Nonperturbative Approach for Dust Acoustic Waves in Plasma with Nonthermal Ions and Trapped Electrons

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Abstract

Dust-ion-acoustic solitary waves in a multi-component unmagnetized dusty plasma containing negatively charged dust particles, nonisothermal electrons and nonthermal ions, have been investigated. The Sagdeev potential approach is applied to study the large amplitude solitary waves. The intermediate integral forms of Korteweg-de Vries (KdV) and modified Korteweg-de Vries (mKdV) equations are derived under different approximations to obtain the solutions of small amplitude solitary waves of different forms. Spiky and Explosive solitary waves as well as double layers are found to exist. The parameters α , β , M , σ , and μ , representing the population of nonthermal ions, ratio of free and trapped electron's temperatures, Mach number, temperature ratio of ion and free electrons, and the density ratio respectively, are found to play a very important role in the formation of solitary waves.

Key words

dusty plasmas, trapped electrons, nonisothermality, nonthermal ion, Sagdeev potential, KdV equation.

I. INTRODUCTION

In the last several decades, nonlinear phenomena in the dusty plasmas have received a great deal of interest due to their occurrence in cometary tails, asteroid zones, planetary rings, intersteller medium, nebulas, earth's ionosphere and magnetosphere, etc.[1−5] Dusty plasma can also be producedin laboratory by Modified Q-machine, de discharges, rf discharges etc.[6] Further, dusty plasma plays an important role in plasma crystals[7], coating and etching of thin film[8] etc. The presence of dust particles in plasma generates many new problems to investigate.[9−11] The solitary waves of dust-ion-acoustic and dust-acoustic modes, arising from the balance between nonlinearity and dispersion, have been extensively studied either by the reductive perturbation technique or the Sagdeev potential approach, or both.[12−22] Barken *et.al.*[23] and Nakamura *et.al.*[24] had observed the dust-ion-acoustic waves in laboratory experiments. Several standard methods such a pseudopotential and reductive perturbation techniques have been adopted to study the properties of waves in plasmas. Washimi and Tanuiti[25] was the first, who derived the nonlinear KdV equation by using reductive perturbation technique to study the plasma acoustic waves. The solution of this KdV equation was obtained in a sec*h* expression, which represents a compressive wave profile. Later on, several methods were developed to solve the KdV equation among which Horota's method, Backlund transformation, Lie group algebra, travelling wave solution method, $\tanh$ method etc. are the main. Sagdeev[26] employed the nonperturbative approach to study the plasma acoustic waves for arbitrary amplitude. He derived an energy integral equation in the form of a pseudopotential to show the motion of the oscillatory charged particles moving in a potential well. Of course, this method was first discussed by Davis. *et.al.*[27] in fluid dynamics. Rao *et. al.*[28] had first theoretically investigated dust acoustic solitary waves through KdV equation by reductive perturbative technique for extremely low phase velocity in unmagnetized dusty plasma with Boltzman ions and electrons. As dust grains are heavier in comparison to ion and electron, so it can come to contact of both, as a result of which dust can be charged both positively and negatively. In most of the investigations in dusty plasma, dust is considered as negatively charged due to the high mobility of electrons, which leads to quick charging due to the contact with the dust particles. Baluku and Hellberg[29] had first theoretically investigated on dust acoustic solitary waves considering negatively charged dust for both large and small amplitude. After that, many researchers have been done on dusty plasma till to date.[30−35] Das and Karmakar[36] have established the existence of modified Korteweg-de Vrise (mKdV) solitons in presence of electron inertia in an interesting situation for different values of electron's drift velocity (v_e') in a dusty plasma. However, these methods are based on certain approximations, as only up to the second order terms are considered, which fails to study the complete nonlinearity of the plasma waves and gives only small amplitude plasma waves.

