On g-(i,j)-Fuzzy Semi-Open Sets in Generalized Fuzzy Bitopological Spaces

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Abstract

In this paper we have studied the notion of g-(i,j)-fuzzy semi-open sets in generalized fuzzy bitopological spaces. We have obtained some significant properties of g-(i,j)-fuzzy semi-open sets and constructed some examples.

Key words: *Fuzzy topological space, Generalized fuzzy topological space, g-(i,j)-fuzzy semi-open sets, g-(i,j)-fuzzy semi-closed sets.*

I. INTRODUCTION

The notion of fuzzy sets was introduced by Zadeh [5] in 1965. In 1968 Chang [2] has introduced the concept of fuzzy topological space as a generalization of topological space. Later on many mathematicians contributed in the development of the theory of fuzzy topological spaces. The generalized form of fuzzy open set known as fuzzy semi open set, was introduced by Azad [1] in the year 1981. Chetty [3] in 2008 has introduced the concept of generalized fuzzy topological spaces (gfts). In this paper we have studied the concept of g-(i,j)-fuzzy semi-open sets in generalized fuzzy bitopological spaces .

II. PRELIMINARIES

*Let X be a universal crisp set. A **fuzzy set** μ on the set X is a mapping $\mu: X \to I$, where $I = [0,1]$. For $x \in X$, the real number $\mu(x)$ in [0,1] is called **grade of membership** of x in the fuzzy set μ. A fuzzy point x_p, $x \epsilon X$ is a fuzzy set on X defined by*

$$x_p(y) = \begin{cases} p \ (p \in (0,1]) & for \ \ y = x, \\ 0 & otherwise \end{cases}$$

for all $y \in X$.

Definition 2.1 [2]: *Let τ be a collection of fuzzy sets on X. Then τ is said to be a **Fuzzy Topology** on X if it satisfies following conditions:*

(i) The fuzzy sets 0 and 1 are in τ, where 0,1:X→ I, are defined as 0(x)=0 and 1(x)=1, for all $x \in X$.

(ii) If $\alpha, \beta \in \tau$ then $\alpha \cap \beta \in \tau$.

(iii) If $\{\alpha_j\}_{j \epsilon J}$, J is an index set in any family of fuzzy sets on X and $\alpha_j \in \tau$, for all $j \in J$ then $\bigcup_{j \in J} \alpha_j \in \tau$.

*The pair (X,τ) is called a **fuzzy topological space**.*

Definition 2.2 [2]: Let (X,τ) be a fuzzy topological space. The members of the collection τ are called **fuzzy open sets** and the complement of fuzzy open sets are called **fuzzy closed sets**. For a fuzzy set α on X the **closure** of α is denoted by Cl(α) and is defined to be the intersection of all fuzzy closed sets in X containing α. The **interior** of α is denoted by Int(α), and is defined to be the union of all fuzzy open sets in X which are contained in α.

Definition 2.3 [3]: Let X be a non empty crisp set. Let τ_g be a collection of fuzzy sets on X such that

(i) The fuzzy sets 0 and 1 are in τ_g, where 0,1:X→ I, are defined as 0(x)=0 and 1(x)=1, for all $x \in X$.

(ii) If $\{\alpha_j\}_{j\epsilon J}$, J is an index set in any family of fuzzy sets on X and $\alpha_j \in \tau_g$, for all $j \in J$ then $\bigcup_{j\in J} \alpha_j \in \tau_g$.

Then τ_g is called a **generalized fuzzy topology** on X and (X, τ_g) is called **generalized fuzzy topological space**. Members of τ_g are called **g-fuzzy-open** sets and their complement are called **g-fuzzy –closed** sets.

Remark: From Definition 2.1 and 2.3 we note that a fuzzy topological space is always a generalized fuzzy topological space. However the converse is not necessarily true.

Example 2.4: X Let X= $\{x_1, x_2\}$ and let α, β, γ be fuzzy sets on X defined as $\alpha(x_1) = 0.5$, $\alpha(x_2) = 0.3$, $\beta(x_1) = 0.1$, $\beta(x_2) = 0.6, \gamma(x_1) = 0.5$ *and* $\gamma(x_2) = 0.6$. Let τ_g= $\{0, \alpha, \beta, \gamma, 1\}$. Then we see that τ_g is generalized fuzzy topology but it is not a fuzzy topology on.

III. GENERALIZED FUZZY BITOPOLOGICAL SPACE

In this section we have studied some properties of g-(i,j)-fuzzy semi-open sets and g-(i,j)-fuzzy semi-closed sets in generalized fuzzy bitopological spaces.

Definition 3.1[4]: Let X be a non empty crisp set and let τ_1 and τ_2 be fuzzy topologies on X. Then the triplet (X,τ_1, τ_2) is called a **fuzzy bitopological space**. The members of $\tau_i (i = 1,2)$ are called $\boldsymbol{\tau_i}$ **-fuzzy open sets** and their complement are called $\boldsymbol{\tau_i}$ **–fuzzy closed sets**.

Definition 3.2: Let X be a non empty crisp set and let τ_{1g} and τ_{2g} be generalized fuzzy topologies on X. Then the triplet (X,τ_{1g}, τ_{2g}) is called a **generalized fuzzy bitopological space**. The members of $\tau_{ig} (i = 1,2)$ are called $\boldsymbol{\tau_{ig}}$ **-fuzzy open sets** and their complement are called $\boldsymbol{\tau_{ig}}$ **–fuzzy closed sets**.

Remark: From Definition 3.1 and 3.2 we note that a fuzzy bitopological space is always a generalized fuzzy bitopological space. However the converse is not necessarily true.

