Certain Fractional Q-Derivative Integral Formulae for the Basic Aleph-Function of Two Variables

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ABSTRACT
In the present paper, we derive two theorem involving fractional q-integral operators of Erdélyi-Kober type and a basic of Aleph-function of two variables. Corresponding assertions for the Riemann-Liouville and Weyl fractional q-integral transforms are also presented. Several special cases of the main results have been illustrated in the concluding section.

1.Introduction.

The fractional q-calculus is the q-extension of the ordinary fractional calculus. The subject deals with the investigations of q-integral and q-derivatives of arbitrary order, and has gained importance due to its various applications in the areas like ordinary fractional calculus, solutions of the q-difference (differential) and q-integral equations, q-transform analysis see ([1] and [10]). Motivated by these avenues of applications, a number of workers have made use of these operators to evaluate fractional q-calculus formulae for various special function, basic analogue of Fox's H-function, general class of q-polynomials etc. One may refer to the recent paper [6]-[7], [13] and [16]-[18] on the subject.

In this paper, we have established two theorems involving the fractional q-integral operator of Erdélyi-Kober type, which generalizes the Riemann-Liouville and Weyl fractional q-integral operators. Several special cases of the main results have been illustrated in the concluding section.

In the theory of q-series, for real or complex a and $|q| < 1$, the q-shifted factorial is defined as :

$$(a;q)_n = \prod_{i=1}^{n-1}(1-aq^i) = \frac{(a;q)_\infty}{(aq^n;q)_\infty} \qquad (n \in \mathbb{N}) \tag{1.1}$$

so that $(a;q)_0 = 1$,

or equivalently

$$(a,q)_n = \frac{\Gamma_q(a+n)(1-q)^n}{\Gamma_q(a)} \qquad (a \neq 0,-1,-2,\cdots). \tag{1.2}$$

The q-analogue of the familiar Riemann-Liouville fractional integral operator of a function $f(x)$ due to Agarwal [2], is given by

$$I_q^\alpha\{f(x)\} = \frac{1}{\Gamma_q(\alpha)}\int_0^x (x-tq)_{\alpha-1}f(t)d_qt \qquad (Re(\alpha)>0, |q|<1). \tag{1.3}$$

Also, the basic analogue of the Kober fractional integral operator, see Agarwal [2] is defined by

$$I_q^{\eta,\alpha}\{f(x)\} = \frac{x^{-\eta-\alpha}}{\Gamma_q(\alpha)}\int_0^x (x-tq)_{\alpha-1}t^\eta f(t)d_qt \quad (Re(\alpha)>0, \eta\in\mathbb{R}, |q|<1). \tag{1.4}$$

a q-analogue of the Weyl fractional integral operator due to Al-Salam [3] is given by

$$K_q^\alpha\{f(x)\} = \frac{q^{\alpha-1)/2}}{\Gamma_q(\alpha)}\int_x^\infty (t-x)_{\alpha-1}f(tq^{1-\alpha})d_qt \ (Re(\alpha)>0, |q|<1). \tag{1.5}$$

In the same paper Al-Salam [3] intoduced the q-analogue of the generalized Weyl fractional integral operator in the following manner

$$K_q^{\eta,\alpha}\{f(x)\} = \frac{q^{-\eta}x^{\eta}}{\Gamma_q(\alpha)}\int_x^{\infty}(t-x)_{\alpha-1}t^{-\eta-\alpha}f(t)d_qt\,(Re(\alpha)>0,\eta\in\mathbb{R},|q|<1). \tag{1.6}$$

Also the basic integral, see Gasper and Rahman [5]) are given by

$$\int_0^x f(t)d_qt = x(1-q)\sum_{k=0}^{\infty}q^kf(xq^k) \tag{1.7}$$

$$\int_x^{\infty} f(t)d_qt = x(1-q)\sum_{k=1}^{\infty}q^{-k}f(zq^{-k}) \tag{1.8}$$

$$\int_0^{\infty} f(t)d_qt = x(1-q)\sum_{k=-\infty}^{\infty}q^kf(zq^k) \tag{1.9}$$

2. Basic of Aleph-function of two variables

Recently Aleph-function of two variables has been introduced and studied by Sharma [15], Kumar Choudary [4] it's an extension of I-function of two variables defined Sharma and Mishra [13] which is a generalization of the H-function of two variables due to Gupta and Mittal. [9,11]. In this paper we introduce a basic of Aleph-function of two variables.

