Union Complex π - Fuzzy Soft Left R-Modules and Ideals

[1]J.Regala Jebalily, [2]G. Subbiah* and [3]V.Nagarajan
[1]Research scholar, Reg.No:12615, Department of Mathematics, S.T.Hindu college, Nagercoil-629 002,Tamilnadu, India.
[2]*Associate Professor in Mathematics, Sri K.G.S. Arts College, Srivaikuntam-628 619, Tamil Nadu, India.
[3]Assistant Professor in Mathematics, S.T.Hindu college, Nagercoil -629 002, Tamilnadu, India.
Affiliated to ManonmaniamSundaranar University, Abishekapatti, Tirunelveli - 627 012, Tamilnadu, India.

Abstract

The main purpose of this study is to observe some of the soft algebraic structures of the complex fuzzy set. We introduce the concept of union complex π- fuzzy soft modules of a given classical module and investigate some of the crucial properties and characterizations of the proposed concept.

Keywords: *fuzzy set, soft set, π- fuzzy soft set, union complex fuzzy modules, inclusion, sum, ideals.*

I.INTRODUCTION

Complex fuzzy set and logic, which is the extension of fuzzy sets and logic respectively, was first proposed by Ramot et.al [9]. According to their definition, a complex fuzzy set is characterized by a complex grade of membership which is a combination of a traditional fuzzy degree of membership referred to as the amplitude term with the addition of an extra term, the phase term. In similarity to the case of intuitionistic fuzzy set, a complex intuitionistic fuzzy set is characterized by a complex grade of membership and complex grade of non-membership .Complex intuitionistic fuzzy sets have been applied in multi attribute decision making problems .Combining fuzzy sets with soft sets, Maji et.al [7] introduced the notion of fuzzy soft sets. This work were further revised and improved by Ahmad and Kharal [6]. They defined arbitrary fuzzy soft union,intersection and proved De Morgan inclusions and De Morgan laws in fuzzy soft set theory. Thereafter, Maji and his co-author [7] introduced the notion of intuitionistic fuzzy soft set which is based on the combination of the intuitionistic fuzzy sets and soft set models and they studied the properties of intuitionistic fuzzy soft set. The algebraic structures of soft set theory have also been studied extensively**.** Aktas and Cagman [2] introduced the basic concepts of soft groups, soft subgroups, normal soft subgroups and soft homomorphism and discussed their basic properties. Feng et.al [5] considered the algebraic structure of semiring and introduced the notion of soft semiring. Some basic algebraic properties of soft semiring and some related notions such as soft ideals, idealistic soft semirings and soft semiring homomorphism were defined and investigated with illustrative examples**.** However, the theory of complex fuzzy sets and complex intuitionistic fuzzy sets are independent of the parameterization tools. Zadeh [14] introduced the concept of fuzzy sets in 1965. A fuzzy set A in U is defined by membership function $\mu_A : U \rightarrow [0,1]$, where U is non empty set, called universe. In 2002, Ramot et.al [11] introduced the concept of complex sets. In 2016,as previous work we [1] introduced the concept of complex fuzzy subgroup. The main purpose of this study is to observe some of the soft algebraic structures of the complex fuzzy set.we introduce the concept of union complex π- fuzzy soft modules of a given classical module and investigate some of the crucial properties and characterizations of the proposed concept.

II. PRELIMINARIES

By a ring, we shall mean an algebraic system (R, +, •), where

(i) (R, +) forms an abelian group,
(ii) (R, •) forms a semi group and
(iii) $x \bullet (y+z) = xy + xz$ and $(x+y) \bullet z = xz + yz$ for all $x,y,z \in R$ (i,e ., left and right distributive laws hold)

Through out this paper, R will always denote a ring.

A left R-module over a ring R consists of an abelian group (M , +) and an operation $R \times M \rightarrow M$ such that for all $r,s \in R$, $x,y \in M$, we have

(i) $r(x+y) = rx + ry$
(ii) $(r+s)x = rx + sx$
(iii) $(rs)x = r(sx)$. It is denoted by R^M.

Clearly R itself is a (left) R-module by natural operation. Suppose M is a left R-module and N is a subgroup of M.Then N is called a sub module (or R-sub module) if, for any $n \in N$ and any $r \in R$, the product rn is in N.
Molodtsov [8] defined the soft set in the following manner; Let U be an initial universe set, E be a set of parameters, P(U) be the power set of U and A is subset of E.
In fact, there exists a mutual correspondence between soft sets and binary relations as shown in [7]. That is, let A and B be non-empty sets and assume that α refers to an arbitrary binary relation between an element of A and an element of B.
A set-valued function $\delta : A \rightarrow B$ can be defined as $\delta (x) = \{ y \in B / (x ,y) \in \alpha \}$.