Example 3.3: Let X= $\{x_1, x_2\}$ and let $\alpha_1, \beta_1, \gamma_1$, be fuzzy sets on X defined as $\alpha_1(x_1) = 0.5$, $\alpha_1(x_2) = 0.3$, $\beta_1(x_1) = 0$, $\beta_1(x_2) = 0.6$,

$\gamma_1(x_1) = 0.5$, $\gamma_1(x_2) = 0.6$, $\alpha_2(x_1) = 0.4$, $\alpha_2(x_2) = 0.3$, $\beta_2(x_1) = 0.6$, $\beta_2(x_2) = 0.2$, $\gamma_2(x_1) = 0.6$ and $\gamma_2(x_2) = 0.3$. Let τ_{1g}= $\{0, \alpha_1, \beta_1\ \gamma_1, 1\}$ and τ_{2g}= $\{0, \alpha_2, \beta_2\ \gamma_2, 1\}$, be two generalized fuzzy topologies on X then (X, τ_{1g}, τ_{2g}) is generalized fuzzy bitopological spaces but it is not a fuzzy bitopological space.

Definition 3.4: A fuzzy set A of a generalized fuzzy bitopological space (X,τ_{1g}, τ_{2g}) is called **g-(i,j)-fuzzy semi-open set** if there exists a τ_{ig}-fuzzy open set U such that $U \le A \le \tau_{ig} - Cl(U)$.

Theorem 3.5: In a generalized fuzzy bitopological spaces (X,τ_{1g}, τ_{2g}) every τ_{ig}-fuzzy open set is g-(i,j)-fuzzy semi-open set.

Proof: Let (X,τ_{1g}, τ_{2g}) be a generalized fuzzy bitopological space. Let U be a τ_{ig} fuzzy open set in X. Then clearly U≤ U≤ τ_{jg} − Cl(U). This implies U is a τ_{ig}-fuzzy semi-open set in X.

Theorem 3.6: Let (X,τ_{1g}, τ_{2g}) be a generalized fuzzy bitopological space. Then any union of g-(i, j)-fuzzy semi-open sets in X is a g-(i, j)-fuzzy semi-open set.

Proof: Let (X,τ_{1g}, τ_{2g}) be generalized fuzzy bitopological space and let $\{A_\alpha\}_{\alpha\epsilon\Lambda}$ be a family of g-(i, j)-fuzzy semi-open sets in X. Suppose that A=$\bigcup_{\alpha\epsilon\Lambda} A_\alpha$. For each $\alpha\epsilon\Lambda$, let U_α be a τ_{ig} fuzzy open set in X such that $U_\alpha \le A \le \tau_{jg} - Cl(U_\alpha)$. Put $U = \cup_{\alpha\epsilon\Lambda} U_\alpha$. Since arbitrary union of τ_{ig} fuzzy open sets is τ_{ig} fuzzy open and $U \le A \le \tau_{ig} - Cl(U)$. Thus A is a g-(i,j)-fuzzy semi-open set in X.

Theorem 3.7: Let (X,τ_{1g}, τ_{2g}) be a generalized fuzzy bitopological space. Then A is g-(i,j)-fuzzy semi-open set in X if and only if A≤ $\tau_{jg} - Cl(\tau_{ig} - int(A))$.

Proof: Let (X,τ_{1g}, τ_{2g}) be a generalized fuzzy bitopological space and A be a g-(i,j)-fuzzy semi-open set in X then there exists a τ_{ig} fuzzy open set U such that U≤ A≤ $\tau_{jg} -$ Cl(U). As U≤ A, we have U= $\tau_{ig} - int(U) \leq \tau_{ig} - int(A)$. This implies τ_{jg}-$Cl(U) \leq \tau_{jg} - Cl\left(\tau_{jg} - int(A)\right)$. Since A≤ $\tau_{jg}\,Cl(U)$, it follows that A≤ $\tau_{jg} - Cl\left(\tau_{ig} - int(A)\right)$. Conversely let A≤ $\tau_{jg} - Cl\left(\tau_{ig} - int(A)\right)$. Suppose U=$\tau_{ig} - int(A)$, then U is a τ_{ig} fuzzy open set in X. Clearly U≤ A≤ $\tau_{jg} -$ Cl(U). Hence A is g-(i,j)-fuzzy semi-open set in X.

Theorem 3.8: Let (X,τ_{1g}, τ_{2g}) be a generalized fuzzy bitopological space and A be a fuzzy set in X. Then A is a g-(i,j)-fuzzy semi-open if and only if for each fuzzy point $x_\beta \epsilon A$ there exists a g-(i,j)-fuzzy semi-open set U such that $x_\beta \epsilon$ U≤ A.

Proof: Let A be a g-(i,j)-fuzzy semi open set in X. Then clearly for each fuzzy point $x_\beta \epsilon A$ there exists a g-(i,j)-fuzzy semi open set A itself satisfying the desired condition. Conversely, let A be a fuzzy set in X having the property that for each fuzzy point $x_\beta \epsilon A$ a g-(i,j) –fuzzy semi-open set $\mu(x_\beta)$ such that $x_\beta \in \mu\left(x_\beta\right) \leq A$. Then we can see that $A = \cup\, \mu\left(x_\beta\right)$. Hence from Theorem 3.6 we find that A is a a g-(i,j) –fuzzy semi-open set in X.

IV. CONCLUSION

In**[6],** (i,j)- semi open set has been defined in fuzzy bitopoligical spaces. In present paper we have generalized the same concept in generalized fuzzy bitopological spaces.

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