**b-Compactness and **b-Connectedness in Topological Spaces

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Abstract

*This paper deals with **b-compact spaces and their properties by using **b-open and **b-closed sets. The notion of **b-connectedness in topological spaces is also introduced and their properties are studied.*

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Keywords: ***b-open sets, **b-closed sets, **b-compact spaces and **b-connectedness.*

I. INTRODUCTION

The notions of compactness and connectedness are useful and fundamental notions of not only general topology but also of other advanced branches of mathematics. Many researchers have investigated the basic properties of compactness and connectedness. The productivity of these notions of compactness and connectedness motivated mathematicians to generalize these notions. b-open[1] sets are introduced by Andrijevic in 1996. The class of b-open sets generates the same topology as the class of pre-open sets .K.Rekha and T.Indira, [7] introduced **b-open sets and **b-closed sets in the year 2012. S.S.Benchalli and Priyanka M.Bansali [3] introduced gb-compactness & gb-connectedness in topological spaces in the year 2011. The aim of this paper is to introduce the concept of **b-compactness & **b-connectedness in topological spaces and is to give some characterizations of **b-compact spaces.

II. PRELIMINARIES

Throughout this paper $(X,\tau),(Y,\sigma)$ are topological spaces with no separation axioms assumed unless otherwise stated. Let $A \subseteq X$. The closure of A and the interior of A will be denoted by $cl(A)$ and $\text{int}(A)$ respectively.

Definition: 2.1 A subset A of a space X is said to be b-open [1] if A $\subseteq$ Cl(Int(A)) $\cup$ Int(Cl(A))
The complement of b-open set is said to be b-closed. The family of all b-open sets (respectively b-closed sets) of (X,τ) is denoted by $bO(X,\tau)$ (respectively $bcl(X,\tau)$).

Definition: 2.2 A subset A of a space X is said to be *b-open [7] if A $\subseteq$ Cl(Int(A)) $\cap$ Int(Cl(A))
The complement of *b-open set is said to be *b-closed. The family of all *b-open sets (respectively *b-closed sets) of (X,τ) is denoted by $*bO(X,\tau)$ (respectively $*bcl(X,\tau)$).

Definition: 2.3 A subset A of a space X is said to be b**-open [4] if
$A \subseteq Int(Cl(Int(A))) \cup Cl(Int(Cl(A)))$
The complement of b**-open set is said to be b**-closed. The family of all b**-open sets (respectively b**-closed sets) of (X,τ) is denoted by $b{*}{*}O(X,\tau)$ (respectively $b{*}{*}cl(X,\tau)$).

Definition: 243 A subset A of a space X is said to be **b-open [7] if
$A \subseteq Int(Cl(Int(A))) \cap Cl(Int(Cl(A)))$
The complement of **b-open set is said to be **b-closed. The family of all **b-open sets (respectively **b-closed sets) of (X,τ) is denoted by ${*}{*}bO(X,\tau)$ (respectively ${*}{*}bcl(X,\tau)$).

Definition: 2.5 Let A be a subset of X . Then
(i) b -interior of A is the union of all b-open sets contained in A .

(ii) b -closure of A is the intersection of all b-closed sets containing A.
(iii) $**b$ -interior of A is the union of all **b-open sets contained in A.
(iv) $**b$ -closure of A is the intersection of all **b-closed sets containing A.

Definition:2.6 [3] A collection $\{A_\alpha\}_{\alpha \in I}$ of subsets of a topological space X is said to be a gb - open cover for X if $\bigcup_{\alpha \in I} A_\alpha = X$ and A_α are gb -open sets in X for each $\alpha \in I$.

Definition:2.7 [3] A space X is said to be gb - compact if every gb - open cover $\{A_\alpha\}$ of X contains a finite sub collection that also covers X.

Definition: 2.8 [3] A topological spaces X is said to be gb -connected if X cannot be expressed as a disjoint union of two non-empty gb -open sets. A subset of X is gb -connected if it is gb -connected as a subspace.

Definition: 2.9 A function $f : X \to Y$ is said to be **b-continuous [11] if $f^{-1}(V)$ is **b-closed in X for every closed set V of Y.

