Analytical Discussion for Existence of Idea of Calculus in Vedic and Post Vedic Periods

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Abstract

It has been tried to find the idea of existence of calculus in Vedic and Post Vedic period. From sulva sutra of Vedic period, it is seen that Vedic Hindus had knowledge of differentiation and integration. Works of different Mathematicians of post Vedic period have been studied and ultimately, we find that calculus had existence in this period.

Keywords: *Calculus, Vedic, Post Vedic, Manjul, Bhaskaracharya.*

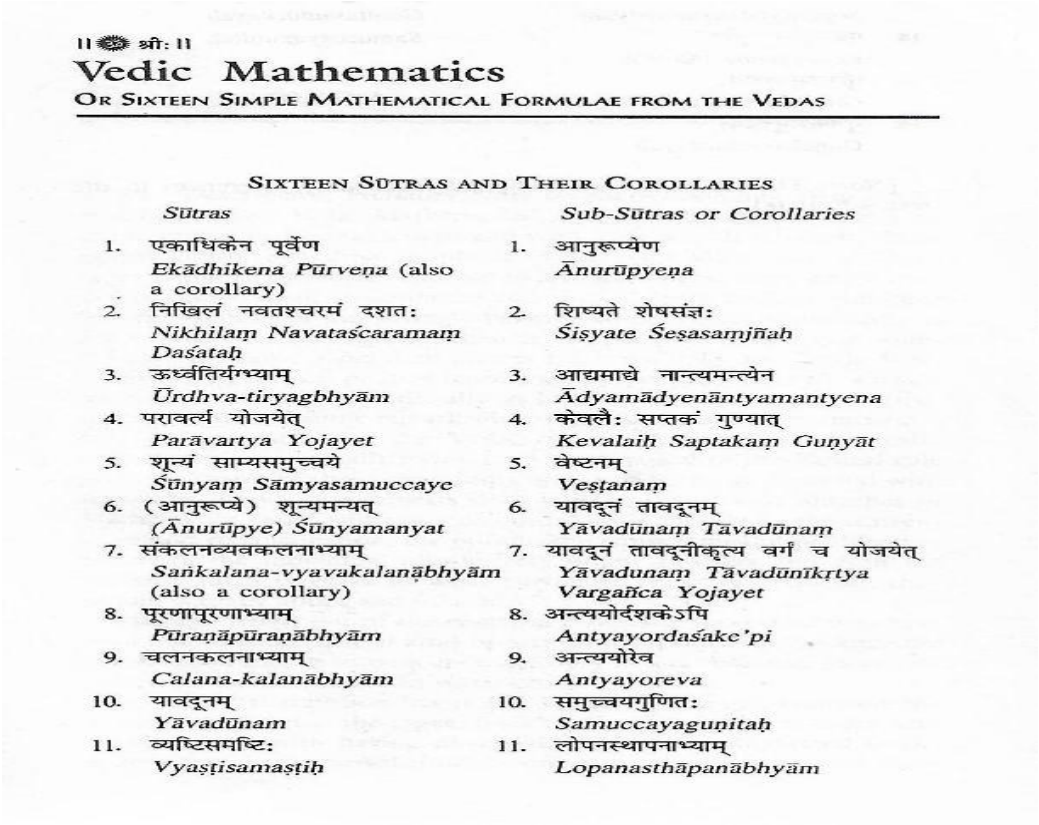
Mathematics Subject Classification: 01A32, 97A30, 97A80

I. INTRODUCTION

Modern Differential Calculus is based on the concept of infinitesimal variation of variables and Integral Calculus is based on summation of infinitesimal variation of variables. Vedic Hindus had knowledge of Astronomy in which application of these concepts were essential. During Post Vedic period, many Indian Scholar worked remarkable works in the field of Mathematics and particularly in Calculus. We shall discuss analytically of their works and try to find the existence of idea of calculus in Vedic and Post Vedic periods.

II. EXISTENCE OF CALCULUS IN VEDIC PERIOD

By eight years labour , Patience and day-night research Jagatguru Swami Sri Bharati Krsna Tirthaji Maharaja of Govardhana Maṭha, Puri, India re-established the "Sutra's" (aphorisms) of Veda. He declared that sutras may be applied to different branches of mathematics. From his book "VEDIC MATHEMATICS" edited by Dr. V. S. Agrawala, we find following sixteen Mathematical formulae of the Vedas.