We note

$$G(q^a) = \left[\prod_{n=0}^{\infty}(1-q^{a+n})\right]^{-1} = \frac{1}{(q^a;q)_{\infty}} \tag{2.1}$$

We have

$$\aleph(z_1,z_2;q) = \aleph^{m_1,n_1;m_2,n_2:m_3,n_3}_{P_i,Q_i,\tau_i;r;P_{i'},Q_{i'},\tau_{i'};r';P_{i''},Q_{i''},\tau_{i''};r''}\left(\begin{array}{c} \mathrm{z}_1 \\ \cdot \\ \cdot \\ \mathrm{z}_2 \end{array};q \;\middle|\; \begin{array}{c} (\mathrm{a}_j,\alpha_j,A_j)_{1,n_1},[\tau_j(a_{ji},\alpha_{ji},A_{ji})]_{n_1+1,P_i}; \\ \cdot \\ \cdot \\ [\tau_j(b_{ji},\beta_{ji},B_{ji})]_{1,Q_i}; \end{array}\right.$$

$$\left.\begin{array}{c} (\mathrm{c}_j,\gamma_j)_{1n_2},[\tau_j(c_{ji'},\gamma_{ji'})]_{n_2+1;P_{i'}};(e_j,E_j)_{1n_2},[\tau_j(e_{ji''},\gamma_{ji''})]_{n_3+1;P_{i''}} \\ \cdot \\ \cdot \\ (\mathrm{d}_j,\delta_j)_{1m_2},[\tau_j(d_{ji'},\delta_{ji'})]_{m_2+1;_{i''}};(f_j,F_j)_{1m_3},[\tau_j(f_{ji''},F_{ji''})]_{m_3+1;Q_{i''}} \end{array}\right) =$$

$$\frac{1}{(2\pi\omega)^2}\int_{L_1}\int_{L_2}\pi^2\phi(s,t;q)\theta_1(s;q)\theta_2(t;q)x^sy^td_qsd_qt, \tag{2.2}$$

where

$$\phi(s,t;q) = \frac{\prod_{j=1}^{n_1}G(q^{1-a_j+\alpha_js+A_jt})}{\sum_{i=1}^{r}\tau_i\left[\prod_{j=1}^{Q_i}G(q^{1-b_{ji}+\beta_{ji}s+B_{ji}t})\prod_{n_1+1}^{P_i}G(q^{a_{ji}-\alpha_{ji}s-A_{ji}t})\right]} \tag{2.3}$$

$$\theta_1(s;q) = \frac{\prod_{j=1}^{m_2}G(q^{d_j-\delta_js})\prod_{j=1}^{n_2}G(q^{1-c_j+\gamma_js})}{\sum_{i'=1}^{r''}\tau_{i'}\left[\prod_{j=m_2+1}^{Q_{i'}}G(q^{1-d_{ji'}+\delta_{ji'}s})\prod_{j=n_2+1}^{P_{i'}}G(q^{c_{ji'}-\gamma_{ji'}s})\right]G(q^{1-s})\sin\pi s} \tag{2.4}$$

$$\theta_2(t;q) = \frac{\prod_{j=1}^{m_3}G(q^{f_j-F_jt})\prod_{j=1}^{n_3}G(q^{1-e_j+E_jt})}{\sum_{i''=1}^{r''}\tau_{i''}\left[\prod_{j=m_3+1}^{Q_{i''}}G(q^{1-f_{ji''}+F_{ji''}t})\prod_{j=n_3+1}^{P_{i''}}G(q^{e_{ji''}-E_{ji''}t})\right]G(q^{1-t})\sin\pi t} \tag{2.5}$$

where x and y (real or complex) are not equal to zero and an empty product is interpreted as unity and the quantities $P_i, P_{i'}, P_{i''}, Q_i, Q_{i'}, Q_{i''}, m_1, m_2, m_3, n_1, n_2, n_3$ are non-negative integers such that

