Applying Floyd's Algorithm for Solving Neutrosophic Shortest Path Problems

1.Dr.V.Jeyanthi, *Associate Professor*
2.Mrs.Radhika V.S, *MPhil Scholar*
Department Of Mathematics,,Sree Narayana Guru College, Coimbatore.

Abstract

Many algorithms have been designed to provide a solution for shortest path problem in a network. The most important algorithms for solving this problems are Dijikstra's algorithm, Floyd's algorithm etc. In this paper Floyd's algorithm has been redesigned to handle the case in which most of the parameters are single valued neutrosophic sets and parameters are uncertain . Floyd's allows the determination of the shortest route between any two nodes in a network.

Key Words : *Floyd's algorithm, single valued neutrosophic number, shortest path problem.*

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I. INTRODUCTION

In order to solve many real life situations which involve uncertainty, incompleteness, vagueness etc **Samarandache [**19,20] proposed the concept of neutrosophic sets and logic. Neutrosophy is a new branch of philosophy that studies the origin,nature and scope of neutralities. The theory of neutrosophic logic can generalize many of the existing logics such as Fuzzy sets[20], Intuitionistic Fuzzy Logics[24] etc. Neutrosophic sets can act as a general frame work for many of the uncertainty analysis in a data set. The main concept of Neutrosophic Logic is to characteristic the logical statement in a 3D space, with each dimension representing the truth (T), the falsehood (F) and the indeterminacy(I).These T,I and F are standard or non-standard subset of $]^{-}0,1^{+}[$.But the neutrosophic set as itself is very difficult to apply in practical problems ,for example real scientific and engineering areas. So Wang et al.[1] proposed the concept of single valued neutrosophic set. Here the classical unit interval [0,1] is used.

For single valued neutrosophic logic, the sum of the components is $0 \leq t+i+f \leq 3$ where all the three components are independent .Single valued neutrosophic sets are easy to express and due to fuzziness of subjective judgment, they are widely used in reality , such as multi-criteria decision making, image processing , medicine etc. Some of the recent works on neutrosophic set theory and its applications can be found in [22].In addition Thamaraiselvi and santhi [21] introduced a mathematical representation of a transportation problem in neutrosophic numbers and also provided the solution method.

The shortest path problem is a problem of finding a shortest path between two vertices, so that the sum of the weight of their corresponding edge is minimized .Edge weight or length can be the quantities from the real life situations such as time, cost etc .Our main aim in the shortest path problem is to minimize time or cost from the starting node to the destination or how to find the shortest path from the starting node to the destination. In conventional shortest path problem we are certain about the weights between different nodes .But in real life situations we are not certain about the quantities used, there always exist uncertainty .Prim's algorithm, Dijkstra's algorithm are different methods to find the shortest distance between nodes. Many algorithms using fuzzy set, intuitionistic fuzzy sets were developed for this [12,21,23]. Floyd's algorithm also helps to find the shortest path from a point in a graph to the destination.

Recently many papers have been published based on neutrosophic graph theory[3,4,13,15,16,17,18].In addition S Broumi Bakali [5,6,7] designed algorithms to find the shortest path of a network, where edge weight are represented using single valued neutrosophic numbers. Dijkstra's algorithm has already been explained using single valued neutrosophic sets.

In this paper a new version of Flody's algorithm is explained for solving shortest path problem on a network where edge weight is characterized by a single valued neutrosophic numbers. In this summing operation and ranking can be done in a very easy form. Details of this paper are explained in the following sections.

SECTION 2: Definitions based on Neutrosophic sets, single valued Neutrosophic sets.
SECTION 3: New version of Floyd's algorithm for solving shortest path problem with connected edges in Neutrosophic data
SECTION 4: Practical example of Floyd's algorithm
SECTION 5: Conclusions

II. PRELIMINARIES

In this section, a review of Neutrosophic sets and single valued neutrosophic sets and explained how to solve Dijkstra's algorithm when the values are single valued neutrosophic numbers are defined .

Definition 2.1: Let K be a set of objects with elements in K denoted by k. The functions T,I,F are the truth-membership function, indeterminacy- membership function and a falsity-membership function respectively of the element kϵK to the set B with the condition $0 \le T_B(x)+I_B(x)+F_B(x) \le 3^+$ where the neutrosophic set B is an object having the form

$$B = \{ <k : T_B(x) ,I_B(x),F_B(x) > k\epsilon K \}.$$

The functions $T_B(x) ,I_B(x),F_B(x)$ are real standard or non standard subsets of $]^-0,1^+[$.