Definition: 2.10 A function $f : X \to Y$ is said to be **b-irresolute [11] if $f^{-1}(V)$ is **b-closed in X for every **b-closed set V of Y.

III. $**b$ -COMPACTNESS

Definition: 3.1
A collection $\{A_\alpha\}_{\alpha \in I}$ of subsets of a topological space X is said to be a **b-open cover for X if $\bigcup_{\alpha \in I} A_\alpha = X$ and A_α are **b-open sets in X for each $\alpha \in I$.

Definition: 3.2
A space X is said to be **b-compact if every **b-open cover $\{A_\alpha\}$ of X contains a finite sub collection that also covers X.

Theorem: 3.3
Let Y be a subspace of X .Then Y is **b-compact if and only if every covering of Y by sets **b-open in X contains a finite sub collection covering Y.
Proof:
Let Y be a subspace of X. Suppose that Y is **b-compact.
To prove: Every covering of Y by sets **b-open in X contains a finite sub collection covering Y.
Let $V = (A_\alpha)$ be a covering of Y by sets **b-open in X . $\Rightarrow A_\alpha$ is **b-open in X.
$\Rightarrow A_\alpha \cap Y$ is **b-open in Y.
$\therefore$ The collection $V_1 = \{A_\alpha \cap Y / A_\alpha \in **bO(X)\}$ is covering of Y by sets **b-open in Y.
Since Y is **b-compact. There exists a finite sub collection $A_{\alpha_1} \cap Y, A_{\alpha_2} \cap Y, ..., A_{\alpha_n} \cap Y$ from V_1 such that $(A_{\alpha_1} \cap Y) \cup (A_{\alpha_2} \cap Y) \cup ... \cup (A_{\alpha_n} \cap Y) = Y$
$\Rightarrow (A_{\alpha_1} \cup A_{\alpha_2} \cup ... \cup A_{\alpha_n}) \cap Y = Y \quad \Rightarrow Y \subset A_{\alpha_1} \cup A_{\alpha_2} \cup ... \cup A_{\alpha_n}$
$\therefore \{A_{\alpha_1}, A_{\alpha_2}, ..., A_{\alpha_n}\}$ covers Y. $\{A_{\alpha_1}, A_{\alpha_2}, ..., A_{\alpha_n}\}$ is a finite sub collection of V that covers Y.
Conversly, Suppose that every covering of Y by sets **b-open in X contains a finite sub collection covering Y. To prove: Y is **b-compact. Let $\{B_\alpha\}$ be a covering of Y by sets **b-open in Y. B_α is **b-open in Y. $\Rightarrow$
$\exists$ a **b-open set A_α in X such that $B_\alpha = A_\alpha \cap Y$
$\therefore B_\alpha \subset A_\alpha$ for each α . $\therefore \cup B_\alpha \subset \cup A_\alpha$
Since $\{B_\alpha\}$ is a cover for Y. $\Rightarrow Y = \cup B_\alpha \subset \cup A_\alpha$

$\therefore \{A_\alpha\}$ is a cover for Y by sets **b-open in X .

By our assumption there exists a finite sub collection $A_{\alpha_1}, A_{\alpha_2}, ..., A_{\alpha_n}$ covering Y .

That is $Y \subset A_{\alpha_1} \cup A_{\alpha_2} \cup ... \cup A_{\alpha_n}$. $\Rightarrow (A_{\alpha_1} \cup A_{\alpha_2} \cup ... \cup A_{\alpha_n}) \cap Y = Y$

That is $(A_{\alpha_1} \cap Y) \cup (A_{\alpha_2} \cap Y) \cup ... \cup (A_{\alpha_n} \cap Y) = Y$.

That is $B_{\alpha_1} \cup B_{\alpha_2} \cup ... \cup B_{\alpha_n} = Y$,

Since $B_{\alpha_i} = A_{\alpha_i} \cap Y$

$\therefore \{B_{\alpha_1}, B_{\alpha_2}, ..., B_{\alpha_n}\}$ is a finite sub collection of $\{B_\alpha\}$ that covers Y .

$\therefore$ Every **b-open covering of Y has a finite sub collection covering Y .

Therefore Y is **b-compact.