$Q_i > 0, Q_{i'} > 0, Q_{i''} > 0; \tau_i, \tau_{i'}, \tau_{i''} > 0 (i = 1, \cdots, r), (i' = 1, \cdots, r'), (i'' = 1, \cdots, r'')$.
All the $A's, \alpha's, \gamma's, \delta's, E's$ and $F's$ are assumed to be positive quantities for standardization purpose ; the definition of basic Aleph-function of two variables given above will however, have a meaning even if some of these quantities are zero. The contour L_1 is in the s-plane and runs from $-\omega\infty$ to $+\omega\infty$ with loops, if necessary, to ensure that the poles of , $G(q^{d_j-\delta_j s})(j = 1, \cdots, m_2)$ are to the right and all the poles of $G(q^{1-a_j+\alpha_j s+A_j t})(j = 1, \cdots, n_1)$, $G(q^{1-c_j+\gamma s})$ $(j = 1, \cdots, n_2)$ lie to the left of L_1. The contour L_2 is in the t-plane and runs from $-\omega\infty$ to $+\omega\infty$ with loops, if necessary, to ensure that the poles of $G(q^{f_j-F_j t})(j = 1, \cdots, m_3)$ are to the right and all the poles of $G(q^{1-a_j+\alpha_j s+A_j t})(j = 1, \cdots, n_1)$, $G(q^{1-e_j+E_j t})(j = 1, \cdots, n_2)$ lie to the left of L_2. For large values of $|s|$ and $|t|$ the integrals converge if $Re(slog(z_1) - log \sin \pi s) < 0$ and $Re(tlog(z_2) - log \sin \pi t) < 0$
The poles of the integrand are assumed to be simple.

we shall note

$$\mathrm{A}_1 = (a_j, \alpha_j, A_j)_{1,n_1}, [\tau_j(a_{ji}, \alpha_{ji}, A_{ji})]_{n_1+1,P_i}; \mathrm{B}_1 = [\tau_j(b_{ji}, \beta_{ji}, B_{ji})]_{1,Q_i} \tag{2.6}$$

$$A_2 = (c_j, \gamma_j)_{1n_2}, [\tau_j(c_{ji'}, \gamma_{ji'})]_{n_2+1;P_{i'}}; (e_j, E_j)_{1n_2}, [\tau_j(e_{ji''}, \gamma_{ji''})]_{n_3+1;P_{i''}} \tag{2.7}$$

$$\mathrm{B}_2 = (d_j, \delta_j)_{1m_2}, [\tau_j(d_{ji'}, \delta_{ji'})]_{m_2+1;_{i''}}; (f_j, F_j)_{1m_3}, [\tau_j(f_{ji''}, F_{ji''})]_{m_3+1;Q_{i''}} \tag{2.8}$$

3. Main formulae

In this section, we will establish two fractional q-integral formulae about the basic aleph-function of two variables.

Theorem 1.

Let $Re(\mu) > 0, |q| < 1, \eta \in \mathbb{R}$ and $I_q^{\eta,\alpha}\{.\}$ be the Kober fractional q-integral operator (1.4), then the following result holds :

$$I_q^{\eta,\mu}\left\{x^{\lambda-1}\aleph^{m_1,n_1;m_2,n_2:m_3,n_3}_{P_i,Q_i,\tau_i;r;P_{i'},Q_{i'},\tau_{i'};r';P_{i''},Q_{i''},\tau_{i''};r''}\left(\begin{array}{c} \mathrm{z}_1 x^\rho \\ \cdot \\ \cdot \\ \mathrm{z}_2 x^\sigma \end{array} ; \mathrm{q} \left| \begin{array}{c} \mathrm{A}_1; A_2 \\ \cdot \\ \cdot \\ \mathrm{B}_1; B_2 \end{array}\right.\right)\right\} = (1-q)^\mu x^{\lambda-1}$$

$$\aleph^{m_1,n_1+1;m_2,n_2:m_3,n_3}_{P_i+1,Q_i+1,\tau_i;r;P_{i'},Q_{i'},\tau_{i'};r';P_{i''},Q_{i''},\tau_{i''};r''}\left(\begin{array}{c} \mathrm{z}_1 x^\sigma \\ \cdot \\ \cdot \\ \mathrm{z}_2 x^\sigma \end{array} ; \mathrm{q} \left| \begin{array}{c} (1\text{-}\lambda-\eta; \rho, \sigma), A_1; A_2 \\ \cdot \\ \cdot \\ \mathrm{B}_1, (1-\lambda-\mu-\eta; \rho, \sigma); \mathrm{B}_2 \end{array}\right.\right) \tag{3.1}$$

where $\rho, \sigma \in \mathbb{N}$, $Re(slog(z_1) - log \sin \pi s) < 0$ and $Re(tlog(z_2) - log \sin \pi t) < 0$.