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Vedic Mathematics

OR SIXTEEN SIMPLE MATHEMATICAL FORMULAE FROM THE VEDAS

SIXTEEN SŪTRAS AND THEIR COROLLARIES

	Sūtras		*Sub-Sūtras or Corollaries*
1.	एकाधिकेन पूर्वेण *Ekādhikena Pūrveṇa* (also a corollary)	1.	आनुरूप्येण *Ānurūpyeṇa*
2.	निखिलं नवतश्चरमं दशतः *Nikhilaṃ Navataścaramaṃ Daśataḥ*	2.	शिष्यते शेषसंज्ञः *Śiṣyate Śeṣasaṃjñaḥ*
3.	ऊर्ध्वतिर्यग्भ्याम् *Ūrdhva-tiryagbhyām*	3.	आद्यमाद्ये नान्त्यमन्त्येन *Ādyamādyenāntyamantyena*
4.	परावर्त्य योजयेत् *Parāvartya Yojayet*	4.	केवलैः सप्तकं गुण्यात् *Kevalaiḥ Saptakaṃ Guṇyāt*
5.	शून्यं साम्यसमुच्चये *Śūnyaṃ Sāmyasamuccaye*	5.	वेष्टनम् *Veṣṭanaṃ*
6.	(आनुरूप्ये) शून्यमन्यत् *(Ānurūpye) Śūnyamanyat*	6.	यावदूनं तावदूनम् *Yāvadūnaṃ Tāvadūnaṃ*
7.	संकलनव्यवकलनाभ्याम् *Saṅkalana-vyavakalanābhyām* (also a corollary)	7.	यावदूनं तावदूनीकृत्य वर्गं च योजयेत् *Yāvadunaṃ Tāvadūnīkṛtya Vargañca Yojayet*
8.	पूरणापूरणाभ्याम् *Pūraṇāpūraṇābhyām*	8.	अन्त्ययोर्दशकेऽपि *Antyayordaśake'pi*
9.	चलनकलनाभ्याम् *Calana-kalanābhyām*	9.	अन्त्ययोरेव *Antyayoreva*
10.	यावदूनम् *Yāvadūnam*	10.	समुच्चयगुणितः *Samuccayaguṇitaḥ*
11.	व्यष्टिसमष्टिः *Vyaṣṭisamaṣṭiḥ*	11.	लोपनस्थापनाभ्याम् *Lopanasthāpanābhyām*

12. शेषाण्यङ्केन चरमेण
Śeṣāṇyaṅkena Carameṇa
13. सोपान्त्यद्वयमन्त्यम्
Sopāntyadvayamantyam
14. एकन्यूनेन पूर्वेण
Ekanyūnena Pūrveṇa
15. गुणितसमुच्चयः
Guṇitasamuccayaḥ
16. गुणकसमुच्चयः
Guṇakasamuccayaḥ

12. विलोकनम्
Vilokanam
13. गुणितसमुच्चयः समुच्चयगुणितः
Gunitasamuccayaḥ
Samuccayaguṇitaḥ

[*Note*: This list has been compiled from stray references in the text.—EDITOR]

Here, we see that the formula- 9 "Calana-Kalanabhyam" which is the Calculus of Vedic Period.

(I) Differential Calculus:

From Vedic Mathematical formula "Gunaka-Samuccaya" we get that the first differential of product of two binomials (x + a) and (x + b) is equal to the sum of these two binomials.

Example.1: $x^2 + 3x + 2 = (x + 1)(x + 2)$, then according to the formula "Gunaka-Samuccaya"

D_1 (The first differential) = First Factor + Second Factor = $(x + 1) + (x + 2)$

i.e., $D_1 = a + b = \Sigma a$, where $a = (x + 1)$, $b = (x + 2)$

Verification by modern Newtonian Calculus:

If $y = uv$, where u and v are functions of x then according to Newtonian Calculus-

$$\frac{dy}{dx} = v\frac{du}{dx} + u\frac{dv}{dx}$$

$$\therefore \frac{d(x^2 + 3x + 2)}{dx} = (x + 2)\frac{d(x + 1)}{dx} + (x + 1)\frac{d(x + 2)}{dx}$$

$$= (x + 2) + (x + 1) = (x + 1) + (x + 2)$$

Example:- (2) $(x^3 + 6x^2 + 11x + 6) = (x + 1)(x + 2)(x + 3)$

Then, $D_1 = ab + bc + ac = \Sigma ab$, where $a = (x + 1)$, $b = (x + 2)$, $c = (x + 3)$

Verification by modern Newtonian Calculus:

$$D_1 = 3x^2 + 12x + 11 = (x^2 + 3x + 2) + (x^2 + 5x + 13) + (x^2 + 4x + 3)$$
$$= (x + 1)(x + 2) + (x + 1)(x + 2) + (x + 1)(x + 2)$$
$$= ab + bc + ac$$
$$= \Sigma ab$$

(II) Integral Calculus:

The Vedic Mathematical formula "Ekadhika Sutra" gives the direction for Integration. For integration of a power of x, it says to add unity to the "purva", i.e., add unity to the original index and divide the coefficient by the new index.