Definition 2.1[8]: Let δ_A and δ_B be two soft sets over a common universe U such that $A \cap B \neq \phi$. The restricted intersection of δ_A and Δ_B is denoted by $\delta_A \cap \Delta_B$, and it is defined by $\delta_A \cap \Delta_B = H_C$, where $C = A \cap B$ and for all $c \in C$, $H(C) = \delta(C) \cap \Delta(C)$.

Definition 2.2[9]: A complex fuzzy set, defined on a universe U, is characterized by a membership $\delta_A(x)$ that assigns any element a complex grade of membership in A. By definition, the values $\delta_A(x)$ may receive all lie within the unit interval [0,1] in the complex plane, and are thus of the form $r_A(x)\, e^{iwA(x)}$, where $i = \sqrt{-1}$, $r_A(x)$ and $w_A(x)$ are both real valued, and $r_A(x) \in [0,1]$, $w_A(x) \in (0,2\pi)$. The complex fuzzy set may be represented as the set of ordered pairs $A = \{(x, \delta_A(x)) : x \in U\}$.

Definition 2.3 [1]: Let $A = \{(x, \delta_A(x)) : x \in U\}$ be a fuzzy soft set. Then the set $\delta_\pi= \{(x, \delta_\pi(x)) / \delta_\pi(x) = 2\pi\, \delta_{\pi(}x), x \in U\}$ is said to be a π- fuzzy soft set.

Definition 2.4[union complex fuzzy soft modules]: Let N be a sub module of M and δ_π be a π-fuzzy soft set over M. Then δ_π is called an union complex fuzzy soft modules of M, denoted by $\delta_\pi <_j M$ if $\delta_\pi(x+y) \leq \max \{\delta_\pi(x), \delta_\pi(y)\}$. $\delta_\pi(mx) \leq \delta_\pi(x)$, for all $x,y \in N$ and $m \in M$.

Example 2.5: consider the ring $R = (Z_{12}, + , \bullet)$, the left R-module $M = (Z_{12}, +)$ with natural operation and the sub module $N_1 = \{0, 6\}$ of M. Let the π - fuzzy soft set $\delta_{N1\pi}$ over M, where
$F : N_1 \rightarrow P(M)$ is a set-valued function defined by
$\delta_{N1\pi} (0) = \{0,4,9\}$ and $\delta_{N1\pi} (6) = \{ 0,3,4,9,11\}$. Then, it can be easily seen that $\delta_{N1\pi} < M$.
Now let the sub module of M be $N_2 = \{ 0,4,8\}$ and the soft set $\Delta_{N2\pi}$ over M, where $\Delta : N_2 \rightarrow P(M)$ is a set –valued function defined by $\Delta(0) = \{0,3,9\}$ and $\Delta(4) = \{ 0,3,5,8,11\}$ and
$\Delta(8) = \{ 0,3,5,8,9,11\}$. Then
$\Delta (2\bullet 4) = \Delta (8) = \{0,3,5,8,9,11\}$ not a subset of $\Delta (4) = \{0,3,5,8,11\}$. Therefore $\Delta_{N2\pi}$ is not a union complex π-fuzzy soft module.

III. SOME STANDARD RESULTS

Theorem-3.1: Let N be a sub module and $A = \{(x, \delta_A(x)) / x \in N \}$ be a homogenous complex fuzzy soft set with membership function $\delta_A(x) = r_A(x)\, e^{iwA(x)}$.Then A is a union complex fuzzy soft modules of N if

(i) The fuzzy soft set $\bar{A} = \{(x , r_A(x)) / x \in N, r_A(x) \in [0,1]\}$ is a union complex fuzzy soft sub modules.
(ii) The π –fuzzy soft set $\Omega = \{(x ,w_A(x)) / x \in N, w_A(x) \in [0,2\pi] \}$ is a union complex π-fuzzy soft sub modules.