Definition 2.2 : Let k be a set of points with generic elements in K denoted by k .A single valued neutrosophic set B is characteristized by truth membership function $T_B(x)$,an indeterminacy membership function $I_B(x)$ and a falsity –membership function $F_B(x)$ .For each point k in K $T_B(x),I_B(x),F_B(x) \in [0,1]$. A single valued neutrosophic number can be written as

$$B = \{<k : T_B(x) ,I_B(x),F_B(x) > k\epsilon K\}$$

Properties 2.3:

Let $B_1$and B_2 be any two sets with elements $(T_1,I_1,F_1),(T_2,I_2,F_2)$,then

(i) $B_1+B_2 = < T_1+T_2-T_1T_2 , I_1I_2 , F_1F_2 >$
(ii) $B_1*B_2= < T_1T_2 , I_1+I_2-I_1I_2 , F_1+F_2+F_1F_2>$

A convenient method for comparing the single valued neutrosophic number is by use of score function and it is defined as $S(B) = \dfrac{2+T-I-F}{3}$

III. SINGLE VALUED NEUTROSOPHIC FLODY'S ALGORITHM

Floyd's algorithm is more general than Dijkstra's algorithm because it determines the shortest route between any two nodes in the network. Here most of the routes are undirected. In this research the distance or weights are represented by single valued neutrosophic numbers.

The algorithm represents an 'm' node network as a square matrix with 'm' rows and 'm' columns. The distance from x to y is denoted by C_{xy} that is the entry (x ,y) of the matrix (i)if x is directly linked to y the distance is finite; (ii) Otherwise it is infinite;

Given three nodes x, y, z in the figure, we want to find the shortest distance from x to z .Here x is directly connected to z by the distance C_{xy} but while we are calculating we found that it is shorter to reach z from x passing through y, if $\mathbf{C_{xy} + C_{yx} < C_{xz}}$. In this case it is optimal to replace the direct route from $x \rightarrow z$ with the indirect route $x \rightarrow y \rightarrow z$. The triple operation network is applied systematically to the network.

General procedure 3.1:
Step 1 : Define the starting matrix C_0 and node sequence matrix S_0 as given below. The diagonal elements are blocked since no loops are allowed. Diagonal cells are marked with (-) symbol.
Set k = 1
Define the matrices as

C_0	1	2	3	….	y	….	m
1	-	C_{12}	C_{13}	…..	C_{1y}	….	C_{1m}

2	C_{21}	-	C_{23}	…..	C_{2y}	….	C_{2m}
3	C_{31}	C_{32}	-	…..	C_{3y}	….	C_{3m}
. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .
X	C_{x1}	C_{x2}	C_{x3}	…..	-	…..	C_{xm}
. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .
m	C_{m1}	C_{m2}	C_{m3}	…..	C_{my}	….	-

S_0	1	2	3	…..	y	….	m
1	-	2	3	…..	y	….	m
2	1	-	3	…..	Y	….	m
3	1	2	-	…..	Y	….	m
:	:	:	:	-	:	….	:
X	1	2	3	…..	-	….	m
:	:	:	:	…..	:	-	:
m	1	2	3	…..	y	….	-

General step k:

Define row k and column k as pivot row and pivot column and apply the triple operation to each element C_{xy} in C_{k-1} for all x and y .If the condition

$C_{xk} + C_k < C_{xy}$ (x‡ k , y ‡ k , z‡ y) is satisfied then

(a) create C_k by replacing C_{xy} in C_{k-1} with

C_{xk} ‡ C_{ky}

(b) create S_k by replacing S_{xy} in S_{k-1} with k.

(c) set k = k+1 and repeat step K.

Note 3.2: If the sum of the elements on the pivot row and pivot column is smaller than the corresponding intersection element, then it is optimal to replace the intersection element , then it is optimal to replace the intersection distance by the sum of the pivot elements. After these m steps , we can determine the shortest route between x and y using the following rules from the matrices C_m and S_m.