Theorem: 3.4

Every **b-closed subspace of a **b-compact space is **b-compact.

Proof:

Let X be a **b-compact space and Y be a **b-closed subspace of X .

To prove: Y is **b-compact. Let $\{B_\alpha\}$ be a **b-open cover for Y .

B_α is **b-open in $Y \Rightarrow$ there exists a **b-open set A_α in X such that $B_\alpha = A_\alpha \cap Y$

$B_\alpha \subset A_\alpha$ for all α .$\Rightarrow \cup B_\alpha \subset \cup A_\alpha$. $\{B_\alpha\}$ is a **b-open cover for Y .

$\therefore Y = \cup B_\alpha \subset \cup A_\alpha$. $\therefore V = \{A_\alpha\}$ is a **b-open cover for Y with **b-open sets in X .

Since Y is **b-closed in X . Therefore, $X - Y$ is **b-open in X .Consider $V_1 = V \cup \{(X - Y)\}$.

V_1 is a **b-open cover for X . Since X is **b-compact. There exists a finite sub collection

$V_2 = \{A_{\alpha_1}, A_{\alpha_2}, ...\}$ from V_1 that covers X . Let $V_3 = V_2 - \{(X - Y)\}$. That is $X - Y \in V_2$ discard it.

Now V_3 covers Y . V_2 is finite $\Rightarrow V_3$ is finite. $\therefore V_3 = \{A_{\alpha_1}, A_{\alpha_2}, ..., A_{\alpha_n}\}$ is a finite sub collection

that covers Y . $\therefore A_{\alpha_1} \cup A_{\alpha_2} \cup ... \cup A_{\alpha_n} \supset Y$. $\therefore (A_{\alpha_1} \cap Y) \cup (A_{\alpha_2} \cap Y) \cup ... \cup (A_{\alpha_n} \cap Y) = Y$

$\therefore B_{\alpha_1} \cup B_{\alpha_2} \cup ... \cup B_{\alpha_n} = Y$ where $B_{\alpha_i} = A_{\alpha_i} \cap Y$. $\therefore \{B_{\alpha_1}, B_{\alpha_2}, ..., B_{\alpha_n}\}$ is a finite sub collection

that covers Y . $\therefore$ Every **b-open covering of Y has a finite sub collection covering Y .

Therefore, Y is **b-compact.

Theorem: 3.5

A **b-continuous image of a **b-compact space is compact.

Proof:

Let $f : X \to Y$ be a **b-continuous map from a **b-compact space X onto a topological space Y . To prove: Y is compact. Let $\{A_\alpha\}_{\alpha \in I}$ be an open covering of Y .

$\Rightarrow$ Each A_α is an open subset of Y . Since f is **b-continuous. $\Rightarrow f^{-1}(A_\alpha)$ is **b-open in X

Then $\{f^{-1}(A_\alpha)\}_{\alpha \in I}$ is a **b-open cover of X .

Since X is **b-compact.

It has a finite sub cover say $f^{-1}(A_1), f^{-1}(A_2), ..., f^{-1}(A_n)$. Since f is on-to.

$\Rightarrow A_1, A_2, ..., A_n$ is a finite cover of Y . $\Rightarrow$ Every open cover of Y has a finite sub cover.

Therefore, Y is compact.

Theorem: 3.6

If a map $f : X \to Y$ is **b-irresolute and a subset B of X is **b-compact relative to X , then the image $f(B)$ is **b-compact relative to Y .

Proof:

Let $\{A_\alpha\}_{\alpha \in I}$ be any collection of **b-open subsets of Y such that $f(B) \subset \cup\{A_\alpha, \alpha \in I\}$

$\Rightarrow B \subset \{f^{-1}(\cup A_\alpha), \alpha \in I\}$. Then $B \subset \cup\{f^{-1}(A_\alpha), \alpha \in I\}$

By hypothesis B is **b-compact relative to X .

Therefore, $\exists$ a finite subset I_0 of $I \ni B \subset \cup\{f^{-1}(A_\alpha), \alpha \in I_0\}$

Therefore we have $f(B) \subset \cup\{A_\alpha, \alpha \in I_0\}$

Here $\{A_\alpha, \alpha \in I_0\}$ is a finite sub collection of $\{A_\alpha, \alpha \in I\}$ which covers $f(B)$

$\Rightarrow$ $f(B)$ is **b-compact relative to Y .

