Fuzzy Identity, Fuzzy Zero of Po Semigroup and Completely Prime Fuzzy, Prime Fuzzy Ideal of Po Semigroup

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Abstract

In this Paper the terms $[f)$, fuzzy left(right) identity, fuzzy left(right) zero of a posemigroup are introduced. It is proved that fuzzy left identity and fuzzy right identity of a posemigroup if it exists then both are same. Also proved that fuzzy left zero and fuzzy right zero of a posemigroup exists then both are equal. Next proved that intersection of arbitrary family of fuzzy posemigroups of a posemigroup S is a fuzzy posemigroup of S. Furthermore, proved that intersection of all fuzzy posemigroups of S containing f is ideal generated by fuzzy subset f. Next proved that f is a fuzzy ideal of a posemigroup S iff$foS \subseteq f, Sof \subseteq f$ and $(f] \subseteq f$. Next demonstrate the union, intersection of arbitrary family of fuzzy ideals of posemigroup S is fuzzy ideal of S. Next proved that every completely prime fuzzy ideal is weakly completely prime fuzzy ideal. Also proved that if f is completely prime fuzzy ideal then f_t is sub semigroup and completely prime ideal of S. Next proved that f_t is completely prime ideal of S then f is weakly completely prime fuzzy and completely prime fuzzy ideal of S. Next proved that equivalent conditions of prime fuzzy ideal, f is prime fuzzy ideal iff 1-f is a fuzzy m-system of S if $1 - f \neq \emptyset$. Finally proved that every maximal fuzzy ideal of S is a prime fuzzy ideal of S.

Mathematical subject classification (2010):20M07; 20M11, 20M12

Keywords: *Fuzzy posemigroup,$[f)$, fuzzy identity of a po semigroup, fuzzy zero of a semigroup, fuzzy posemigroup generated by f, fuzzy ideal, completely prime fuzzy ideal, weakly completely prime fuzzy ideal, prime fuzzy ideal, m-system, Maximal ideal.*

I. INTRODUCTION

The algebraic theory of semigroups was widely studied by Clifford[2,3]. The ideal theory in general semigroups was developed by Anjaneyulu[1]. Since then a series of researchers have been extending the concepts and results of abstract algebra. Padmalatha, A. Gangadhara Rao and A.Anjaneyulu[10] introduced posubsemigroup, posubsemigroup generated by a subset, two sided identity of a posemigroup, zero of a posemigroup, po ideal ,po ideal generated by a subset. On the other hand, P.M.Padmalatha ,A.Gangadahara Rao, P.RamyaLatha [12] introduced completely prime, prime ideal of a posemigroup.

The concept of a fuzzy set was introduced by Zadeh in 1965[6]. This idea opened up new thoughts and applications in a wide range of scientific fields. A. Rosenfeld applied the notion of fuzzy subset to several areas of mathematics, among other disciplines. N. Kuroki, J N Mordeson developed the fuzzy semigroups concept. N.Kehayopulu, M.Tsingelis introduced the notion of fuzzy subset of a posemigroups[7-9]. Motivated by the study of N.Kehayopulu, M.Tsingelis work in posemigroups we attempt in the paper to study the fuzzy identity and fuzzy zero of a posemigroup, operations on fuzzy posemigroups, results on completely prime fuzzy ideals and prime fuzzy ideals of posemigroup.

II. PRELIMINARIES

***Definition 2.1*:**[10]A semigroup S with an ordered relation $\leq$ is said to be *posemigroup*if S is a partially ordered set such that $a \leq b \Rightarrow ax \leq bx, xa \leq xb,\ \forall a, b, x \in S$.

Definition 2.2:[13]A function f from posemigroupS to the closed interval [0, 1] is called a *fuzzy subset* of S, where [0,1] is the interval of real numbers.The posemigroup S itself is a fuzzy subset of S such that $S(x) = 1, \forall x \in S$.It is denoted by S or 1.

***Definition 2.3*:**[5]Let A be a nonempty subset of S. We denote by f_A the characteristic mapping of A, that is mapping of S into [0,1] defined by $f_A(x) = \begin{cases} 1 \; if \; x \in A \\ 0 \; if \; x \notin A \end{cases}$
Then f_A is a fuzzy subset of S.

***Definition 2.4*:**[5]A fuzzy subset $f$of a posemigroup S is called *fuzzy subsemigroup*of S if $f(xy) \geq f(x) \wedge f(y), \forall$ x, y $\in$ S .

***Definition 2.5*:**[10]Let S be a posemigroup. For $a \in S$, we define $(a] = \{b \in S \,/\, b \leq a\}$

***Definition 2.6*:**[10]Let S be a posemigroup. For $a \in S$, we define $[a) = \{b \in S \,/\, b \geq a\}$

***Definition 2.7*:**[13]Let *f*be a fuzzy subset of a posemigroup S. We define$(f]$ by$(f](x) = \bigvee_{x \leq y} f(y), \forall x \in S$ where $y \in S$.

Note 2.8: The set of all fuzzy subsets of S is denoted by F(S).