Proof
To prove the above theorem, we consider the left hand side of equation (3.1) (say I) and make use of the definitions (1.4) and (2.2), we obtain

$$I = \frac{x^{-\eta-\alpha}}{\Gamma_q(\alpha)} \int_0^x (x-yq)_{\alpha-1} y^\eta \frac{y^{\lambda-1}}{(2\pi\omega)^2} \int_{L_1}\int_{L_2} \pi^2 \phi(s,t;q)\theta_1(s;q)\theta_2(t;q) x^s y^t d_q s d_q t \, d_q y$$

interchanging the order of integrations which is justified under the conditions mentioned above, we obtain

$$I = \frac{x^{-\eta-\alpha}}{\Gamma_q(\alpha)(2\pi\omega)^2} \int_{L_1}\int_{L_2} \pi^2 \phi(s,t;q)\theta_1(s;q)\theta_2(t;q) x^s y^t \int_0^x (x-yq)_{\alpha-1} y^\eta \{y^{\rho s+\sigma t+\lambda-1}\} d_q y d_q s d_q t$$

The above equation writes

$$I=\frac{x^{-\eta-\alpha}}{(2\pi\omega)^2}\int_{L_1}\int_{L_2}\pi^2\phi(s,t;q)\theta_1(s;q)\theta_2(s;q)x^sy^tI_q^{\eta,\mu}\{x^{\rho s+\sigma t+\lambda-1}\}d_qsd_qt$$

Now using the result due to Yadav and Purohit ([16], p. 440, eq. (19))

$$I_q^{\eta,\mu}\{x^{\lambda-1}\}=\frac{\Gamma_q(\lambda+\eta)}{\Gamma_q(\lambda+\eta+\mu)}x^{\lambda-1},(Re(\lambda+\mu)>0). \tag{3.2}$$

Subsituting (3.2) in the above equation, we obtain

$$I=\frac{x^{-\eta-\alpha}}{\Gamma_q(\alpha)(2\pi\omega)^2}\int_{L_1}\int_{L_2}\pi^2\phi(s,t;q)\theta_1(s;q)\theta_2(t;q)x^sy^t\frac{\Gamma_q(\rho s+\sigma t+\lambda+\eta)}{\Gamma_q(\rho s+\sigma t+\lambda+\eta+\mu)}x^{\rho s+\sigma t+\lambda-1}d_qsd_qt$$

Now, interpreting the q-Mellin-Barnes double integrals contour in terms of the basic Aleph-function of two variables, we get the desired result (3.1).

Theorem 2.

If $Re(\mu)>0,|q|<1,\eta\in\mathbb{R}$, then the generalized Weyl q-integral operator for the basic aleph-function of two variables is given by

$$K_q^{\eta,\mu}\{x^{\lambda-1}\aleph^{m_1,n_1;m_2,n_2:m_3,n_3}_{P_i,Q_i,\tau_i;r;P_{i'},Q_{i'},\tau_{i'};r';P_{i''},Q_{i''},\tau_{i''};r''}\left(\begin{array}{c}z_1x^{-\rho}\\ \cdot\\ \cdot\\ z_2x^{-\sigma}\end{array};q\left|\begin{array}{c}A_1;A_2\\ \cdot\\ \cdot\\ B_1;B_2\end{array}\right.\right)\Bigg\}=(1-q)^{\mu}x^{\lambda-1}q^{-\mu\lambda}$$

$$\aleph^{m_1,n_1+1;m_2,n_2:m_3,n_3}_{P_i+1,Q_i+1,\tau_i;r;P_{i'},Q_{i'},\tau_{i'};r';P_{i''},Q_{i''},\tau_{i''};r''}\left(\begin{array}{c}z_1(xq^{-\mu})^{\rho}\\ \cdot\\ \cdot\\ z_2(xq^{-\mu})^{\sigma}\end{array};q\left|\begin{array}{c}(1+\lambda-\eta;\rho,\sigma),A_1;A_2\\ \cdot\\ \cdot\\ B_1,(1+\lambda-\mu-\eta;\rho,\sigma);B_2\end{array}\right.\right) \tag{3.3}$$

where $\rho,\sigma\in\mathbb{N}$, $Re(slog(z_1)-log\sin\pi s)<0$ and $Re(tlog(z_2)-log\sin\pi t)<0$.