IV. $**b$ -CONNECTEDNESS

Definition: 4.1

Let X be a topological space. A **b-separation of X is a pair A, B of disjoint non-empty **b-open subsets of X whose union is X .That is $X = A \cup B$ where $A \neq \phi, B \neq \phi, A \cap B = \phi$ & A, B are **b-open subsets of X .

Note: 4.2

Suppose that A and B form a **b-separation of X .

That is $X = A \cup B$, where A, B are **b-open subsets of X and disjoint.

$\Rightarrow$ A and B are both**b-open and **b-closed.

Since A and B are **b-closed.

$\Rightarrow$ $**bcl(A) = A$ and $**bcl(B) = B$

$**bcl(A) \cap B = A \cap B = \phi$

$**bcl(B) \cap A = A \cap B = \phi$

Hence $A \cap **bcl(B) = \phi$ and $B \cap **bcl(A) = \phi$.

Definition: 4.3

Two subsets A and B of a topological space X are said to be **b-separated if $A \neq \phi, B \neq \phi$ and $A \cap **bcl(B) = \phi$ and $B \cap **bcl(A) = \phi$.

Definition: 4.4

A topological space X is said to be **b-disconnected if $X = A \cup B$, where A and B are any two non-empty **b-separated sets.

Thus, X is **b-disconnected if

(i) $X = A \cup B$

(ii) $(A \cap **bcl(B)) \cup (B \cap **bcl(A)) = \phi$

Definition: 4.5

A topological space X is said to be **b-connected if it is not **b-disconnected.

Example: 4.6

Let $X = \{a, b, c\}$

$\tau = \{X, \phi, \{a\}, \{b, c\}\}$

The collection of **b-open sets of $X = \{X, \phi, \{a\}, \{b, c\}\}$

Let $A = \{a\}$ & $B = \{b, c\}$ be non-empty **b-open subsets of X

$A \cap B = \phi$ & $X = A \cup B$

$\Rightarrow$ A & B form a **b-separation of X

Example: 4.7

The collection of **b-closed sets of $X = \{X, \phi, \{b, c\}, \{a\}\}$

Since $A = \{a\} \Rightarrow **bcl(A) = \{a\}$ &

$B = \{b, c\} \Rightarrow **bcl(B) = \{b, c\}$

$A \cap **bcl(B) = \phi$ and $B \cap **bcl(A) = \phi$

$\Rightarrow A \& B$ form a **b-separation of X

Example: 4.8

Let $X = \{a, b, c\}$

$\tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}\}$

The collection of **b-open sets of $X = \{X, \phi, \{a\}, \{b\}, \{a, b\}\}$

Let $A = \{a\}$ & $B = \{b\}$ be non-empty **b-open subsets of X

$A \cap B = \phi$ & $X \neq A \cup B$

$\Rightarrow$ The space X is not **b-separated

$\Rightarrow X$ is**b-connected.

Theorem: 4.9

Let C be a **b-connected subset of a topological space X .

Let $X = A \cup B$, where A and B are **b-separated subsets of X .Then either $C \subset A$ or $C \subset B$.

Proof:

Let C be a **b-connected subset of X and $X = A \cup B$.

$\Rightarrow C \subset X = A \cup B \Rightarrow C \subset A \cup B$

$C = C \cap (A \cup B)$

$C = (C \cap A) \cup (C \cap B)$ …………(1)

Since A and B are **b-separated sets.

Hence $A \cap **bcl(B) = \phi$ and $B \cap **bcl(A) = \phi$. Since C is **b-connected.

Hence, $C \cap A = \phi$ or $C \cap B = \phi$ [From (1)]. Therefore, $C \subset B$ or $C \subset A$

Theorem: 4.10

Let $\{C_\lambda : \lambda \in \Lambda\}$ be a family of **b-connected subsets of a topological space X such that $\bigcap_{\lambda \in \Lambda} C_\lambda \neq \phi$. Then $C = \bigcup_{\lambda \in \Lambda} C_\lambda$ is **b-connected.