***Definition 2.9*:**[13] Let S be a posemigroup. For $x \in S$, we define $A_x = \{(y, z) \in S \times S / x \leq yz\}$. Let f and g be two fuzzy subsets of S. The *product* fog is defined by

$$(fog)(x) = \begin{cases} \bigvee_{(y,z) \in A_x} (f(y) \wedge g(z) \; if \; A_x \neq \emptyset \\ \quad 0 \; otherwise \end{cases}$$

The above one can also be defined as *(fog)(x)* = $\begin{cases} \bigvee_{x \leq yz} \big(f(y) \wedge g(z)\big) \; if \; x \leq yz \text{ exists} \\ 0 \quad \text{otherwise} \end{cases}$

***Definition 2.10*:**[10]Anonempty subset A of a posemigroup$S$ is said to be *po left ideal* of S if (i) $b \in S, a \in A \Rightarrow ba \in A (ii) a \in A$ and $s \in S$ such that $s \leq a \Rightarrow s \in A$.

***Definition 2.11*:**[10]Anonempty subset A of a posemigroup$S$ is said to be *po right ideal* of S if (i) $b \in S, a \in A \Rightarrow ab \in A (ii) a \in A$ and $s \in S$ such that $s \leq a \Rightarrow s \in A$.

***Definition 2.12*:**[10]Anonempty subset A of a posemigroup$S$ is said to be *po two sided ideal or po ideal* of S if (i) $b \in S, a \in A \Rightarrow ba \in A, ab \in A (ii) a \in A$ and $s \in S$ such that $s \leq a \Rightarrow s \in A$.

***Definition 2.13*:**[13]Let S be a posemigroup. A fuzzy subset $f$of S is called a *fuzzy left ideal*of S if (i) *x ≤ y then f(x) ≥ f(y)* (ii) *f(xy) ≥ f(y)*, ∀ *x,y* ∈ *S.*

***Lemma2.14*:**[7]*Let S be a posemigroup and f be a fuzzy subset of S. Then f is a fuzzy left ideal of S if and only if f satisfies that (i) x ≤ y then f(x) ≥ f(y)* ∀ *x, y*∈*S (ii) Sof*⊆*f.*

***Definition 2.15*:**[13] Let S be a posemigroup. A fuzzy subset $f$of S is called a *fuzzy right ideal*of S if (i) x ≤ y then f(x) ≥ f(y) (ii) f(xy) ≥ f(x), $\forall x, y \in S$.

***Lemma 2.16*:**[7]*Let S be a posemigroup and f be a fuzzy subset of S. Then f is a fuzzy right ideal of S if and only if f satisfies that (i) x ≤ y then f(x) ≥ f(y)* ∀ *x,y*∈ *S (ii) foS*⊆*f.*

***Definition 2.17*:**[13] Let S be a posemigroup. A fuzzy subset f of S is called a *fuzzy ideal*of S if (i) *x ≤ y then f(x) ≥ f(y)* (ii) *f(xy) ≥ f(y) and f(xy) ≥ f(x)*, ∀ *x,y*∈*S.*

***Lemma 2.18*:**[7] *Let S be a posemigroup and f be a fuzzy subset of S. Then f is a fuzzy ideal of S if and only if f satisfies that (i) x ≤ y then f(x) ≥ f(y)* ∀ *x,y*∈*S (ii) foS*⊆*f and Sof*⊆*f.*

***Note 2.19*:** Let S be a posemigroup. A fuzzy subset f of S is said to be *fuzzy ideal* of S if f is both fuzzy right and fuzzy left ideal of S.

***Lemma 2.20*:**[7]*Let S be a posemigroup and $\emptyset \neq A \subseteq S$. Then A is a left ideal of S if and only if the characteristic mapping f_A of A is a fuzzy left ideal of S.*

***Lemma 2.21*:**[7]*Let S be a posemigroup and $\emptyset \neq A \subseteq S$. Then A is a right ideal of S if and only if the characteristic mapping f_A of A is a fuzzy right ideal of S.*

***Lemma 2.22*:**[7]*Let S be a posemigroup and $\emptyset \neq A \subseteq S$. Then A is an ideal of S if and only if the characteristic mapping f_A of A is a fuzzy ideal of S.*

***Proposition 2.23*:**[13]Let*f,g,h* be fuzzy subsets of *S. Then the following statements are true.*
a. $f \subseteq (f], \forall f \in F(S)$ *b. If* $f \subseteq g$ *then* $(f] \subseteq (g]$
c.$(f]o(g] \subseteq (fog], \forall f, g \in F(S)$*d.* $(f] = ((f]], \forall f \in F(S)$
e.For any fuzzy ideal f of S,$f = (f]$
f. If f,gare fuzzy ideals of S, then fog ,f∪g are fuzzy ideals of S.
g. $fo(g \cup h] \subseteq (fog \cup foh]$ *h.*$(g \cup h]of \subseteq (gof \cup hof]$.
i. If a_λ is an ordered fuzzy point of S, then $a_\lambda = (a_\lambda]$.

***Definition 2.24*:**[13]Let S be a posemigroup, $a \in S$and$\lambda \in [0,1]$. An *ordered fuzzy point* $\boldsymbol{a}_\lambda$,$\boldsymbol{a}_\lambda : S \to [0,1]$ defined by $a_\lambda(x) = \begin{cases} \lambda \; if \; x \in (a] \\ 0 \; if \; x \notin (a] \end{cases}$
clearlya_λ is a fuzzy subset of S.For every fuzzy subset *f* of S, we also denote $a_\lambda \subseteq f$ by $a_\lambda \in f$

***Definition 2.25*:**[5] Let f be a fuzzy subset of X. Let $t \in [0,1]$. Define$f_t = \{x \in X / f(x) \geq t\}$. We call f_t a t-cut or a level set.