Proof: (i) Let A be an union complex fuzzy soft sub modules and $x, y \in N$. Then we have
$r_A(x+y)\, e^{iwA(x+y)}= \delta_A(x+y)$
$\leq \max \{\delta_A(x), \delta_A(y)\} = \max \{r_A(x)\, e^{iwA(x)}, r_A(y)\, e^{iwA(y)}\}$
$= \max \{ (\delta_A(x), \delta_A(y))\, e^{i\max\{ wA(x) ,wA(y)\}} \}$ (since A is homogenous)
So
$r_A(x+y) \leq \max \{r_A(x), r_A(y)\}$ and $w_A(x+y) \leq \max \{w_A(x), w_A(y)\}$.
On the other hand
$r_A(mx)\, e^{iwA(mx)} = \delta_A(mx) \leq \delta_A(x)$.
Therefore $\bar{A}$ is a union complex fuzzy soft modules. So A is a union complex fuzzy soft modules.
(ii) Let $\bar{A}$ be a union fuzzy soft sub modules and Ω be a union complex π- fuzzy soft sub modules. Then we have
$r_A(x+y) \leq \max \{ r_A(x), r_A(y)\}$ and $w_A(x+y) \leq \max \{ w_A(x), w_A(y)\}$
$r_A(mx) \leq r_A(x)$ and $w_A(mx) \leq w_A(x)$.
Now

$\delta_A(x+y) = r_A(x+y)\, e^{iwA(x+y)}$
$\leq \max \{ (r_A(x), r_A(y))\, e^{i\max \{wA(x), wA(y)\}}\}$
$= \max \{\delta_A(x), \delta_A(y)\}$
On the other hand
$\delta_A(mx) = r_A(mx)\, e^{iwA(mx)} \leq r_A(x)\, e^{iwA(x)} \leq \delta_A(x)$. So A is a union complex fuzzy soft modules.

Theorem-3.2: Let $\{A_i / i\in I\}$ be a collection of union complex fuzzy soft modules of N such that A_j is homogenous with A_k for all j, k ∈ I. Then $\cap A_i$ is a union complex fuzzy soft sub modules of N and i∈ I.
Proof: For all i∈ I. we have $r_{Ai}(x)$ is a complex fuzzy soft modules and $w_{Ai}(x)$ is complex π- fuzzy soft modules by theorem-3.1.
Now let x,y∈ N.Then
$\delta_{\cap Ai}(x+y) = r_{\cap Ai}(x+y)\, e^{iwAi(x+y)}$
$= \max \{r_{Ai}(x+y)\, e^{i\max \{wAi(x+y)\}}\}$
$\leq \max \{ \max \{r_{Ai}(x), r_{Ai}(y)\} e^{i\max \{ \max \{wAi(x), wAi(y)\}\}}\}$
$= \max \{ \max\{r_{Ai}(x), \max \{r_{Ai}(y)\} e^{i\max \{ \max \{wAi(x)\}, \max \{wAi(y)\}\}}\}$
Since A_j is homogenous with A_k for all j,k∈ I.
$= \max \{\max \{r_{Ai}(x)\, e^{i\max \{wAi(x)\}}\}, \max \{r_{Ai}(y)\, e^{i\max\{wAi(y)\}}\} \}$
$= \max\{\delta_{\cap Ai}(x), \delta_{\cap Ai}(y)\}$,
On the other hand
$\delta_{\cap Ai}(mx) = r_{\cap Ai}(mx)\, e^{iw\cap Ai(mx)}$
$\leq r_{\cap Ai}(x)\, e^{iw\cap Ai(x)}$
$\leq \delta_{\cap Ai}(x)$. The proof is completed.

Theorem-3.3: A complex fuzzy soft set A is a union complex fuzzy soft modules of N if and only if A^C is a intersection complex fuzzy soft modules of N.
Proof: Let A be a union complex fuzzy soft modules. Then we have
$\delta_A{}^C(x+y) = (1- r_A(x+y))\, e^{i\,(2\pi + wA(x+y))}$
$\leq (1 - \max\{r_A(x), r_A(y)\})\, e^{i(2\pi+\max \{wA(x), wA(y)\})}$
$= \min \{ 1- r_A(x), 1-r_A(y)\}\, e^{i\min \{(2\pi+wA(x)), (2\pi+wA(y))\}}$
$\geq \min \{(1-r_A(x))\, e^{i(2\pi+wA(x))}, (1- r_A(y))\, e^{i(2\pi+ wA(y))} \}$
$= \min \{ \delta_A(x), \delta_A(y)\}$.
On the other hand
$\delta_A{}^C(mx) = (1- r_A(mx))\, e^{i(2\pi+ wA(mx))}$
$\leq (1- r_A(x))e^{i(2\pi+ wA(x))}$
$\geq r_A{}^C(x)\, e^{i(2\pi+ wA(x))} \leq \delta_A{}^C(x)$.
Conversly, let A^C be a union complex fuzzy soft modules. Then we have
$\delta_A(x+y) = r_A(x+y)\, e^{iwA(x+y)}$
$= (1- (1- r_A(x+y)))\, e^{i\,(2\pi+wA(x+y))}$
$\leq (1 - \max\{(1- r_A(x)),(1- r_A(y))\})\ e^{i(2\pi+\max \{wA(x), wA(y)\})}$
$= \min \{ r_A{}^C(x), r_A{}^C(y)\}\, e^{i\min \{wAC(x), wAC(y)\}}$
$= \min \{(1-r_A(x))\, e^{iwA(x)}, (1- r_A(y))\, e^{iwA(y)} \}$
$= \min \{ \delta_A(x), \delta_A(y)\}$,
On the other hand
$\delta_A(mx) = r_A(mx))\, e^{iwA(mx)} = r_A(x)\, e^{iwA(x)} = \delta_A(x)$.