Proof:

Let $\{C_\lambda : \lambda \in \Lambda\}$ be a family of **b-connected subsets of a topological space X such that $\bigcap_{\lambda \in \Lambda} C_\lambda \neq \phi$.

Since $C = \bigcup_{\lambda \in \Lambda} C_\lambda$. To prove: C is **b-connected

Assume that C is **b-disconnected. Let $C = A \cup B$, where A and B are **b-separated subsets of X .

Each C_λ is **b-connected and $C_\lambda \subset A \cup B$ for each λ

By theorem 4.9,

Either $C_\lambda \subset A$ or $C_\lambda \subset B$ for each λ . Therefore $\bigcup_{\lambda \in \Lambda} C_\lambda \subset A$ or $\bigcup_{\lambda \in \Lambda} C_\lambda \subset B$ for each λ

That is $C \subset A$ or $C \subset B$ …………………(1)

Since $\bigcap_{\lambda \in \Lambda} C_\lambda \neq \phi$. Let $x \in \bigcap_{\lambda \in \Lambda} C_\lambda$

Then $x \in C_\lambda$ for each λ and So $x \in C$. Hence $x \in A$ (or) $x \in B$. But A and B are disjoint.

Therefore x cannot belong to both A and B . Let $x \in A$, $x \notin B$.

Then, by (1), $C \not\subset B$. Thus we would have $C \subset A$.

This contradicts the fact that C is **b-disconnected.

So C must be **b-connected.

Theorem: 4.11

Let X be a space such that each pair of points in X is contained in a **b-connected subset of X. Then X is **b-connected.

Proof:

Let x be a given point of X and $y \neq x$ be an arbitrary point in X.

By hypothesis, there exists a **b-connected subset C_y containing x and y.

Also $X = \cup\{C_y : y \in X\}$, $\cap\{C_y : y \in X\} = \phi$. Since each C_y is **b-connected.

By theorem 4.10, X is **b-connected.

Theorem: 4.12

Let C be a **b-connected subset of a space X and Let D be a subset such that $C \subset D \subset **bcl(C)$. Then D is **b-connected.

Proof:

To prove: D is **b-connected. Assume that D is **b-disconnected.

Let $D = A \cup B$. Where A and B are **b-separated subsets of X.

Then $A \neq \phi$ and $B \neq \phi$. Since C is **b-connected and $C \subset D \Rightarrow C \subset A \cup B$

By theorem 4.9,

Either $C \subset A$ (or) $C \subset B$. Suppose $C \subset A$. Then $**bcl(C) \subset **bcl(A)$

$\Rightarrow **bcl(C) \cap B \subset **bcl(A) \cap B = \phi$, Since A and B are **b-separated

But $B \subset D$ and $D \subset **bcl(C)$. $\Rightarrow B \cap D \subset B \cap **bcl(C)$

Hence $\phi \subset B = B \cap D \subset B \cap **bcl(C) = \phi$. Therefore, $\phi \subset B \subset \phi$

$\Rightarrow B = \phi$. Which is a contradiction to our assumption.

Hence D must be **b-connected.

Theorem: 4.13

The **b-closure of a **b-connected set is **b-connected.

Proof:

Let C be a **b-connected set. Take $D = **bcl(C)$. Since $C \subset **bcl(C) = D$

$\Rightarrow C \subset D \subset **bclC)$. By theorem 4.12,

D is **b-connected. $**bcl(C)$ is **b-connected.

Hence **b-closure of a **b-connected set is **b-connected.

Theorem: 4.14

If C is a **b-dense subset of a space X and if C is also **b-connected, then X is **b-connected.

Proof:

Let C be **b-dense in X. $\Rightarrow **bcl(C) = X$. $\Rightarrow C \subset **bcl(C)$

$\Rightarrow C \subset X \subseteq **bcl(C)$. By theorem 4.12, X is **b-connected.

Theorem: 4.15

If $f : X \to Y$ is an on-to **b-continuous map and X is **b-connected, then Y is connected.

Proof:

Let $f : X \to Y$ be an on-to **b-continuous map and X be **b-connected.