It has been found that the nonisothermality occurs due to the presence of free and trapped electrons. As the velocity of the trapped electrons is closed to the acoustic speed, so the effect of trapped electrons might not be similar with the free electron. Schamel[37−38] made the first investigation to study the effect of trapped electrons on ion acoustic solitary waves. Nejoh[39] had investigated the effect of dust charging and the influence of the ion density and temperature on electrostatic nonlinear waves in a dusty plasma with trapped electron. Labany[40] had studied the effect of trapped electron temperature, dust charge variations, and grain radius on the nonlinear dust-ion-acoustic waves (DIAWs) in dusty plasmas having trapped electrons. He showed that the amplitude and width of the soliton depend mainly on the trapped electron temperature, dust charge variation, and grain radius. Further, Labany and Taibany[41−42] had investigated the effect of variable dust charge, dust temperature, and trapped electron on small amplitude dust acoustic waves. They found that both compressive and rarefactive solitons as well as double layers exist depending on the nonisothermal parameters. Waleed[43] had investigated three dimensional dust-ion-acoustic solitons in dusty plasma consisting of positive ions, negatively charged dust particles, and two temperature trapped electrons. He showed that only compressive soliton can propagate in this system by obtaining the approximate solution of modified Kadontsev-Pelviashivili equation. He found that the trapped electrons have no effect on the soliton width, but the ratio of free and trapped electron temperatures increases the soliton amplitude. Das *et.al.*[44] made a quasipotential analysis for ion acoustic solitary waves and double layers in plasma with the consideration of trapped electrons. The contribution of nonthermal ion on dust acoustic solitary waves was investigated by Dorranion *et.al.*[45] In this investigation it is reported that the amplitude of the solitary waves decreases with the increased values of the population of nonthermal ion, but the width increases. Adhikary *et.al.*[46] had investigated the effect of nonthermal ion on dusty plasma by deriving KdV and mKdV equations.

In this paper, we have investigated the effect of trapped electrons on dust acoustic solitary waves. Various substantial and characteristic changes on soliton amplitudes and growth are observed in this investigation due to the presence of nonisothermal electrons and nonthermal ions. In this situation we have considered that some of the electrons are free and others are moving in a potential well. The electrons, moving in the potential well, are continuously loosing energy and as such bounce back and forth within the potential well. As a result of which, they become trapped in the potential well.

II. BASIC EQUATIONS

To study the nonlinear dust acoustic wave phenomena, we have considered an unmagetized, collisionless plasma consisting of negatively charged dust grains, nonisothermal electrons and non thermal ions. The basic equations of governing the state of this plasma are as follows,

$$\frac{\partial n_d}{\partial t} + \frac{\partial (n_d v_d)}{\partial x} = 0 \tag{1}$$

$$\frac{\partial v_d}{\partial t} + v_d \frac{\partial v_d}{\partial x} = -\frac{\partial \phi}{\partial x} \tag{2}$$

$$\frac{\partial^2 \phi}{\partial x^2} = n_d + (\mu - 1) n_e - \mu n_i \qquad (3)$$

$$n_e = 1 + \sigma\phi - \frac{4}{3} b (\sigma\phi)^{\frac{3}{2}} + \frac{1}{2} (\sigma\phi)^2 - \frac{8}{15} c (\sigma\phi)^{\frac{5}{2}} + \frac{1}{6} (\sigma\phi)^3 - \ldots\ldots \qquad (4)$$

$$n_i = \left(1 + \beta\phi + \beta(\phi)^2\right) \exp(\phi) \qquad (5)$$

where, $b = \frac{1 - \beta_1}{\sqrt{\pi}}$, $c = \frac{1 - \beta_1^{\,2}}{\sqrt{\pi}}$, $\beta = \frac{4\alpha}{1 + 3\alpha}$, α being the parameter defining the population of non thermal ions. $\mu = \frac{n_{i0}}{z_d n_{d0}}$, $\sigma = \frac{T_i}{T_{ef}}$, T_i is the temperature of ion. The temperatures of the free and trapped electrons are taken as T_{ef} and T_{et} , and their ratio is defined by β_1 , that is $\beta_1 = \frac{T_{ef}}{T_{et}}$.