Definition 3.4: Let A be a complex fuzzy soft set of X. For α ∈[0,1] and β ∈ [0,2π], the set
$A(\alpha, \beta) = \{ x \in X / r_A(x) \leq \alpha , w_A(x) \leq \beta \}$is called a level subset of the complex fuzzy soft set A. In particular if β = 0. Then we get the level subset $A^\alpha = \{ x\in X / r_A(x) \leq \alpha \}$ and if α = 0. Then we get the level subset $A^\beta = \{x \in X / w_A(x) \leq \beta \}$.

Theorem-3.5: Let A be an union complex fuzzy soft modules of N, if $r_A(e) \leq \alpha$ and $w_A(e) \leq \beta$. Then the level subset A(α, β) is a sub module of N.
Proof: e ∈A(α, β), so A(α, β) ≠ ϕ.
Let x,y∈ A(α, β).Then we have
$r_A(x) \leq \alpha$ and $w_A(x) \leq \beta$, also $r_A(y) \leq \alpha$ and $w_A(y) \leq \beta$.
Now
$r_A(x+y)\, e^{iwA(x+y)} = \delta_A(x+y)$

$\leq$ max { $\delta_A(x)$, $\delta_A(y)$}
= max { $r_A(x)\, e^{iwA(x)}$, $r_A(y)\, e^{iwA(y)}$}
= max {$(r_A(x), r_A(y))\, e^{imax\,\{wA(x),\, wA(y)\}}$}
This implies
$r_A(x+y) \leq \max\,\{r_A(x), r_A(y)\} \leq \max\,\{\alpha , \alpha\} = \alpha$. and
$w_A(x+y) \leq \max\,\{w_A(x), w_A(y)\} \leq \max\,\{\beta , \beta\} = \beta$.
So $x+y \in A(\alpha, \beta)$.
On the other hand we have
$r_A(mx)\, e^{iwA(mx)} = \delta_A(mx) \leq \delta_A(x) \leq \alpha$. So $mx \in A(\alpha, \beta)$. Therefore $A(\alpha, \beta)$ is a sub modules of N.

Corollary 3.6: Let A be an union complex fuzzy soft modules of N, if $r_A(e) \leq \alpha$ and $w_A(e) \leq \beta$. Then the level subsets $A^\alpha = \{x \in N / r_A(x) \leq \alpha\}$ and $A^\beta = \{x \in N / w_A(x) \leq \beta\}$ are two sub modules of N.
Proof: It is obivious.

Definition 3.7: Let I be an ideal of N and $\delta_{I\pi}$ be π- fuzzy soft set over N. Then $\delta_{I\pi}$ is called a union complex π – fuzzy soft ideal of N, denoted by $\delta_{I\pi} \sim N$ if
(I_1) $\delta_{I\pi}(x+y) \geq \min\{\delta_{I\pi}(x) , \delta_{I\pi}(y)\}$
(I_2) $\delta_{I\pi}(mx) \geq \delta_{I\pi}(x)$, for all $x,y \in I$ and $m \in M$.