***Definition 2.26*:**[12]A po (left/right) ideal of A of a posemigroup S is said to be *completely prime (left/right) ideal* of S provided $x, y \in S$ and $xy \in A$ implies either $x \in A$ *or* $y \in A$.

III. FUZZY IDENTITY AND FUZZY ZERO OF A PO SEMIGROUP

***Definition 3.1*:** Let *f* be a fuzzy subset of a posemigroup S. We define $[f)$ by $[f)(x) = \bigvee_{x \geq y} f(y), \forall x \in S$ where$y \in S$.

Proposition 3.2: *Let f,g,hbe* fuzzy*subsets of S. Then the following statements are true.*
a)$f \subseteq [f), \forall f \in F(s)$ *b) If* $f \subseteq g$ *than* $[f) \subseteq [g)$
***Proof*: a)**$\forall x \in S$, Since $[f)(x) = \bigvee_{x \geq y} f(y)$
Since$x \geq x \Rightarrow [f)(x) = \bigvee_{x \geq x} f(x) \geq f(x)$
Therefore $f \subseteq [f)$
b) Let $f \subseteq g$ then $\forall x \in S, f(x) \leq g(x)$
Thus $[f)(x) = \bigvee_{x \geq y} f(y) \leq \bigvee_{x \geq y} g(y) = [g)(x)$
Therefore $[f) \subseteq [g)$.

***Definition 3.3*:** An*ordered fuzzy point* $a_\lambda$of a posemigroup S with identity is said to be *fuzzy left identity* of S if $a_\lambda of = f$and $f \subseteq a_\lambda$, ⍰$f \in F(S)$ where$\lambda \in (0,1], a \in S$.

***Definition 3.4*:** An*ordered fuzzy point* a_λ of a posemigroup S with identity is said to be *fuzzy right identity* of S if $foa_\lambda = f$and $f \subseteq a_\lambda$, ⍰$f \in F(S)$ where$\lambda \in (0,1], a \in S$.

***Definition 3.5*:** An *ordered fuzzy point* a_λ of a po semigroup S with identity is said to be *fuzzy identity* of S if it is both fuzzy left identity and fuzzy right identity.

***Definition 3.6*:** A fuzzy subset*f* of a posemigroup Swith identity is said to be *fuzzy left identity* of S if $fof_1 = f_1$and $f_1 \subseteq f, \forall f_1 \in F(s)$.

***Definition 3.7*:** A fuzzy subset*f* of posemigroup Swith identity is said to be *fuzzy right identity* of S if $f_1of = f_1$and $f_1 \subseteq f, \forall f_1 \in F(s)$.

Theorem 3.8: *If a_λ is a fuzzy left identity and b_λ is a fuzzy right identity of a posemigroup S with identity then* $a_\lambda = b_\lambda$ where $\lambda \in (0,1], a, b \in S$.
***Proof**:* Since a_λ is a fuzzy left identity of S, $a_\lambda of = f \ and \ f \subseteq a_\lambda, \forall f \in F(s)$

$$\Longrightarrow a_\lambda ob_\lambda = b_\lambda \ and \ b_\lambda \subseteq a_\lambda$$

since b_λ is a fuzzy right identity of S, $fob_\lambda = f \ and \ f \subseteq b_\lambda, \forall f \in F(s)$

$$\Longrightarrow a_\lambda ob_\lambda = a_\lambda and \ a_\lambda \subseteq b_\lambda$$

There fore $a_\lambda = b_\lambda$.

Definition 3.9: An ordered fuzzy point a_λ of a posemigroup S is said to be a *fuzzy left(right) zero* of S if $a_\lambda of = a_\lambda (foa_\lambda = a_\lambda) \ and \ a_\lambda \subseteq f, \forall f \in F(s)$ where $\lambda \in [0,1]$.

Definition 3.10:An ordered fuzzy point a_λ of a po semigroup S is said to be *fuzzy zero* of S if $a_\lambda of = foa_\lambda = a_\lambda \ and \ a_\lambda \subseteq f, \forall f \in F(s)$ where $\lambda \in [0,1]$.

Theorem 3.11: *If a_λ is a fuzzy left zero and b_λ is a fuzzy right zero of a posemigroup S then $a_\lambda = b_\lambda$ where $\lambda \in [0,1]$*
Proof: Let a_λ be fuzzy left zero of S
$\Rightarrow a_\lambda of = a_\lambda \ \forall f \in F(s) \ and \ a_\lambda \subseteq f$
$\Rightarrow a_\lambda ob_\lambda = a_\lambda$
Since b_λ is a fuzzy right zero of S
$fob_\lambda = b_{\lambda,} \forall f \in F(s) \ and b_\lambda \subseteq f$
$\Rightarrow a_\lambda ob_\lambda = b_\lambda$. Therefore $a_\lambda = b_\lambda$

Theorem 3.12:*Any fuzzy semigroup has atmost 1 fuzzy zero element*
Proof:Let a_λ, b_λ be any two fuzzy zeros of a posemigroup S.
$\Rightarrow a_\lambda, b_\lambda$ be considered as fuzzy left and fuzzy right zeros of S respectively.
By Theorem 2.35, $a_\lambda = b_\lambda$
Therefore Fuzzy semigroup has atmost 1 fuzzy zero element.

Note 3.13: The fuzzy zero element of a posemigroup usually denotd by '0'.