We express the system of equations (1) – (5) in nondimensional form by normalizing densities n_e, n_i, n_d of electron, ion and dust are normalized by the unperturbed densities n_{e0}, n_{i0}, n_{d0} respectively. The plasma potential ϕ is normalized by $\frac{T_i}{e}$. Dust velocity v_d is normalized by $\left(\frac{z_d T_i}{m_d}\right)^{\frac{1}{2}}$, z_d and m_d being the number of charged dust particle and mass of the dust particle. The space and time variables x and t are normalized by the Debye length $\lambda_D = \left(\frac{T_i}{4\pi e^2 z_d n_{d0}}\right)^{\frac{1}{2}}$ and inverse of the dust plasma frequency

$$\omega_{pd}^{-1} = \left(\frac{m_d}{4\pi e^2 z_d^{\,2} n_{d0}}\right)^{\frac{1}{2}}.$$

The overall charge neutrality condition is given by

$$z_d n_{d0} + n_{e0} - n_{i0} = 0 \qquad (6)$$

III. DERIVATION OF SAGDEEV POTENTIAL EQUATION

To derive the Sagdeev Potential equation from equations (1) - (5) , we use the transformation, $\xi = x - Mt$, where M is the wave Mach number.

Under this transformation, the partial derivative operators with respect to space and time variables are reduced to the ordinary derivative operator as, $\frac{\partial}{\partial x} = \frac{d}{d\xi}$ and $\frac{\partial}{\partial t} = -M \frac{d}{d\xi}$.

Using this in the equation (1) and integrating with respect to ξ with boundary conditions, $n_d \to 1, v_d \to 0$ as $|\xi| \to \infty$, we get

$$v_d = M\left(1 - \frac{1}{n_d}\right).$$

Integrating equation (2) with respect to ξ with boundary conditions, $v_d \to 0, \phi \to 0$ as $|\xi| \to \infty$, and using $v_d = M\left(1 - \frac{1}{n_d}\right)$, the dust density is obtained as,

$$n_d = \left(1 - \frac{2\phi}{M^2}\right)^{-\frac{1}{2}}.$$

Equation (3) can be expressed as,

$$\frac{d}{d\xi}\left[\frac{1}{2}\left(\frac{d\phi}{d\xi}\right)^2\right] = \left\{n_d + (\mu - 1)n_e - \mu n_i\right\}\frac{d\phi}{d\xi}$$

Integrating with respect to ξ with boundary conditions, $\phi \to 0, \frac{d\phi}{d\xi} \to 0$ as $|\xi| \to \infty$, we get

$$\frac{1}{2}\left(\frac{d\phi}{d\xi}\right)^2 + S(\phi) = 0 \qquad (7)$$

where,

$$S(\phi) = M^2\left[\left(1 - \frac{2\phi}{M^2}\right)^{\frac{1}{2}} - 1\right] + \frac{\mu - 1}{\sigma}\left[1 - \exp(\sigma\phi)\right]$$
$$+ \frac{8b(\mu - 1)}{15\sigma}(\sigma\phi)^{\frac{5}{2}} + \frac{16c(\mu - 1)}{105\sigma}(\sigma\phi)^{\frac{7}{2}}$$
$$+ \mu\left[\left(1 + \beta - \beta\phi + \beta\phi^2\right)\exp(\phi) - (1 + \beta)\right] \qquad (8)$$

The equation (7) is an energy equation, showing that the total energy of a classical particle having unit mass is zero, and is called Sagdeev Potential equation. The term $S(\phi)$ is a pseudopotential, called Sagdeev potential, and is given by equation (8).

IV. CONDITIONS OF EXISTENCE OF SOLITARY WAVES

The Sagdeev potential $S(\phi)$ given by (8), is a function of ϕ and depends upon the parameters μ, σ, b, c, β and M, which are already defined.

The conditions for the existence of solitary waves are,

(i) $S(\phi) = 0$ and $S'(\phi) = 0$ at $\phi = 0$

(ii) $S''(\phi) < 0$ at $\phi = 0$

(iii) $S(\phi) = 0$ at $\phi = \phi_m$, where $|\phi_m|$ is the maximum value ϕ.

and, $S(\phi)$ is negative between $\phi = 0$ and $\phi = \phi_m$, so that the solitary wave oscillates between $\phi = 0$ and $\phi = \phi_m$.

Obviously, the condition (i) is satisfied. That is, $S(0) = 0$ and $S'(0) = 0$.