Example 3.8: Let $N = (Z_{12}, + , \bullet)$, $I_1 = \{0,6\}|$ and the soft set δ_{I1} over N, when $\delta : I_1 \rightarrow P(N)$ is a set –valued function defined by $\delta(0) = Z_{12}$ and $\delta(6) = \{1,7\}$. It can be easily given that $\delta_{I1} \sim N$.Let $I_2 = \{0,4,8\}$ and the soft set Δ_{I2} over $\Delta : I_2 \rightarrow P(N)$ is a set-valued function defined by
$\Delta(0) = Z_{12}$, $\Delta(4) = \Delta(8) = \{3,9\}$. It can be easily illustrated that $\Delta_{I2} \sim N$.
However if we define the soft set H_{I2} over N such that the soft set $H : I_2 \rightarrow P(N)$ is a set-valued function defined by $H(0) = Z_{12}$, $H(4) = \{1,2,3\}$ and $H(8) = \{1,2\}$, then $H(5.4) = H(8) = \{1,2\}$ not greater than or equal to $H(4) = \{1,3\}$. It follows that H_{I2} is a union complex π- fuzzy
soft ideal of N.

Theorem-3.9: If $\delta_{I1} \sim N$ and $\Delta_{I2} \sim N$, then $\delta_{I1} \cup \Delta_{I2} \sim N$.
Proof: Since $I_1, I_2 \sim N$, then $I_1 U I_2 \sim N$. By definition 3.7, $\delta_{I1} \cup \Delta_{I2} = (H , I_1 U I_2)$, where
$H(x) = \delta(x) \cup \Delta(x)$ for all $x \in I_1 U I_2 \neq \phi$. Then for all $x,y \in I_1 \cup I_2$ and for all $m \in M$.
$H(x+y) = \min\{\delta(x+y), \Delta(x+y)\}$
$\geq \min\{ \min\{\delta(x), \delta(y)\}, \min\{\Delta(x), \Delta(y)\}\}$
$= \min\{\min\{\delta(x), \Delta(x)\}, \min\{\delta(y), \Delta(y)\}\}$
$= \min\{(\delta U \Delta)(x), (\delta U \Delta)(y)\}$
$= \min\{ H(x), H(y)\}$,
and
$H(mx) = \min\{\delta(mx), \Delta(mx)\}$
$\geq \min\{ \delta(x), \Delta(x)\}$
$= (\delta U \Delta)(x) = H(x)$. Therefore $\delta_{I1} \cup \Delta_{I2} = (H, I_1 U I_2) \sim N$.

Theorem-3.10: If $\delta_{I1} \sim N_1$ and $\Delta_{I2} \sim N_2$, then $\delta_{I1} \times \Delta_{I2} \sim N_1 \times N_2$.
Proof: Since I_1 and I_2 are ideals of N_1 and N_2, respectively, then $I_1 \times I_2$ is an ideal of $N_1 \times N_2$.By definition 3.7, $\delta_{I1} \times \Delta_{I2} = (Q , I_1 \times I_2)$, where $Q(x,y) = \delta_{I1} \times \Delta_{I2}$ for all $(x,y) \in I_1 \times I_2$. Then for all $(x_1, y_1), (x_2, y_2) \in I_1 \times I_2$ and $(m_1, m_2) \in N_1 \times N_2$.
$Q((x_1, y_1) + (x_2, y_2)) = Q(x_1+x_2 , y_1+y_2)$
$= \delta(x_1+x_2) \times \Delta(y_1+y_2)$
$\geq \min\{ \min\{\delta(x_1), \delta(x_2)\}, \min\{\Delta(y_1), \Delta(y_2)\}\}$
$= \min\{ \min\{ \delta(x_1) , \Delta(y_1)\}, \min\{\delta(x_2) , \Delta(y_2)\}\}$
$= \min\{ (\delta\times\Delta)(x_1, y_1) , (\delta\times\Delta)(x_2, y_2) \}$
$= \min\{ Q(x_1,y_1) , Q(x_2, y_2)\}$
$Q((m_1,m_2)(x_1,y_1)) = Q(m_1x_1, m_2y_1)$
$= \delta(m_1x_1) \times \Delta(m_2y_1)$
$\geq \min\{ \delta(x_1) , \Delta(y_1)\} = (\delta\times\Delta)(x_1, y_1) = Q(x_1, y_1)$.
Therefore $\delta_{I1} \times \Delta_{I2} \sim N_1 \times N_2$.

Note 3.11: It is worth noting that if I_1 and I_2 are two ideals of (N, +, •), then the sum of these two ideals is defined by$I_1 + I_2 = \{ i_1 + i_2 \}$, $i_1 \in I_1$ and $i_2 \in I_2$.