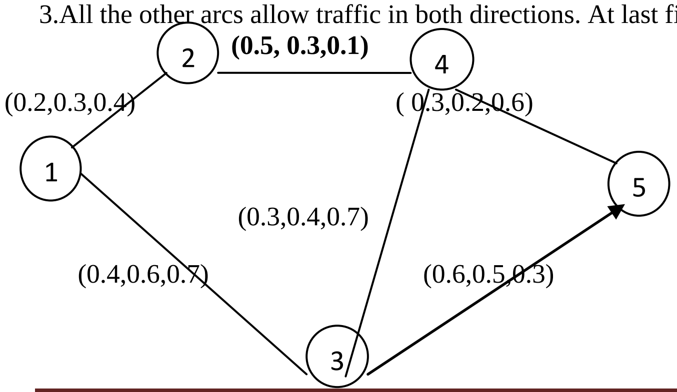
(i) From C_k and C_{xy} we get the shortest distance between nodes x and y.

(ii) From S_n determine the intermediate node k = S_{xy} that yields the route x → k→y.

(iii) If S_{xk} = k and S_{ky} = y then stop the process otherwise repeat the process between nodes x and k and between nodes k and y.

ILLUSTRATIVE EXAMPLE 3.3:

Find the shortest route between every two nodes. The distance are represented by single valued neutrosophic numbers and given on arcs. Arc (3,5) is directional so that no traffic is allowed from node 5 to node 3.All the other arcs allow traffic in both directions. At last find the shortest distance from node 1 to node 5?



Solution:

Step 1 : First let us convert the single valued neutrosophic numbers into a single digit for comparison by using the score function $S(B) = \frac{2 + T - I - F}{3}$

Construct matrices C_0 and S_0 and using the corresponding score function .In the initial matrix C_{14}, C_{15},C_{32},C_{23},C_{51},C_{52},C_{53},C_{25} all these cells have no direct connections ,so we can substitute all these cells as infinity .In the network no traffic is allowed from node 5 to node 3 .

S(0.2,0.3,0.4) = 0.5 S(0.3,0.4,0.7)= 0.4

S(0.4,0.6,0.7)= 0.37 S(0.6,0.5,0.3)=0.6

S(0.5,0.3,0.1) = 0.7 S(0.3,0.2,0.6)= 0.5

C_0	1	2	3	4	5
1	-	0.5	0.37	∞	∞
2	0.5	-	∞	0.7	∞
3	0.37	∞	-	0.4	0.6
4	∞	0.7	0.4	-	0.5
5	∞	∞	∞	0.5	-

S_0	1	2	3	4	5
1	-	2	3	4	5
2	1	-	3	4	5
3	1	2	-	4	5
4	1	2	3	-	5
5	1	2	3	4	-

Step 2 : set k=1 ,in the above matrix the lightly shaded first row and first column represents the pivot row and pivot column .The darker cells C_{23} and C_{32} are the only cells that can be improved by triple operation. Thus C_1 and S_1 are obtained by the operation,

1. Replace C_{23} with $C_{21}+C_{13} = 0.5+0.37 = 0.87$ and set $S_{23} = 1$
2. Replace C_{32} with $C_{31}+C_{12} = 0.37+0.5 = 0.87$ and set $S_{32} = 1$

These changes are shown in the next matrix

C_1	1	2	3	4	5
1	-	0.5	0.37	∞	∞
2	0.5	-	**0.87**	0.7	∞
3	0.37	**0.87**	-	0.4	0.6
4	∞	0.7	0.4	-	0.5
5	∞	∞	∞	0.5	-

S_1	1	2	3	4	5
1	-	2	3	4	5
2	1	-	**1**	4	5
3	1	**1**	-	4	5
4	1	2	3	-	5
5	1	2	3	4	-

Step 3 : Set K=2 ,here the pivot row and pivot column is the second row and second column ,the darker cells C_{41} and C_{14} can be improved by triple operation. The resulting changes are shown in bold C_2 and S_2.

C_2 and S_2 are obtained by

1. Replace C_{14} with $C_{12}+C_{24} = 0.5+0.7 = 1.2$ and set $S_{14} = 2$

C_2	1	2	3	4	5
1	-	0.5	0.37	**1.2**	∞
2	0.5	-	**0.87**	0.7	∞

3	0.37	**0.87**	-	0.4	0.6
4	**1.2**	0.7	0.4	-	0.5
5	∞	∞	∞	0.5	-

2. Replace C_{41} with $C_{42}+C_{24}$ = 0. 7+0.5 = 1.2 and set S_{41} = 2

S_2	1	2	3	4	5
1	-	2	3	**2**	5
2	1	-	**1**	4	5
3	1	**1**	-	4	5
4	**2**	2	3	-	5
5	1	2	3	4	-

Step 4 : Set K=3 ,the pivot row and column are third row and third column . The darker cells C_{15} and C_{25} can be improved by triple operation. Thus C_3 and S_3 are given by the matrices