Expanding $S(\phi)$ in Taylor's series about $\phi = 0$,

$$S(\phi) = S(0) + \phi S'(0) + \frac{\phi^2}{2}S''(0) + \ldots\ldots\ldots$$

Because of the condition (ii),

$$M < \frac{1}{\sqrt{\mu(1+2\beta)+(1-\mu)\sigma}}$$

Let, $\phi = \phi_m = \frac{M^2}{2}$, (beyond which n_d is no longer real). Let $M = M_c$ be the critical Mach number, where $S(\phi) = 0$ for $\phi = \phi_m$. That is,

$$S(\phi_m) = 0.$$

where, $S(\phi_m)$ is given by,

$$S(\phi_m) = -M_c^2 + \frac{\mu-1}{\sigma}\left[1-\exp\left(\frac{\sigma M_c^2}{2}\right)\right] + \frac{8b(\mu-1)}{15\sigma}\left(\frac{\sigma M_c^2}{2}\right)^{\frac{5}{2}} + \frac{16c(\mu-1)}{105\sigma}\left(\frac{\sigma M_c^2}{2}\right)^{\frac{7}{2}}$$
$$+\mu\left[\begin{array}{l}\left\{1+\beta-\beta\left(\frac{M_c^2}{2}\right)+\beta\left(\frac{M_c^4}{4}\right)\right\}\exp\left(\frac{M_c^2}{2}\right)\\ -(1+\beta)\end{array}\right] \quad (9)$$

V. SOLUTION FOR SMALL AMPLITUDE SOLITARY WAVE

In this part of the paper, we describe the solution of the equation (7) under different approximations to study the small amplitude solitary waves.

Considering the terms up to the $\left(\frac{5}{2}\right)^{th}$ order, the equation (7) becomes,

$$\left(\frac{d\varphi}{d\xi}\right)^2 = a_1\varphi^2 - a_2\varphi^{\frac{5}{2}} \quad (10)$$

where,

$$a_1 = \mu\sigma - (1+\mu)(\beta-1) - \frac{1}{M^2}$$

$$a_2 = \frac{16b}{15}\mu\sigma^{\frac{3}{2}}$$

The solution of the equation (11) is given by,

$$\phi = \left(\frac{a_1}{a_2}\right)^2 \sec h^4\left(\frac{\sqrt{a_1}}{4}\xi\right) \quad (11)$$

The solution (12) satisfies the mKdV equation,

$$\frac{\partial\phi}{\partial t} + A\phi^{\frac{1}{2}}\frac{\partial\phi}{\partial x} + B\frac{\partial^3\phi}{\partial x^3} = 0 \quad (12)$$

where, $A = \frac{15Ma_2}{8a_1}$ $\quad B = \frac{M}{\sqrt{a_1}}$

Hence, the solution (11) represents a compressive solitary wave profile, whose amplitude is

$$\phi_0 = \left(\frac{a_1}{a_2}\right)^2$$

Again, considering terms up to the 3rd order, the equation (7) can be written as,

$$\left(\frac{d\phi}{d\xi}\right)^2 = a_1\phi^2 - a_2\phi^{\frac{5}{2}} - a_3\phi^3 \tag{13}$$

where,

$$a_1 = \mu\sigma - (1+\mu)(\beta-1) - \frac{1}{M^2}, \quad a_2 = \frac{16b}{15}\mu\sigma^{\frac{3}{2}}, \quad a_3 = \frac{1}{3}\left\{\frac{3}{M^4} - (1+\mu) + \mu\sigma^2\right\}$$

The solution of (13) is given by,

$$\phi = \left[\frac{a_2}{2a_1} + \left(\frac{a_2^2}{4a_1} + \frac{a_3}{a_1}\right)^{\frac{1}{2}} \cosh\left(\frac{\sqrt{a_1}}{2}\xi\right)\right]^{-2} \tag{14}$$

The solution (14) satisfies the equation,

$$\frac{\partial\phi}{\partial t} + \left(\phi + A\phi^{\frac{1}{2}}\right)\frac{\partial\phi}{\partial x} + B\frac{\partial^3\phi}{\partial x^3} = 0 \tag{15}$$

The equation (15) is another form of the KdV equation with having a mixed nonlinearity of KdV and mKdV equations.

Thus, (14) represents a solitary wave profile with maximum amplitude,

$$\phi_{\max} = \frac{a_2}{2a_1} + \left(\frac{a_2^2}{4a_1} + \frac{a_3}{a_1}\right)^{\frac{1}{2}}$$

The solution of the equation (15) had already obtained by Tagare and Chakraborty [47] for amall amplitude ion acoustic solitary wave.