IV. OPERATIONS ON FUZZY POSEMIGROUPS

Definition4.1:*Let $\{f_i\}_{i\in I}$ be family of fuzzy subsets of an ordered semigroup S, where I is an index set. Defineintersection* and *union* $\underset{i\in I}{\cap} f_i$ and $\underset{i\in I}{\cup} f_i$ as follows.

$$(\underset{i\in I}{\cap} f_i) = \underset{i\in I}{\wedge} f_i(x) = \min\{f_1(x), f_2(x), \dots \dots \dots\}, \forall x \in S$$

$(\underset{i\in I}{\cup} f_i)(x) = \underset{i\in I}{\vee} f_i(x) = \max\{f_1(x), f_2(x), \dots \dots \dots\}, \forall x \in S$.

Definition 4.2: A fuzzy subset f of a posemigroup S is called *fuzzy posemigroup* of S if (i) $x \leq y \Rightarrow f(x) \geq f(y)$ (ii) $f(xy) \geq f(x) \wedge f(y)$, $\forall$ x, y $\in$ S .

Theorem 4.3:*The intersection of two fuzzy posemigroups of a posemigroup S is a fuzzy posemigroup of S.*
Proof: Let f_1, f_2 be two fuzzy posemigroups of a posemigroup S.
a) Consider $(f_1 \cap f_2)(xy) = f_1(xy) \wedge f_2(xy) \geq f_1(x) \wedge f_1(y) \wedge f_2(x) \wedge f_2(y) \geq f_1(x) \wedge f_2(x) \wedge f_1(y) \wedge f_2(y)$

$$\geq (f_1 \cap f_2)(x) \wedge (f_1 \cap f_2)(y), \forall x, y \in S$$

b) Let $x \leq y$
Consider $(f_1 \cap f_2)(x) = f_1(x) \wedge f_2(x) \geq f_1(y) \wedge f_2(y) = (f_1 \cap f_2)(y)$
$\Rightarrow f_1 \cap f_2$ is a fuzzy posemigroup of S.

Theorem 4.4: *The intersection of arbitrary family of fuzzy posemigroups of a posemigroup S is a fuzzy posemigroup of S.*
Proof: Let $f_1, f_{2,} f_3 \dots \dots$ be the family of fuzzy posemigroups of a posemigroup S.
a) Consider $(\underset{i\in I}{\cap} f_i)(xy) = f_1(xy) \wedge f_2(xy) \dots \dots \dots \geq f_1(x) \wedge f_1(y) \wedge f_2(x) \wedge f_2(y) \dots \dots \dots \geq f_1(x) \wedge f_2(x) \wedge f_1(y) \wedge f_2(y) \dots \dots \dots$

$$\geq (\underset{i\in I}{\cap} f_i)(x) \wedge (\underset{i\in I}{\cap} f_i)(y)$$

b) Let $x \leq y$,
Consider $(\underset{i\in I}{\cap} f_i)(x) = f_1(x) \wedge f_2(x) \dots \dots \dots \geq f_1(y) \wedge f_2(y) \wedge \dots \dots \dots$

$$\geq (\cap_{i\in I} f_i)(y)$$

⇒ The intersection of arbitrary family of fuzzy posemigroups of a po semigroup S is a fuzzy posemigroup of S.

Definition 4.5**:** Let S be a po semigroup and f be a fuzzy subset of S. The smallest fuzzy po semigroup of S containing f is called a *fuzzy posemigroup of S generated by* f.It is denoted by (f).

Theorem 4.6**:** Let S be a po semigroup and f is a fuzzy subset of S.Then (f) =Theintersection of all fuzzy posemigroups of Scontaining$f$.

Proof**:** Let $\Delta = \{g/g \text{ is a fuzzy po semigroup of } S \text{ and } f \subseteq g\}$

since$S$ itself is a fuzzy posemigroup and $f \subseteq S$

$$\Rightarrow S \in \Delta \Rightarrow \Delta \neq \emptyset$$

Let $F^* = \cap_{g\in\Delta} g_1 \Rightarrow F^* \neq \emptyset$by Theorem 4.3, F^* is a fuzzy po semigroup of S.

Since$F^* \subseteq g_1, \forall g_1 \in \Delta$, F^* is the smallest fuzzy posemigroup of Scontaining f.

Therefore$F^* = (f)$.

V. FUZZY IDEALS OF PARTIALLY ORDERED SEMIGROUPS:

Theorem 5.1**:** *A fuzzy subset f of a po semigroup S is a fuzzy left ideal of S iff (i)$Sof \subseteq f$ (ii)$(f] \subseteq f$*

Proof**:** Let *f* be fuzzy left ideal.

(i) Consider $(Sof)(x) = \vee_{x\leq yz}[S(y) \wedge f(z)] = \vee_{x\leq yz} f(z) \leq f(x) \Rightarrow Sof \subseteq f$

(ii) Consider $(f](x) = \vee_{x\leq y} f(y), \forall x \in S$

$\Rightarrow (f](x) = \vee_{x\leq y} f(y) \leq f(x)$ since $x \leq y \Rightarrow f(x) \geq f(y), \forall x, y \in S$

Therefore $(f] \subseteq f$

Conversely, suppose that (i) $Sof \subseteq f$(ii)$(f] \subseteq f$

a) Consider $f(xy) = f(a)$ where $a = xy$

$$\geq (Sof)(a) = \vee_{a\leq bc} S(b) \wedge f(c) \geq S(x) \wedge f(y) = 1 \wedge f(y) = f(y)$$

$$\Rightarrow f(xy) \geq f(y), \forall x, y \in S$$

b) Let$x \leq y \Rightarrow f(x) \geq (f](x) = \vee_{x\leq y} f(y) \geq f(y)$since $(f] \subseteq f$

$$\Rightarrow f(x) \geq f(y)$$

Therefore f is a fuzzy left ideal of S.