Definition 3.12: Let δ_{I1} and Δ_{I2} be two union complex fuzzy soft ideals of N. If $I_1 U\ I_2 = \{0\}$, then the sum of soft ideals δ_{I1} and Δ_{I2} is defined by
$\delta_{I1} + \Delta_{I2} = \{H , I_1 + I_2\}$, where $H(x+y) = \delta(x) + \Delta(y)$ for all $x+y \in I_1 + I_2$.

Theorem-3.13: If $\delta_{I1} \sim N$ and $\Delta_{I2} \sim N$ where $I_1 U\ I_2 = \{0\}$, then $\delta_{I1} + \Delta_{I2} \sim N$.
Proof: Since I_1 and I_2 are ideals of N, then I_1+I_2 is an ideal of N. By definition 3.12, let $\delta_{I1} + \Delta_{I2} = \{H, I_1 + I_2\}$, where $H(x+y) = \delta(x) + \Delta(y)$ for all $x+y \in I_1 + I_2$. It is seen that H is well defined because $I_1 U\ I_2 = \{0\}$. Then for all x_1+y_1 , $x_2+y_2 \in I_1 + I_2$ and $m \in N$.
$H((x_1+y_1) + (x_2+y_2)) = H ((x_1+x_2) + (y_1+y_2))$
$= \delta (x_1+x_2) + \Delta (y_1+y_2)$
$\geq \min \{\min \{\delta(x_1), \delta(x_2)\}, \min \{\Delta(y_1), \Delta(y_2)\}\}$
$= \min \{ \delta(x_1) + \Delta(y_1) , \delta(x_2) + \Delta(y_2)\}$
$= \min \{H(x_1+y_1) ,\ H(x_2+y_2)\}$,
$H(m(x_1+y_1)) = H (mx_1+my_1) = \delta(mx_1)+\Delta(my_1) \geq \delta(x_1) +\Delta(y_1) = H (x_1+y_1)$.
Therefore $\delta_{I1} + \Delta_{I2} \sim N$.
To illustrative the above theorem, we have the following example.

Example 3.14: we take $\delta_{I1} \sim Z_{12}$ and $\Delta_{I2} \sim Z_{12}$ in example 3.8. By definition 3.12, $\delta_{I1} + \Delta_{I2} =$
$\{Q , I_1 + I_2\}$,where $Q(x+y) = \delta(x) + \Delta(y)$ for all $x+y \in I_1 + I_2 = \{0,2,4,6,8,10\}$. It can be easily seen that $Q_{I1+I2} \sim N$.we show the operations for some elements of I_1+I_2,
$Q((6+4) + (6+8)) = Q((6+6) + (4+8)) = Q(0+0) = \delta(0) + \Delta(0) = Q(0) = Z_{12.}$
$Q(3 \bullet (6+4)) = Q(6 +0) = \delta(6) + \Delta(0) = Z_{12}$.

Theorem-3.15: Let N be a sub module and $A = \{(x, \delta_A(x)) / x \in N \}$ be a homogenous complex fuzzy soft set with membership function $\delta_A(x) = r_A(x)\ e^{iwA(x)}$.Then A is a complex fuzzy soft ideal of I if

1. The fuzzy soft set $\bar{A} = \{ (x , r_A(x)) / x \in I, r_A(x) \in [0,1]\}$ is a complex fuzzy soft ideal.
2. The π –fuzzy soft set $\Omega = \{ (x ,w_A(x)) / x \in I, w_A(x) \in [0,2\pi] \}$ is a complex π-fuzzy soft ideal.

Proof: (i)Let A be an complex fuzzy soft sub ideal and x, y $\in$ I. Then we have
$r_A(x+y)\ e^{iwA(x+y)} = \delta_A(x+y)$
$\geq \min \{ \delta_A(x), \delta_A(y)\} = \min \{r_A(x)\ e^{iwA(x)}, r_A(y)\ e^{iwA(y)} \}$
$= \min \{(\delta_A(x), \delta_A(y))\ e^{i\min\{ wA(x) ,wA(y)\}} \}$ (since A is homogenous)
So
$r_A(x+y) \geq \min \{r_A(x), r_A(y)\}$ and
$w_A(x+y) \geq \min \{ w_A(x), w_A(y)\}$.
On the other hand
$r_A(mx)\ e^{iwA(mx)} = \delta_A(mx) \geq \delta_A(x)$.
So $\bar{A}$ is a complex fuzzy soft ideal and A is a complex π – fuzzy soft sub ideal.
(ii) Let $\bar{A}$ be a fuzzy soft ideal and Ω be a complex π- fuzzy soft ideal. Then we have
$r_A(x+y) \geq \min \{ r_A(x), r_A(y)\}$ and $w_A(x+y) \geq \min \{ w_A(x), w_A(y)\}$
$r_A(mx) \geq r_A(x)$ and $w_A(mx) \geq w_A(x)$.
Now
$\delta_A(x+y) = r_A(x+y)\ e^{iwA(x+y)}$
$\geq \min\{(r_A(x), r_A(y))\}\ e^{i\min \{wA(x), wA(y)\}}$
$= \min \{\delta_A(x), \delta_A(y)\}$
On the other hand
$\delta_A(mx) = r_A(mx)\ e^{iwA(mx)} \geq r_A(x)\ e^{iwA(x)} \geq \delta_A(x)$. So A is a complex fuzzy soft ideal.