In absence of nonisothermality, $b = 0$, and the solution (14) reduces to,

$$\phi = \frac{a_1}{a_3}\operatorname{sech}\left(\frac{\sqrt{a_1}}{2}\xi\right)$$

which is similar to the well known sech solution of the general KdV equation,

$$\frac{\partial\phi}{\partial t} + A_1\phi\frac{\partial\phi}{\partial x} + A_2\frac{\partial^3\phi}{\partial x^3} = 0 .$$

Thus, under both the above approximations, the presence of the trapped electrons, which leads to the nonisothermality of the plasma, generates the same solitary wave structures in dust acoustic mode, as in the case of isothermality .

For a next better approximation of the solitary wave solution of the equation (7), we consider the terms up to the $\left(\frac{7}{2}\right)^{\text{th}}$ order, so that the equation (7) becomes,

$$\left(\frac{d\phi}{d\xi}\right)^2 = a_1\phi^2 - a_2\phi^{\frac{5}{2}} - a_3\phi^3 - a_4\phi^{\frac{7}{2}} \tag{16}$$

where,

$$a_1 = \mu\sigma - (1+\mu)(\beta - 1) - \frac{1}{M^2}$$

$$a_2 = \frac{16b}{15}\mu\sigma^{\frac{3}{2}}$$

$$a_3 = \frac{1}{3}\left\{\frac{3}{M^4} - (1+\mu) + \mu\sigma^2\right\}$$

$$a_4 = \frac{32c}{105}\mu\sigma^{\frac{5}{2}}$$

Under certain assumptions, the equation (16) can be expressed in a perfect cube as,

$$\left(\frac{d\psi}{d\xi}\right)^2 = \frac{k}{4}\psi^2(\phi_0 - \psi)^3 \tag{17}$$

where,

$\phi(\xi) = \psi^2(\xi), \phi_0 = -\frac{a_3}{3a_4}, k = a_4$ and a_1, a_2, a_3, a_4

must satisfy the relations, $3a_1a_3 = -a_2^2$

and $3a_2a_4 = a_3^2$.

The solution of (17) is given by,

$$\psi = \psi_0 \sec h^2\left[\pm\frac{1}{2}\sqrt{k\phi_0}(\phi_0 - \psi)\xi\right]$$

where $\psi_0 = \sqrt{\phi_0}$ and $0 < \psi(\xi) < \phi_0$

Or equivalently,

$$\phi = \phi_0 \sec h^4\left[\pm\frac{1}{2}\sqrt{k\phi_0}\left(\phi_0 - \sqrt{\phi}\right)\xi\right] \tag{18}$$

where, $0 < \sqrt{\phi} < \phi_0$.

If, $\sqrt{\phi} < 0$, then the above solution becomes,

$$\phi = \phi_0 \cos ech^4\left[\pm\frac{1}{2}\sqrt{k\phi_0}\left(\phi_0 - \sqrt{\phi}\right)\xi\right] \tag{19}$$

The equation (18) and (19), represent two different wave profiles. Since, both the solutions depend upon the parameters involved in a_1, a_2, a_3, a_4, so the solitary wave structures exclusively depends upon the values of the parameters.

The wave profile represented by (18) is called the Spiky solitary wave and the wave profile represented by (19) is called the explosive wave.

Also, for the entire region $0 < \sqrt{\phi} < \phi_0$, the solution (18) can be expressed as,

$$\phi = \phi_0 \tanh^4 \left[\pm \frac{1}{2} \sqrt{k \phi_0} \sqrt{\phi} \left(\phi_0 - \sqrt{\phi} \right)^{\frac{1}{2}} \xi \right] \tag{20}$$

The solution (20) represents a double layer in the region $0 < \sqrt{\phi} < \phi_0$. Hence, under the prescribed values of the parameters as discussed in the first part (for large amplitude solitary wave), there are possibilities of occurrence of spiky and explosive solitary waves, as well as double layer also.

VI. RESULTS AND DISCUSSIONS

To discuss the effects of different parameters on the solitary wave we plot $S(\phi)$ vs. ϕ .