Theorem 5.2**:** *A fuzzy subset f of a po semigroup S is a fuzzy right ideal of S iff (i)$foS \subseteq f$ (ii)$(f] \subseteq f$*

Proof**:** Proof follows from Theorem 5.1.

Corollary5.3**:** *A fuzzy subset f of a po semigroup S is a fuzzy ideal of S iff (i)$foS \subseteq f, Sof \subseteq f$ (ii)$(f] \subseteq f$*

Corollary5.4**:** Let f be a fuzzy subset of a posemigroup S. If f is fuzzy left (right/2-sided) ideal of Sthen$f = (f]$.

Theorem 5.5**:** *The intersection of any two fuzzy ideals of a posemigroupS is a fuzzy left ideal ofS*

Proof**:** Let f, g be any two fuzzy ideals of S

$\Rightarrow f, g$ are fuzzy left ideals.

a) Consider $(f \cap g)(xy) = f(xy) \wedge g(xy) \geq f(y) \wedge g(y) = (f \cap g)(y)$

$$\Rightarrow (f \cap g)(xy) \geq (f \cap g)(y)$$

b) Let $x \leq y \Rightarrow f(x) \geq f(y)$ and $g(x) \geq g(y)$

Consider $(f \cap g)(x) = f(x) \wedge g(x) \geq f(y) \wedge g(y) = (f \cap g)(y)$

$\Rightarrow (f \cap g)$ is a fuzzy left ideal of S

Theorem 5.6**:** *The intersection of arbitrary family of fuzzy left(right/2-sided) ideals of a posemigroupS is a fuzzy left(right/2-sided) ideal of S*

Proof**:** Let $\{f_\alpha\}_{\alpha\in\Delta}$ be a family of fuzzy left ideals of S and $f = \cap_{\alpha\in\Delta} f_\alpha$.

Let$x, y \in S$. Consider $f(xy) = \cap_{\alpha\in\Delta} f_\alpha(xy) \geq \cap_{\alpha\in\Delta} f_\alpha(y) = f(y)$

$$\Rightarrow f(xy) \geq f(y)$$

Let$x \leq y$. Consider$f(x) = \cap_{\alpha\in\Delta} f_\alpha(x) \geq \cap_{\alpha\in\Delta} f_\alpha(y) = f(y)$

$$\Rightarrow f(x) \geq f(y)$$

Therefore the intersection of arbitrary family of fuzzy left ideals of a po semigroup is a fuzzy left ideal of$S$.

Similarly the intersection of arbitrary family of fuzzy right (2-sided) ideals of a po semigroup is a fuzzy right (2-sided) ideal of S.

Theorem 5.7: *The union of any two fuzzy left (right/2-sided) ideals of a posemigroup$S$ is a fuzzy left(right/2-sided) ideal of S*

Proof: Let f, g be any 2 fuzzy left ideals of a po semigroup S.

(i) Let$x, y \in S$. Consider $(f \cup g)(xy) = f(xy) \vee g(xy) \geq f(y) \vee g(y) = (f \cup g)(y)$

$$\Rightarrow (f \cup g)(xy) \geq (f \cup g)(y)$$

(ii) Let $x \leq y \Rightarrow f(x) \geq f(y)$ and $g(x) \geq g(y)$

Consider $(f \cup g)(x) = f(x) \vee g(x) \geq f(y) \vee g(y) = (f \cup g)(y)$

$$\Rightarrow (f \cup g)(x) \geq (f \cup g)(y)$$

Therefore the union of any two fuzzy leftideals is a fuzzy left ideal of S.

Similarly, the union of any two fuzzy right(2-sided) ideals of a posemigroup$S$ is a fuzzy right(2-sided) ideal of S.

Theorem 5.8: *The union of arbitrary family of fuzzy left (right/2-sided) ideals of a posemigroup$S$ is a fuzzy left(right/2-sided) ideal of S.*

Proof: Let $\{f_\alpha\}_{\alpha\in\Delta}$ be a family of fuzzy left ideal of S and$f = \underset{\alpha\in\Delta}{\cup} f_\alpha$.

Let$x, y \in S$. Consider $f(xy) = \underset{\alpha\in\Delta}{\cup} f_\alpha(xy) \geq \underset{\alpha\in\Delta}{\cup} f_\alpha(y) = f(y)$

$$\Rightarrow f(xy) \geq f(y)$$

Let $x \leq y$. Consider $f(x) = \underset{\alpha\in\Delta}{\cup} f_\alpha(x) \geq \underset{\alpha\in\Delta}{\cup} f_\alpha(y) = f(y)$

Therefore the union of arbitrary family of fuzzy left ideals of a posemigroup is a fuzzy left ideal of S.

Similarly the union of arbitrary family of fuzzy right (2-sided) ideals of a posemigroup is a fuzzy right (2-sided) ideal of S.