Proof

Using the definition of the generalized Weyl fractional q-integral operator (1.6) in the left hand side of (3.3), writing the function in the form by (2.2), interchanging the order of integrations which is justified under the conditions mentioned above, using the result (3.2) and interpreting the q-Mellin-Barnes double integrals contour in terms of the basic Aleph-function of two variables, we get the desired result (3.3).

4. Special cases.

In this section, we shall see several corollaries.

Corollary 1.

Let $Re(\mu)>0,|q|<1,\eta\in\mathbb{R}$ and $I_q^{\mu}\{.\}$ be the Riemann-Liouville fractional q-integral operator (1.3), then the following result holds :

$$I_q^{\mu}\{x^{\lambda-1}\aleph^{m_1,n_1;m_2,n_2:m_3,n_3}_{P_i,Q_i,\tau_i;r;P_{i'},Q_{i'},\tau_{i'};r';P_{i''},Q_{i''},\tau_{i''};r''}\left(\begin{array}{c}z_1x^{\rho}\\ \cdot\\ \cdot\\ z_2x^{\sigma}\end{array};q\left|\begin{array}{c}A_1;A_2\\ \cdot\\ \cdot\\ B_1;B_2\end{array}\right.\right)\Bigg\}=(1-q)^{\mu}x^{\lambda+\mu-1}$$

$$\aleph^{m_1,n_1+1;m_2,n_2:m_3,n_3}_{P_i+1,Q_i+1,\tau_i;r;P_{i'},Q_{i'},\tau_{i'};r';P_{i''},Q_{i''},\tau_{i''};r''}\left(\begin{array}{c|c} \mathrm{z}_1 x^{\sigma} & (1\text{-}\lambda;\rho,\sigma),A_1;A_2 \\ \cdot & \cdot \\ \cdot \quad ;\mathrm{q} & \cdot \\ \mathrm{z}_2 x^{\sigma} & \mathrm{B}_1,(1-\lambda-\mu;\rho,\sigma);B_2 \end{array}\right) \tag{3.4}$$

where $\rho,\sigma \in \mathbb{N}$, $Re(slog(z_1) - log\sin\pi s) < 0$ and $Re(tlog(z_2) - log\sin\pi t) < 0$.

Corollary 2

$$K_q^{\mu}\{x^{\lambda-1}\aleph^{m_1,n_1;m_2,n_2:m_3,n_3}_{P_i,Q_i,\tau_i;r;P_{i'},Q_{i'},\tau_{i'};r';P_{i''},Q_{i''},\tau_{i''};r''}\left(\begin{array}{c|c} \mathrm{z}_1 x^{-\rho} & \mathrm{A}_1;A_2 \\ \cdot & \cdot \\ \cdot \quad ;\mathrm{q} & \cdot \\ \mathrm{z}_2 x^{-\sigma} & \mathrm{B}_1;B_2 \end{array}\right)\Bigg\} = (1-q)^{\mu}x^{\lambda+\mu}q^{-\mu\lambda-\mu(\mu+1)/2}$$

$$\aleph^{m_1,n_1+1;m_2,n_2:m_3,n_3}_{P_i+1,Q_i+1,\tau_i;r;P_{i'},Q_{i'},\tau_{i'};r';P_{i''},Q_{i''},\tau_{i''};r''}\left(\begin{array}{c|c} \mathrm{z}_1 (xq^{-\mu})^{\rho} & (1+\lambda+\mu;\rho,\sigma),A_1;A_2 \\ \cdot & \cdot \\ \cdot \quad ;\mathrm{q} & \cdot \\ \mathrm{z}_2 (xq^{-\mu})^{\sigma} & \mathrm{B}_1,(1+\lambda;\rho,\sigma);B_2 \end{array}\right) \tag{3.5}$$

where $\rho,\sigma \in \mathbb{N}$, $Re(slog(z_1) - log\sin\pi s) < 0$ and $Re(tlog(z_2) - log\sin\pi t) < 0$.

Now $\tau_i,\tau_{i'},\tau_{i''}(i = 1,\cdots,r),(i' = 1,\cdots,r'),(i'' = 1,\cdots,r'') \to 0$, the basic aleph-function of two variables reduces in basic I-function of two variables (basic analogue of I-function of two variables defined by Sharma and mishra [14]).