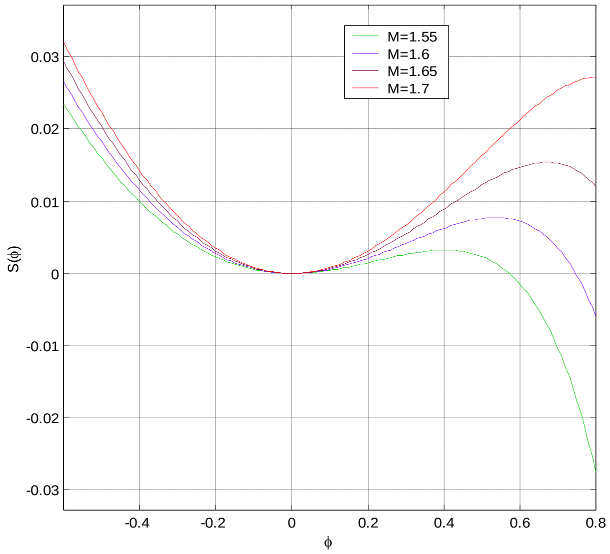

Figure 1(a) $S(\phi)$ vs. ϕ for different values of M $(= 1.55,\ 1.6,\ 1.65,\ 1.7)$ and for fixed values $b = 0.05,\ c = 0.15,\ \mu = 0.01,\ \alpha = 0.15,\ \sigma = 0.51$

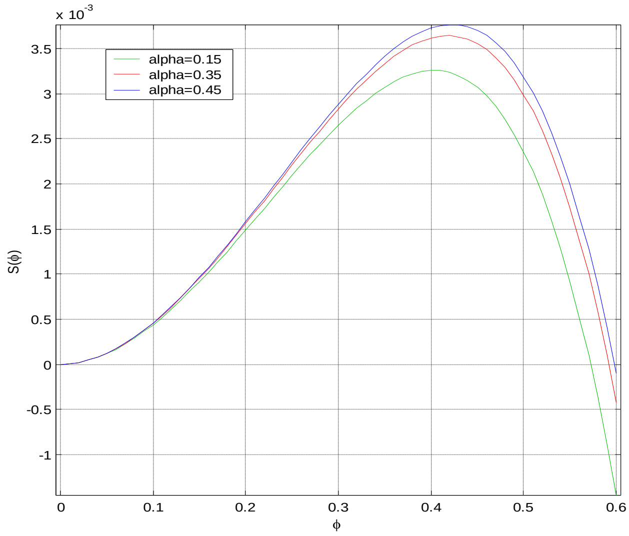

Figure 1(b) $S(\phi)$ vs. ϕ for different values of $\alpha\,(= 0.15,\ 0.35,\ 0.45,\ )$ and for fixed values $b = 0.05,\ c = 0.15,\ \mu = 0.01,\ \sigma = 0.51,\ M = 1.55$

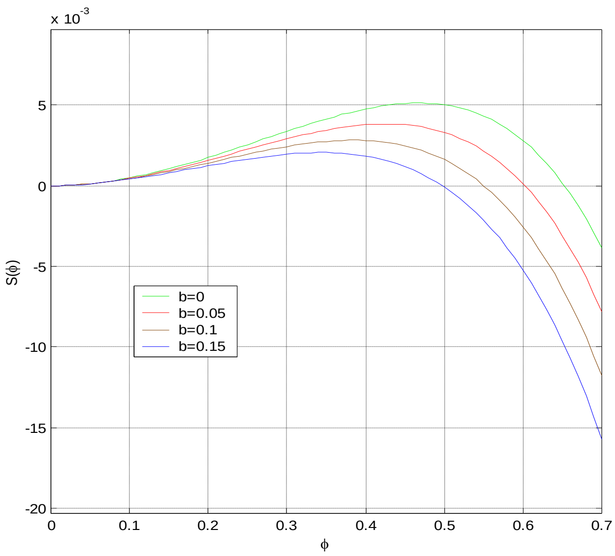

Figure 1(c) $S(\phi)$ vs. ϕ for different values of $b\,(= 0,\ 0.05,\ 0.1,\ 0.15)$ and for fixed values $c = 0.1,\ \mu = 0.01,\ \alpha = 0.45,\ \sigma = 0.51,\ M = 1.55$