VI. COMPLETELY PRIME FUZZY IDEALS AND PRIME FUZZY IDEALS OF PO SEMIGROUP:

Definition 6.1: A fuzzy ideal f of a posemigroup S is called *completely prime fuzzy ideal*if$\forall$ two ordered fuzzy points x_t, y_r of S ($\forall t, r \in (0,1]$) such that $x_t o y_r \subseteq f$ then $x_t \subseteq f$or $y_r \subseteq f$ where $x, y \in S$.

Definition 6.2: A fuzzy ideal *f* of a posemigroup S is called *weakly completely prime fuzzy ideal* if $\forall$ two ordered fuzzy points x_t, y_t of S ($\forall t \in (0,1]$) such that $x_t o\ y_t \subseteq f$ then $x_t \subseteq f$ or $y_t \subseteq f$ where $x, y \in S$.

Theorem 6.3: *Let f be a fuzzy ideal of posemigroup S. If f is completely prime fuzzy ideal then f is weakly completely prime fuzzy ideal.*

Proof:Let *f* be completely prime fuzzy ideal of S. Let x_t, y_t be any two fuzzy points of S such that $x_t\ o\ y_t \subseteq f$, $t \in (0,1]$. ⇒Either $x_t \subseteq f$ or $y_t \subseteq f$ since *f* is completely prime fuzzy ideal.

⇒*f* is weakly completely prime fuzzy ideal.

Theorem 6.4: *Let f be a fuzzy ideal of a posemigroup S. If f is completely prime fuzzy ideal then f_t is a subsemigroup of S and also completely prime ideal of S if $f_t \neq \emptyset$.*

Proof:Let f be completely prime fuzzy ideal of S.

(a) let $x, y \in f_t \Rightarrow f(x) \geq t, f(y) \geq t \Rightarrow f(x) \vee f(y) \geq t$

$$\Rightarrow f(xy) \geq f(x) \vee f(y) \geq t \Rightarrow xy \in f_t.$$

Therefore $f_{\ t}$is a subsemigroup of S.

(b) Let $x, y \in S\ sch\ that\ xy \in f_t \Rightarrow f(xy) \geq t$

Since *f* is completely prime fuzzy ideal, by th*f* is weakly completely prime fuzzy ideal.

$$\Rightarrow f(xy) = f(x) \vee f(y)$$

$$\Rightarrow f(x) \vee f(y) = f(xy) \geq t \Rightarrow f(x) \vee f(y) \geq t, \forall x, y \in S$$

⇒Either$f(x) \geq t$ or $f(y) \geq t$

$\Rightarrow x \in f_t$or $y \in f_t$

Therefore $f_t$is completely prime ideal of S.

Theorem 6.5: *Let f be fuzzy ideal of a posemigroup S. If $f_t$is completely prime ideal of a posemigroupS ,$\forall t \in (0,1]$ then f is weakly completely prime fuzzy ideal of S.*

Proof:Assume that f_t is completely prime ideal of S

For any two fuzzy points $x_t, y_t\ of\ S \ni x_t o y_t \subseteq f$ for $t \in (0,1]$

Let $f(ab) = l\ where\ l \in (0,1] \Rightarrow ab \in f_l$ since f_l is completely prime

$\Rightarrow$ either $a \in f_l or\ b \in f_l \Rightarrow f(a) \geq l$ or $f(b) \geq l$
Now $\forall t, x_t o y_t \subseteq f \Rightarrow (x_t o y_t)(ab) \leq f(ab) = l$
$\Rightarrow (x_t o y_t)(ab) = \vee [x_t(a) \wedge y_t(b)] \leq t, \forall a, b$
$\Rightarrow x_t(a) \wedge y_t(b) \leq \vee (x_t(a) \wedge y_t(b) \leq t \leq f(ab) = l$
$\Rightarrow x_t(a) \wedge y_t(b) \leq l, \forall a, b \in S$
$\Rightarrow x_t(a) \leq l$ or $y_t(b) \leq l$
$\Rightarrow x_t(a) \leq l \leq f(a)$ or $y_t(b) \leq l \leq f(b)$
$\Rightarrow x_t \subseteq f$ or $y_t \subseteq f, \forall a, b \in S$
Therefore *f* is weakly completely prime fuzzy ideal of S.

Theorem 6.6: *Let f be a fuzzy ideal of posemigroup S. If* f_t *is completely prime ideal of S,*$\forall t \in (0,1]$ *then f is completely prime fuzzy ideal of S.*
***Proof*:** Assume that f_t is completely prime ideal of S.
Assume that for any 2 fuzzy points x_t, y_r of S such that $x_t o y_r \subseteq f$ where $t, r \in (0,1]$
Let $a, b \in S \Rightarrow ab \in S$. Now $(x_t o y_r)(ab) \leq f(ab)$
Let $f(ab) = min\{t, r\} = u(say) \Rightarrow ab \in f_u$
Since f_u is completely prime ideal of S.
$\Rightarrow$Either $a \in f_u$ or $b \in f_u \Rightarrow f(a) \geq u$ or $f(b) \geq u$
clearly $x_t(a) \wedge y_r(b) \leq \vee [x_t(a) \wedge y_r(b)]$
$= (x_t o y_r)(ab) \leq t \wedge r = u$
$\Rightarrow x_t(a) \wedge y_r(b) \leq u$
$\Rightarrow x_t(a) \leq u \leq f(a)$ or $y_r(b) \leq u \leq f(b)$
$\Rightarrow x_t \subseteq f$ or $y_r \subseteq f$
Therefore *f* is completely prime fuzzy ideal of S.

