma(\alpha + q + 1)} + \frac{1}{\Gamma(\alpha + 1)} \right] \\ \Delta_2 &= \frac{1}{\Gamma(\alpha + 2)} + (|z_1| + |z_4|) \frac{\zeta^{\alpha+p+1}}{\Gamma(\alpha + p + 2)} + (|z_2| + |z_3|) \left[\frac{|b| \xi^{\alpha+q+1}}{\Gamma(\alpha + q + 2)} + \frac{1}{\Gamma(\alpha + 2)} \right] \end{aligned}$$

Now we prove that $\mathcal{G}B_r \subset B_r$.

For $u \in B_r$, we have

$$\begin{aligned} |(\mathcal{G}u)(t)| &\leq \sup_{t \in J} \{I^\alpha (|g(s, u(s), Hu(s)) - g(s, 0, 0)| + |g(s, 0, 0)|)(t) \\ &\quad + (|z_1| + |z_4|) I^{\alpha+p} (|g(s, u(s), Hu(s)) - g(s, 0, 0)| + |g(s, 0, 0)|)(\zeta) \\ &\quad + (|z_2| + |z_3|) [|b| I^{\alpha+q} (|g(s, u(s), Hu(s)) - g(s, 0, 0)| + |g(s, 0, 0)|)(\xi) \\ &\quad + I^\alpha (|g(s, u(s), Hu(s)) - g(s, 0, 0)| + |g(s, 0, 0)|)(1)] \} \\ &\leq I^\alpha (|g(s, u(s), Hu(s)) - g(s, 0, 0)| + |g(s, 0, 0)|)(1) \\ &\quad + (|z_1| + |z_4|) I^{\alpha+p} (|g(s, u(s), Hu(s)) - g(s, 0, 0)| + |g(s, 0, 0)|)(\zeta) \\ &\quad + (|z_2| + |z_3|) [|b| I^{\alpha+q} (|g(s, u(s), Hu(s)) - g(s, 0, 0)| + |g(s, 0, 0)|)(\xi) \\ &\quad + I^\alpha (|g(s, u(s), Hu(s)) - g(s, 0, 0)| + |g(s, 0, 0)|)(1)] \\ &\leq (L_g r + M_1) \left\{ \frac{1}{\Gamma(\alpha + 1)} + (|z_1| + |z_4|) \frac{\zeta^{\alpha+p}}{\Gamma(\alpha + p + 1)} + (|z_2| + |z_3|) \left[\frac{|b| \xi^{\alpha+q}}{\Gamma(\alpha + q + 1)} + \frac{1}{\Gamma(\alpha + 1)} \right] \right\} \\ &\quad + \\ &\quad (L_g L_h r + L_g M_2) \left\{ \frac{1}{\Gamma(\alpha + 2)} + (|z_1| + |z_4|) \frac{\zeta^{\alpha+p+1}}{\Gamma(\alpha + p + 2)} \right. \\ &\quad \left. + (|z_2| + |z_3|) \left[\frac{|b| \xi^{\alpha+q+1}}{\Gamma(\alpha + q + 2)} + \frac{1}{\Gamma(\alpha + 2)} \right] \right\} \end{aligned}$$

$$\begin{aligned} &\leq (L_f r + M_1)\Delta_1 + (L_f L_g r + L_f M_2)\Delta_2 \\ &\leq r L_f (\Delta_1 + L_g \Delta_2) + M_1 \Delta_1 + L_f M_2 \Delta_2 \\ &\leq r \end{aligned}$$

Thus $\mathcal{G}B_r \subset B_r$.