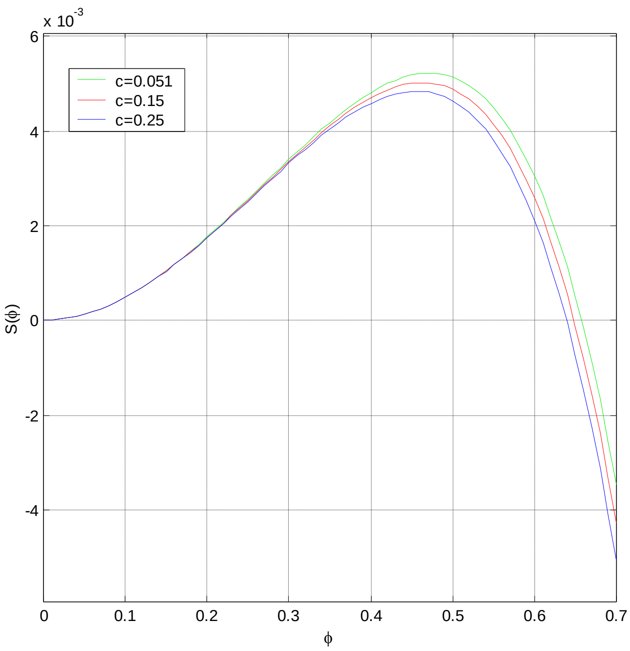

Figure 1(d) $S(\phi)$ vs. ϕ for different values of $c\,(= 0.05,\ 0.15,\ 0.25)$ and for fixed values $b = 0,\ \mu = 0.01,\ \alpha = 0.45,\ \sigma = 0.51,\ M = 1.55$

From figure 1(a) it is seen that as the Mach number increases $S(\phi)$ also increases for fixed values of the parameters. That is, the amplitude of the soliton increases with the increased values of the wave Mach number.

Further, the soliton remains compressive as long as the lowest value of the Mach number is approximately greater than 1.5.

In the figure 2(b) also $S(\phi)$ increases for the increased values of α (population of nonthermal ion) for the fixed values of the parameters. That is, the population of nonthermal ion increases the amplitude of the soliton.

On the other hand, the nonisothermal parameters b and c have played a reverse role in the propagation of solitary waves. In Figure 1(c), $S(\phi)$ decreases as b increases. Similarly in figure 1(d) also, as c increases, $S(\phi)$ decreases. This shows that, as the nonisothermality increases, the amplitude of the soliton decreases. As the nonisothermality occurs due to the presence of trapped electrons, therefore the trapped electrons have played an important role in the propagation of solitary waves.

Thus, the contribution of nonisothermality, nonthermal ion and the Mach number are responsible for generating compressive dust acoustic solitary waves in the present plasma model.

VII. CONCLUSION

The theoretical study of nonlinear plasma waves helps in understanding the astrophysical and geophysical phenomena as well as the satellite observations. Wu *et.al.*[48] have investigated on solitary waves based upon the Freja satellite observations and found the existence of different wave structures in space plasma. Our theoretical results also showed the possibilities of occurance of such waves as well as double layers in space.

By deriving Sagdeev potential equation, we find the compressive dust acoustic solitary waves for large amplitude, which exclusively depend upon the parameters as discussed above. From the Sagdeev potential equation we have derived different intermediate integral forms of KdV and mKdV equations for different approximations. In the solutions (11) and (14), there are only weak non isothermality is involved, and they show the solitary waves of small amplitude and of compressive in nature.

But, the equation (17) contains strong non isothermality, where the effect of the trapped electrons is highly considered. The equation (17) gives three different solutions for different regions. As such, they describe different wave profiles. The $\sec h^4$ and $\cos ech^4$ solutions, given by (18) and (19) give respectively the spiky and explosive solitary waves and the $\tan h^4$ solution given by (20) gives the double layer.

Thus, the present plasma model with negatively charged dust, nonisothermal electrons and nonthermal ions, the nonlinear plasma wave structures in the form of solitary waves are found to exist in plasma as dust acoustic solitary wave, and supports the observations made in space.

As a future scope of the present study, other periodic progressive wave modes apart from the spiky and explosive waves as well as double layers can be investigated, due to the effect of trapped electrons and non thermal ions.

The new finding in this study is that, the higher contribution of nonisothermality arises the possibility of occurrence of different solitary wave structures, which are not found in the observations of Freja satellite. Thus, the experiments on the satellite observations could be motivated by our present work for new findings in space plasma.

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