***Definition 6.7*:**[5]Let S be a posemigroup. A fuzzy ideal f of S is said to be *prime fuzzy* if $\forall$ 2 fuzzy ideals g and h of S, $goh \subseteq f$ then either $g \subseteq f$ or $h \subseteq f$.

***Definition 6.8*:** The fuzzy ideal generated by a_λ, denoted by (a_λ) is
$\forall x \in S, (a_\lambda)(x) = \begin{cases} \lambda\ if\ x \in (a) \\ 0\ if\ x \notin (a) \end{cases}$
where (a) is an ideal of S generated by a, $i.e\ (a) = (a \cup aS \cup Sa \cup SaS] = (S^1 a S^1]$
***Definition 6.9*:** Let a_λ be ordered fuzzy point of S, $\lambda \in (0,1]$
$\forall\ x \in S, (Soa_\lambda oS)(x) = \begin{cases} \lambda\ if\ x \in (SaS] \\ 0\ if\ x \notin (SaS] \end{cases}$ and Soa_λoS is a fuzzy ideal of S.

***Theorem 6.10*:** *Let f be a fuzzy ideal of a po semigroup S. Then the following are equivalent*
(a) f is prime fuzzyideal
(b) $\forall$ 2 ordered fuzzy points $x_r, y_t of\ S\ if < x_r > o < y_t > \subseteq f$ then $x_r \subseteq f$ or $y_t \subseteq f$ for $r, t \in (0,1]$
(c) $\forall$ 2 ordered fuzzy points $x_r, y_t of\ S$ if $Sox_r oSoy_t oS \subseteq f \Rightarrow x_r \subseteq f or\ y_t \subseteq f\ (rt > 0)$
***Proof*:** $(\boldsymbol{a}) \Rightarrow (\boldsymbol{b})$:
Let f be a prime fuzzy ideal of S .
Let x_r, y_t be any 2 ordered fuzzy points of S $\ni t, r \in (0,1]\ \& < x_r > o < y_t > \subseteq f$
Since $< x_r >, < y_t >$ are fuzzy ideals of S Either $x_r \subseteq f\ or\ y_t \subseteq f$
$(\boldsymbol{b}) \Rightarrow (\boldsymbol{c})$:
Assume that$\forall$ 2 ordered fuzzy points x_r, y_t of S if $< x_r > o < y_t > \subseteq f$ then either $x_r \subseteq f$ or $y_t \subseteq f$.
Suppose $Sox_r oSoy_t oS \subseteq f$
$\Rightarrow Sox_r oSoSoy_t oS \subseteq f$
since $(Sox_r oS)o(Soy_t oS) \leq r \wedge t$ and also $< x_r > o < y_t > \leq r \wedge t \implies x_r \subseteq f\ or\ y_t \subseteq f$
$(\boldsymbol{c}) \Rightarrow (\boldsymbol{a})$:
Assume that for any two ordered fuzzy points x_r, y_t of $SSox_r oSoy_t oS \subseteq f$ then $x_r \subseteq f$ or $y_t \subseteq f (rt > 0)$
Let *g*,*h* be 2 fuzzy ideals of S $\ni goh \subseteq f$
if possible suppose that $g \nsubseteq f$ and $h \nsubseteq f$ then $\exists\ x, y \in S \ni g(x) > f(x)$and $h(y) > f(y)$
Let $r = g(x), t = h(y)$then $r, t > 0, (rt > 0), x_r \subseteq g, y_t \subseteq h$ then
Sox_roSoy_toS $\subseteq (Sox_r$oS)o(Soy_toS) $\leq (x_r)o(y_t) \subseteq goh \subseteq f$
therefore either $x_r \subseteq f$ or $y_t \subseteq f$
say $x_r \subseteq f\ then\ f(x) \geq r = g(x) \Rightarrow f(x) \geq g(x)$, a contradiction

⇒ either $g \subseteq f$ or $h \subseteq f$
Therefore$f$ is a prime fuzzy ideal of S.

Theorem 6.11: *Let f be a fuzzy ideal of a posemigroup S. f is prime fuzzyiff fuzzy ideals $f_1, f_2, f_3, \ldots\ldots\ldots f_n, n \in N$, if $f_1 o f_2 o \ldots\ldots\ldots o f_n \subseteq f \Rightarrow f_i \subseteq f$ for some $i = 1,2,3, \ldots\ldots, n$*
Proof: Let *f* be prime fuzzy, assume that $f_1 o f_2 o \ldots\ldots\ldots o f_n \subseteq f$ where$f_1, f_2, f_3, \ldots\ldots\ldots f_n$are fuzzy ideals
if n=1 then clearly $f_1 \subseteq f$
if n=2 then $f_1 o f_2 \subseteq f$ since f is prime fuzzy, either $f_1 \subseteq f$ or $f_2 \subseteq f$
by induction on n, $f_1 o f_2 o \ldots\ldots\ldots o f_n \subseteq f \Rightarrow f_i \subseteq f$ $forsome$ $i = 1,2,3, \ldots\ldots, n$
conversely suppose if $f_1 o f_2 o \ldots\ldots\ldots o f_n \subseteq f \Rightarrow f_i \subseteq f$ $forsome$ $i = 1,2,3, \ldots\ldots, n$
since $n \in N$ $take$ $n = 2$ then clearly *f* is primefuzzy ideal.