IV.PROPERTIES OF UNION COMPLEX FUZZY SOFT IDEAL STRUCTURES

Theorem-4.1: Let $\{ A_i / i \in I \}$ be a collection of union complex fuzzy soft ideals of a sub module N such that A_j is homogenous with A_k for all j, k $\in$ I. Then $\cap\ A_i$ is a complex fuzzy soft ideals of N and $i \in I$.
Proof: For all $i \in I$, we have r_{Ai} (x) is a complex fuzzy soft ideals and $w_{Ai}(x)$ is a complex π- fuzzy soft ideals by theorem-3.9.
Now let $x,y \in N$.Then

$\delta_{\cap Ai}(x+y) = r_{\cap Ai}(x+y)\, e^{iwAi(x+y)}$
$= \min \{r_{Ai}(x+y)\, e^{imin\{wAi(x+y)\}}\}$
$\geq \min \{ \min (r_{Ai}(x), r_{Ai}(y))\, e^{imin\{\min\{wAi(x), wAi(y)\}\}}\}$
$= \min \{(\min r_{Ai}(x), \min r_{Ai}(y))\, e^{imin\{\min\{wAi(x), \min\{wAi(y)\}\}}\}$
Since A_j is homogenous with A_k for all $j,k \in I$.
$= \min \{\min r_{Ai}(x)\, e^{imin\{wAi(x)\}}, \min r_{Ai}(y)\, e^{imin\{wAi(y)\}}\}$
$= \min\{\delta_{\cap Ai}(x), \delta_{\cap Ai}(y)\}$,
On the other hand
$\delta_{\cap Ai}(mx) = r_{\cap Ai}(mx)\, e^{iw\cap Ai(mx)}$
$\geq r_{\cap Ai}(x)\, e^{iw\cap Ai(x)}$
$\geq \delta_{\cap Ai}(x)$. The proof is completed.

Theorem 4.2: A complex fuzzy soft set A is a complex fuzzy soft ideal of I if and only if A^C is a complex fuzzy soft sub modules of I.
Proof: Let A be a complex fuzzy soft ideal. Then, we have
$\delta_A{}^C(x+y) = (1- r_A(x+y))\, e^{i(2\pi + wA(x+y))}$
$\geq (1 - \min\{r_A(x), r_A(y)\})\, e^{i(2\pi+\max\{wA(x), wA(y)\})}$
$= \max \{1- r_A(x), 1-r_A(y)\}\, e^{imin\{(2\pi+wA(x)), (2\pi+wA(y))\}}$
$= \min \{(1-r_A(x))\, e^{i(2\pi+wA(x))}, (1- r_A(y))\, e^{i(2\pi+ wA(y))}\}$
$= \max \{ \delta_A{}^C(x), \delta_A{}^C(y)\}$.
On the other hand
$\delta_A{}^C(mx) = (1- r_A(mx))\, e^{i(2\pi+ wA(mx))}$
$\geq (1- r_A(x))\, e^{i(2\pi+ wA(x))}$
$= r_A{}^C(x)\, e^{i(2\pi+ wA(x))} \leq \delta_A{}^C(x)$.
Conversly, let A^C be a complex fuzzy soft ideal. Then, we have
$\delta_A(x+y) = r_A(x+y)\, e^{iwA(x+y)}$
$= (1- (1- r_A(x+y))\, e^{i(2\pi + (2\pi+wA(x+y))}$
$\geq(1 - \min\{(1- r_A(x)),(1- r_A(y))\})\, e^{imin\{2\pi+wA(x), 2\pi+wA(y)\}}$
$= \max \{ r_A(x), (r_A(y)\}\, e^{imax\{wA(x), wA(y)\}}$
$= \max \{r_A(x)\, e^{iwA(x)}, r_A(y)\, e^{iwA(y)}\}$
$= \max \{ \delta_A(x), \delta_A(y)\}$,
On the other hand
$\delta_A(mx) = r_A(mx)\, e^{iwA(mx)} \geq r_A(x)\, e^{iwA(x)} = \delta_A(x)$.