Next we will prove that \mathcal{G} is a contraction mapping on B_r . For $u, v \in y$ and $t \in J$, we have

$$\begin{aligned} |\mathcal{G}u(t) - \mathcal{G}v(t)| &\leq \sup_{t \in J} \{ I^\alpha |g(s, u(s), Hu(s)) - g(s, v(s), Hv(s))|(t) \\ &\quad + (|z_1| + |z_4|) I^{\alpha+p} |g(s, u(s), Hu(s)) - g(s, v(s), Hv(s))|(\zeta) \\ &\quad + (|z_2| + |z_3|) [|b| I^{\alpha+q} |g(s, u(s), Hu(s)) - g(s, v(s), Hv(s))|(\xi) \\ &\quad + I^\alpha |g(s, u(s), Hu(s)) - g(s, v(s), Hv(s))|(1)] \} \\ &\leq L_g \left\{ \left(\frac{1}{\Gamma(\alpha+1)} + (|z_1| + |z_4|) \frac{\zeta^{\alpha+p}}{\Gamma(\alpha+p+1)} + (|z_2| + |z_3|) \left[\frac{|b|\xi^{\alpha+q}}{\Gamma(\alpha+q+1)} + \frac{1}{\Gamma(\alpha+1)} \right] \right) \right. \\ &\quad + L_h \left(\frac{1}{\Gamma(\alpha+1)} + (|z_1| + |z_4|) \frac{\zeta^{\alpha+p}}{\Gamma(\alpha+p+1)} \right. \\ &\quad \left. \left. + (|z_2| + |z_3|) \left[\frac{|b|\xi^{\alpha+q}}{\Gamma(\alpha+q+1)} + \frac{1}{\Gamma(\alpha+1)} \right] \right) \right\} \|u - v\| \\ &\leq \Psi_2 \|u - v\| \end{aligned}$$

Since $\Psi_2 < 1$. Hence, the operator \mathcal{G} is contraction. Then \mathcal{G} has a unique fixed point which is a solution of the boundary value problem (1)-(2) on J .

This completes the proof.

IV. EXAMPLES

Example 4.1. Consider the following fractional integro-differential equation

$${}^c D^{6/5} u(t) = \frac{1}{5} \left(\frac{|u(t)|}{1 + |u(t)|} \right) \cos^2 t + \frac{1}{5} \int_0^t \frac{e^{-3s}}{9} \cos^3 s ds, \quad t \in J \tag{10}$$

with Riemann-Liouville fractional integral conditions

$$u(0) = I^{1/3} u \left(\frac{1}{2} \right), \quad u(1) = I^{1/4} u \left(\frac{1}{6} \right) \tag{11}$$

Here $\alpha = 6/5$, $p = 1/3$, $q = 1/4$, $\zeta = 1/2$, $\xi = 1/6$, $a = b = 1$, and also $L_g = 1/5$, $L_h = 1/9$. Using the given

data, we find that values $z_1 = 4.8048$, $z_2 = 1.8467$, $z_3 = 0.5034$, $z_4 = 1.5634$ and $z = 0.1885$

Thus

$$\begin{aligned} L_g \left\{ \left(\frac{1}{\Gamma(\alpha+1)} + (|z_1| + |z_4|) \frac{\zeta^{\alpha+p}}{\Gamma(\alpha+p+1)} + (|z_2| + |z_3|) \left[\frac{|b|\xi^{\alpha+q}}{\Gamma(\alpha+q+1)} + \frac{1}{\Gamma(\alpha+1)} \right] \right) \right. \\ + L_h \left(\frac{1}{\Gamma(\alpha+1)} + (|z_1| + |z_4|) \frac{\zeta^{\alpha+p}}{\Gamma(\alpha+p+1)} \right. \\ \left. \left. + (|z_2| + |z_3|) \left[\frac{|b|\xi^{\alpha+q}}{\Gamma(\alpha+q+1)} + \frac{1}{\Gamma(\alpha+1)} \right] \right) \right\} \\ \approx 0.3760 < 1 \end{aligned}$$

The conditions (H1) and (H2) are satisfied. Thus, by theorem 3.3, the problem (10) and (11) has a unique solution on J .

V. CONCLUSION

In this paper, we have established the existence and uniqueness of solutions for fractional integro-differential equations with Riemann-Liouville fractional integral conditions (1) – (2) in a Banach space. Existence result of the problem is derived by the Krasnoselskii’s fixed point theorem, while the uniqueness result is proved by the application of Banach’s Contraction Principle.

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