We consider the theorem 1 and the above condition, we get

Corollary 3.

Let $Re(\mu) > 0, |q| < 1, \eta \in \mathbb{R}$ and $I_q^{\eta,\alpha}\{.\}$ be the Kober fractional q-integral operator (1.4), then the following result holds :

$$I_q^{\eta,\mu}\{x^{\lambda-1}I^{m_1,n_1;m_2,n_2:m_3,n_3}_{P_i,Q_i;r;P_{i'},Q_{i'};r';P_{i''},Q_{i''};r''}\left(\begin{array}{c|c} \mathrm{z}_1 x^{\rho} & \mathrm{A}_1;A_2 \\ \cdot & \cdot \\ \cdot \quad ;\mathrm{q} & \cdot \\ \mathrm{z}_2 x^{\sigma} & \mathrm{B}_1;B_2 \end{array}\right)\Bigg\} = (1-q)^{\mu}x^{\lambda-1}$$

$$I^{m_1,n_1+1;m_2,n_2:m_3,n_3}_{P_i+1,Q_i+1;r;P_{i'},Q_{i'};r';P_{i''},Q_{i''};r''}\left(\begin{array}{c|c} \mathrm{z}_1 x^{\sigma} & (1\text{-}\lambda-\eta;\rho,\sigma),A_1;A_2 \\ \cdot & \cdot \\ \cdot \quad ;\mathrm{q} & \cdot \\ \mathrm{z}_2 x^{\sigma} & \mathrm{B}_1,(1-\lambda-\mu-\eta;\rho,\sigma);B_2 \end{array}\right) \tag{4.1}$$

where $\rho,\sigma \in \mathbb{N}$, $Re(slog(z_1) - log\sin\pi s) < 0$ and $Re(tlog(z_2) - log\sin\pi t) < 0$.

We consider the theorem 2 and the above condition, we get

Corollary 4

If $Re(\mu) > 0, |q| < 1, \eta \in \mathbb{R}$, then the generalized Weyl q-integral operator for the basic aleph-function of two variables is given by

$$K_q^{\eta,\mu}\{x^{\lambda-1}I^{m_1,n_1;m_2,n_2:m_3,n_3}_{P_i,Q_i;r;P_{i'},Q_{i'};r';P_{i''},Q_{i''};r''}\left(\begin{array}{c|c} \mathrm{z}_1 x^{-\rho} & \mathrm{A}_1;A_2 \\ \cdot & \cdot \\ \cdot \quad ;\mathrm{q} & \cdot \\ \mathrm{z}_2 x^{-\sigma} & \mathrm{B}_1;B_2 \end{array}\right)\Bigg\} = (1-q)^{\mu}x^{\lambda-1}q^{-\mu\lambda}$$

$$I^{m_1,n_1+1;m_2,n_2:m_3,n_3}_{P_i+1,Q_i+1;r;P_{i'},Q_{i'};r';P_{i''},Q_{i''};r''}\left(\begin{array}{c} z_1(xq^{-\mu})^{\rho} \\ \cdot \\ \cdot \\ z_2(xq^{-\mu})^{\sigma} \end{array} ; q \;\middle|\; \begin{array}{c} (1+\lambda-\eta;\rho,\sigma), A_1; A_2 \\ \cdot \\ \cdot \\ B_1, (1+\lambda-\mu-\eta;\rho,\sigma); B_2 \end{array}\right) \qquad (4.2)$$

where $\rho,\sigma \in \mathbb{N}$, $Re(slog(z_1) - log\sin\pi s) < 0$ and $Re(tlog(z_2) - log\sin\pi t) < 0$.

If $r = r' = r'' = 1$, the basic I-function of two variables reduces in H-function of two variables defined by Saxena et al. [12], we obtain the same results, see Yadav et al. [19] for more details.

5. Conclusion.

The importance of our all the results lies in their manifold generality. By specialising the various parameters as well as variables in the basic Aleph-function of two variables, we obtain a large number of results involving remarkably wide variety of useful basic functions (or product of such basic functions) which are expressible in terms of basic H-function [13], Basic Meijer's G-function, Basic E-function, basic hypergeometric function of one and two variables and simpler special basic functions of one and two variables. Hence the formulae derived in this paper are most general in character and may prove to be useful in several interesting cases appearing in literature of Pure and Applied Mathematics and Mathematical Physics.

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