1. Replace C_{15} with $C_{13}+C_{35}$ = 0.37+0.6 = 0.97 and set S_{15} = 3
2. Replace C_{25} with $C_{23}+C_{35}$ = 0.87+0.6 = 1.47 and set S_{25} = 3

C_3	1	2	3	4	5
1	-	0.5	0.37	**1.2**	**0.97**
2	0.5	-	**0.87**	0.7	**1.47**
3	0.37	**0.87**	-	0.4	0.6
4	**1.2**	0.7	0.4	-	0.5
5	∞	∞	∞	0.5	-

S_3	1	2	3	4	5
1	-	2	3	**2**	**3**
2	1	-	**1**	4	**3**
3	1	**1**	-	4	5
4	**2**	2	3	-	5
5	1	2	3	4	-

Step 5: Set K= 4, here fourth row and column are the pivot row and column. The new matrix C_4 and S_4 are given by replacing

1. Replace C_{15} with $C_{14}+C_{45}$ = 1.2+0.5=1.7 and set S_{15}= 4
2. Replace C_{23} with $C_{24}+C_{43}$ = 0.7+0.4=1.1 and set S_{23}= 4
3. Replace C_{25} with $C_{24}+C_{45}$ =0.7+0.5=1.2 and set S_{25} =4
4. Replace C_{32} with $C_{34}+C_{42}$ = 0.4+0.7=1.1 and set S_{32}= 4
5. Replace C_{51} with $C_{54}+C_{41}$= 0.5+1.2=1.7 and set S_{51}=4
6. Replace C_{52} with $C_{54}+C_{42}$ = 0.5+0.7=1.2 and set S_{52}= 4
7. Replace C_{53} with $C_{54}+C_{43}$= 0.5+0.4=0.9 and set S_{53} = 4
8. Replace C_{35} with $C_{34}+C_{45}$= 0.4+0.5=0.9 and set S_{35} = 4

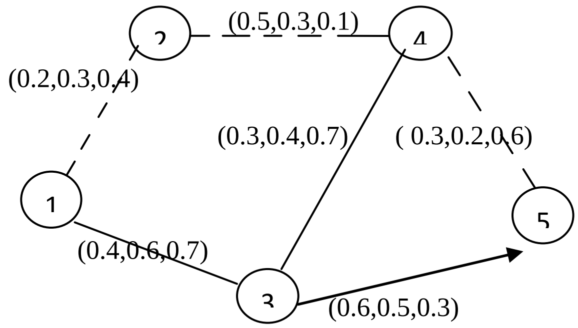
C_4	1	2	3	4	5
1	-	0.5	0.37	**1.2**	**1.7**
2	0.5	-	**1.1**	0.7	**1.1**
3	0.37	**1.1**	-	0.4	**0.9**
4	**1.2**	0.7	0.4	-	0.5
5	**1.7**	**1.2**	**0.9**	0.5	-

S_4	1	2	3	4	5
1	-	2	3	**2**	**4**
2	1	-	**4**	4	**4**
3	1	**4**	-	4	**4**
4	**2**	2	3	-	5
5	**4**	**4**	**4**	4	-

Step 6: Set K=5 as shown by the shaded row and column in C_4. No further improvements are possible. Hence $C_5$and S_5 are same as $C_4$and S_4.

The final matrix contain all the information needed to determine the shortest route from node 1 to node 5 .To determine the shortest route check whether S_{xy} =y or not. Otherwise x and y are linked through at least one other intermediate node .The shortest route is 1 →2→4→5.

That is S_{12}=2, S_{24}=4, S_{45}=5 and the value of the route is 1.7. Therefore the process ends.



IV. CONCLUSIONS

This paper explains how we can extend the Fuzzy Floyd's algorithm to find the shortest route in the algorithm with single valued neutrosophic numbers. In the algorithm the weights or the values of the path are uncertain. Two important issues are explained, one is how to determine the addition of weights of two edges, and the other is how to compare the weights of two different paths. The proposed method can be applied to many real life situations such as traffic control, transportation network etc. In future we can do research in the applications of this algorithm.

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