Theorem* 6.12:**[13]Let f be a fuzzy ideal of a posemigroup S. Then if f is completely prime fuzzy then f is a primefuzzyideal of S.*

Theorem 6.13:[13]*Let S be a commutative posemigroup and f is a fuzzy ideal of S. Then f is completely prime fuzzy if and only if f is prime fuzzy ideal.*

Definition 6.14: Let f be a fuzzy subset of a posemigroup S. f is said to be *fuzzy m-system* of *S* provided if $f(x) > t_1, f(y) > t_2 \Rightarrow \exists c, s \in S \ni f(c) > t_1 \vee t_2$ and $c \leq xsy$where $x, y \in S$.

Theorem 6.15: *A fuzzy ideal f of aposemigroupS is a prime ideal iff*$1 - f$ *is a fuzzy m-system of S provided* $1 - f \neq \emptyset$.
Proof: Suppose that f is a prime fuzzy ideal of S.
Let $\forall t, S \in [0,1], a, b \in S$ if $(1 - f)(a) > t, (1 - f)(b) > S$
⇒ $f(a) < 1 - t, f(b) < 1 - s$ then $a_{1-t} \nsubseteq f$ and$b_{1-s} \nsubseteq f$
⇒ $a_{1-t} o b_{1-s} \nsubseteq f$ Now $a_{1-t} o f_{\{x\}} o b_{1-s} = (axb)_{(1-t)\wedge(1-s)} \nsubseteq f$

$$\Rightarrow f(axb) < (1 - t) \wedge (1 - s) = 1 - (t \vee s)$$
$$(1 - f)(axb) > t \vee s$$

Therefore $1 - f$is a fuzzy m-system of S.
conversely suppose $1 - f$ is a fuzzy m-system of S
Let g,h be two fuzzy ideals of S∋ $goh \subseteq f$
If possible suppose $g \nsubseteq f$ and $h \nsubseteq f$
⇒ ∃ an ordered fuzzy points $x_\lambda \in g, y_\mu \in h$ and $x_\lambda \notin f$ and $y_\mu \notin f$
⇒ $f(x) < \lambda$ and $f(y) < \mu$
⇒ $(1 - f)(x) > 1 - \lambda, (1 - f)(y) > 1 - \mu$
⇒ $\exists c, s \in S \ni (1 - f)(c) > (1 - \lambda) \vee (1 - \mu) = 1 - (\lambda \wedge \mu)$$and$ $c \leq xsy$
⇒ $f(c) < \lambda \wedge \mu$ since $c \leq xsy \Rightarrow f(c) \geq f(xsy) \Rightarrow f(xsy) < \lambda \wedge \mu$
but $x_\lambda \subseteq g, y_\mu \subseteq h \Rightarrow x_\lambda o y_\mu \subseteq goh \subseteq f$

$$\Rightarrow x_\lambda o y_\mu \subseteq f \Rightarrow (x_\lambda o y_\mu)(t) \leq f(t), \forall t \in s \Rightarrow \lambda \wedge \mu \leq f$$

But $xsy \in S$ and $f(xsy) < \lambda \wedge \mu$
⇒ $\lambda \wedge \mu > f(xsy)$which is contradiction
Therefore either $g \subseteq f$ or $h \subseteq f$
⇒ f is primefuzzy ideal.

Definition 6.16: A fuzzy ideal f of a po semigroup S is called *maximal* if there doesn't exist any proper fuzzy ideal $g$of $S \ni f \subset g$.

Theorem 6.17: *Let S be a posemigroup. Every maximal fuzzy ideal f of Sis a prime fuzzy ideal of S.*
Proof: Let f be a fuzzy maximal ideal of S. Let g, h be two fuzzy ideals of $S \ni goh \subseteq f$
Suppose if possible $g \nsubseteq f$ and $h \nsubseteq f$
⇒ $g \cup f$ is a fuzzy ideal of S and $f \subset f \cup g \subseteq S$
since f is maximal, $g \cup f = S$ Similarly if $h \nsubseteq f$ $then$ $h \cup f = S$
Now $S = SoS = (g \cup f) \circ (h \cup f) = (f \cup g) \circ (f \cup h)$

$$= f \cup (goh)$$
$$\subseteq f \cup f = f$$

⇒ $S \subseteq f$ it is a contradiction

Therefore either $g \subseteq f$ or $h \subseteq f$
$\Rightarrow f$ is primefuzzy ideal of S.

Definition 6.18: Let S be a posemigroup. $S$is called *fuzzy semi simple* if $\forall t \in (0,1]$ if $a_t \subseteq < a_t >^2$.

Theorem 6.19: *Let S be a posemigroup and f is maximal fuzzy ideal of S then f is fuzzy semisimple.*
Proof: Let f be maximal fuzzy ideal.$\Rightarrow$ $f$isprimefuzzy ideal.
If $a_t \in 1 - f$ then $< a_t > \nsubseteq f$ $\Rightarrow < a_t >^2 \nsubseteq f$ since f is primefuzzy ideal.
Now $S = f \cup < a_t > = f \cup < a_t >^2$since f is fuzzy maximal.
Therefore $a_t \subseteq < a_t >^2$. $\Rightarrow S$is fuzzy semi simple.

VII. ACKNOWLEDGEMENT

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