Theorem 4.3: Let A be an complex fuzzy soft ideal of I, if $r_A(e) \geq \alpha$ and $w_A(e) \geq \beta$. Then the level subset $A(\alpha, \beta)$ is a ideal of N.
Proof: $e \in A(\alpha, \beta)$, so $A(\alpha, \beta) \neq \phi$.
Let $x,y \in A(\alpha, \beta)$.Then we have
$r_A(x) \geq \alpha$ and $w_A(x) \geq \beta$, also $r_A(y) \geq \alpha$ and $w_A(y) \geq \beta$.
Now
$r_A(x+y)\, e^{iwA(x+y)} = \delta_A(x+y)$
$\geq \min \{ \delta_A(x), \delta_A(y)\}$
$= \min \{ r_A(x)\, e^{iwA(x)}, r_A(y)\, e^{iwA(y)}\}$
$= \min \{(r_A(x), r_A(y))\}\, e^{imin\{wA(x), wA(y)\}}$
This implies
$r_A(x+y) \geq \min \{r_A(x), r_A(y)\} \geq \min \{\alpha , \alpha \} = \alpha$. and
$w_A(x+y) \geq \min \{w_A(x), w_A(y)\} \geq \min \{\beta , \beta \} = \beta$.
So $x+y \in A(\alpha, \beta)$.
On the other hand we have
$r_A(mx)\, e^{iwA(mx)} = \delta_A(mx) \geq \delta_A(x) \geq \alpha$. So $mx \in (\alpha, \beta)$. Therefore $A(\alpha, \beta)$ is a ideal of N.

Corollary 4.4: Let A be a complex fuzzy soft ideal of N, if $r_A(e) \geq \alpha$ and $w_A(e) \geq \beta$. Then the level subsets $A^\alpha = \{x \in N / r_A(x) \geq \alpha \}$ and $A^\beta = \{x \in N / w_A(x) \geq \beta \}$ are two ideals of N.
Proof: It is obivious.

V. CONCLUSION

Complex intuitionistic fuzzy sets have been applied in multi attribute decision making problems. we introduce the concept of union complex π- fuzzy soft module of a given classical module and investigate some of the crucial properties and characterizations of the proposed concept.

REFERENCES

[1] M.Alsarahead, A.Ahmad, Complex fuzzy subgroups, Applied Mathematical Sciences, Aceepted.
[2] Aktas H. and CagmanN.Soft sets and soft groups. Information Sciences, 1(2010): 2726-2735.
[3] Ali M. I., Feng F., Lui X., Min W. K. and Shabir M. On some new operations in soft set theory. Computers and Mathematics with Applications, 57 (2009): 1547- 1553.
[4] Cagman N, EnginogluS.,Soft set theory and uni-int decision making, Eur. J. Oper. Res.,207(2010), 848-855.
[5] FengF., Jun Y. B. and Zhao X. Soft semirings. Computers and Mathematics with Applications, 56(2008): 2621- 2628.
[6] Kharal A. and Ahmad B . Mappings on soft class. New Mathematics and Natural Computation, 7(3)(2011): 471-481.
[7] Maji P. K., Biswas R. and Roy A. R. Intuitionistic fuzzy soft sets. The Journal of Fuzzy Mathematics, 9(3)(2001): 677-693.
[8] Molodtsov D. Soft set Theory-First results. Computers and Mathematics with Applications, 37(1999): 19-31.
[9] D.Ramot, M.Fried man , G.Longholz and A.Kandel, Complex fuzzy logic, IEEE Transaction on fuzzy systems, 11 (2005), 450-461.
[10] D.Ramot, M.Fried man , G.Longholz and A.Kandel, Complex fuzzy logic, IEEE Transaction on fuzzy systems, 10 (2002), 171- 186.
[11] Sezgin A. and Atagun A.O. On operations of soft sets. Computers and Mathematics with Applications, 60 (2011A): 1840-1849.
[12] Sezgin A. and Atagun A.O. Soft groups and normalistic soft group. Computers and Mathematics with Applications, 62(2011B): 685-698.
[13] L.A.Zadeh, fuzzy sets, Information and control, 8 (1965), 338-353.