To prove: Y is connected. Suppose Y is not connected

Let $Y = A \cup B$ where A and B aredisjoint non-empty open sets in Y.

$\Rightarrow f^{-1}(Y) = f^{-1}(A \cup B) \Rightarrow f^{-1}(Y) = f^{-1}(A) \cup f^{-1}(B) \Rightarrow X = f^{-1}(A) \cup f^{-1}(B)$

Since f is **b-continuous and A ,B are open sets in Y.

$\Rightarrow f^{-1}(A)$ and $f^{-1}(B)$ are **b-open sets in X. $f^{-1}(A) \cap f^{-1}(B) = f^{-1}(A \cap B) = f^{-1}(\phi) = \phi$

$\Rightarrow X = f^{-1}(A) \cup f^{-1}(B)$ where $f^{-1}(A)$ and $f^{-1}(B)$ are disjoint non-empty **b-open sets in X .

$\Rightarrow X$ is not **b-connected. Which is a contradiction to our assumption. Hence Y is connected.

Theorem: 4.16

If $f : X \to Y$ is an on-to **b-irresolute map and X is **b-connected, then Y is **b-connected.

Proof:

Let $f : X \to Y$ be an on-to **b-irresolute map and X be **b-connected.

To prove: Y is **b-connected. Suppose Y is not **b-connected

Let $Y = A \cup B$ where A and B are disjoint non-empty **b-open sets in Y .

$\Rightarrow f^{-1}(Y) = f^{-1}(A \cup B) \Rightarrow f^{-1}(Y) = f^{-1}(A) \cup f^{-1}(B) \Rightarrow X = f^{-1}(A) \cup f^{-1}(B)$

Since f is **b-irresolute and A and B are **b-open sets in Y .

$\Rightarrow f^{-1}(A)$ and $f^{-1}(B)$ are **b-open sets in X . $f^{-1}(A) \cap f^{-1}(B) = f^{-1}(A \cap B) = f^{-1}(\phi) = \phi$

$\Rightarrow X = f^{-1}(A) \cup f^{-1}(B)$ where $f^{-1}(A)$ and $f^{-1}(B)$ are disjoint non-empty **b-open sets in X .

$\Rightarrow X$ is not **b-connected. Which is a contradiction to our assumption.

Hence Y is **b-connected.

Example: 4.17

Let $X = \{a, b, c\}$

$\tau = \{X, \phi, \{a\}, \{a, b\}\}$

The collection of *b-open sets of $X = \{X, \phi, \{a\}, \{a, b\}, \{a, c\}\}$

The collection of b**-open sets of $X = \{X, \phi, \{a\}, \{c\}, \{a, b\}, \{a, c\}\}$

The collection of **b-open sets of $X = \{X, \phi, \{a\}, \{a, b\}, \{a, c\}\}$

$\Rightarrow$ The space X is b**-separated, But not *b-separated and not **b-separated.

Here, the space X is b**-separated, *b-connected and **b-connected.

Example: 4.18

Let $X = \{a, b, c\}$

$\tau = \{X, \phi, \{a\}\}$

The collection of *b-open sets of $X = \{X, \phi, \{a\}, \{a, b\}, \{a, c\}\}$

The collection of b**-open sets of $X = \{X, \phi, \{a\}, \{a, b\}, \{a, c\}\}$

The collection of **b-open sets of $X = \{X, \phi, \{a\}, \{a, b\}, \{a, c\}\}$

$\Rightarrow$ The space X is not *b-separated, not b**-separated, not **b-separated.

Here, the space X is *b-connected, b**-connected and **b-connected.

Example: 4.19

Let $X = \{a, b, c\}$

$\tau = \{X, \phi, \{a\}, \{b, c\}\}$

The collection of *b-open sets of $X = \{X, \phi, \{a\}, \{b, c\}\}$

The collection of b**-open sets of $X = \{X, \phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}\}$

The collection of **b-open sets of $X = \{X, \phi, \{a\}, \{b, c\}\}$

$\Rightarrow$ The space X is *b-separated, b**-separated, **b-separated.

V. CONCLUSION

As an extension of this paper, b**-compactness and b**-connectedness in Topological spaces can be defined and can obtain theorems based on the above defined concepts.

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