Example: Integrate $28x^3$

Solution:

$$Integration = \frac{28}{4}x^{3+1} = 7x^4$$

Here, original index is 3 and by "Ekadhika Sutra" we shall have to add 1 with original 3 i.e., 3+1 will be new power of x and also to divide the new index 4.

III. CALCULUS IN POST VEDIC PERIOD

Manjul, an eminent scholar of Astronomy was born in 932 A.D. His famous contribution in Astronomy was Precession of Equinoxes. Before him no one could find such type of accurate calculation. In his book "Laghumanas", he has considered a function whose form in modern Mathematics is of type $u = v \pm e \sin A$ and then he showed that the infinitesimal increment of this function is $du = dv \pm e (\cos A)\, dA$.
Bhaskaracharya or Bhskar- II was born in 1114 A.D at Bizapur in South India. From his result we get that when x approaches to y then Siny – Sinx approaches to (y - x) Cosy which shows the derivative of sine in modern form as $d(\sin\theta) = \cos\theta\, d\theta$. Bhaskara-II used this formula to find the position angle of ecliptic which is useful for the prediction result of an eclipse.

He wrote the famous book "Sidhanta Siromoni" in 1150 A.D. In "Sidhanta Siromoni" he mentioned that
(1) During the case of maximum value, the differential of the function vanishes.
(2) If a function vanishes at two points then at some intermediate point of differential vanishes.
No.1 is the concept of Maxima and Minima of modern Calculus and No.2 brings the concept of Rolle's theorem of modern Calculus. For this he showed that when a planet is at maximum distance or minimum distance the equation of centre vanishes. So, it is concluded that for some intermediate position the differential of the equation of the centre is equal to zero. Hence, the flavour of the general mean value theorem comes out and which is derived from Rolle's Theorem.

During the process of finding area and volume of sphere Bhaskara-II used the integration by subdividing the sphere into a number of small divisions.

He has given the instantaneous motion of a planet. The term "instantaneous motion" for a moving body is the seed idea of derivative of the displacement with respect to time at a certain instant. It reflects the flavour of Calculus.

These concepts have been included in modern mathematics.

IV. FINDINGS AND CONCLUSION

we see that during the process of factorization and by using Vedic Mathematical formula "Gunaka-Samuccaya" the people used to apply the idea of differentiation which reflects that people of Vedic period had the idea of differentiation of a variable i.e., idea of differential Calculus. As the Mathematicians of western countries give the credit of discovery of Differential Calculus to Newton and Leibniz but Vedic Hindus had this knowledge in another form.

Also, we find that Vedic Hindus had the knowledge of Integration. They used to apply Vedic Mathematical formula "Ekadhika Sutra" in the case of integration.

Hence, we may draw a conclusion that the ancient Indian Mathematicians had the concept of Calculus.
In Post Vedic period, Manjul and Bhaskaracharya used the idea of derivative in trigonometric functions during the study of subject Astronomy. During the period of finding the position of Celestial objects, the small variation of position of objects had to be calculated. In such case, the need of concept of calculus was felt and consequently derivative and anti derivative were used automatically i.e., the concept of calculus was used.
Manjul's discovery, $u = v \pm e \sin A$ implies $du = dv \pm e (\cos A)\, dA$ and Bhaskaracharya's discovery $d(\sin\theta) = \cos\theta\, d\theta$ show that Indians had the idea differentials much more before Newton and Leibniz.
Also, Bhaskaracharya's Concept of the condition for Maxima and his concept of modern Rolle's theorem concludes that he had deep knowledge of differentiation. His concept of integration by collecting of sum of infinitesimal areas of volumes describes that he had the deep knowledge of integration.
By the words of Sanchayan Sarkar (2017), a research fellow of University of Pittsburgh, we would like to draw the conclusion in much better way.

"one can do calculus using Vedic maths but it will not follow the usual Greek Model of Science. Calculus was present much earlier in the Vedic Texts , but the expressions will be different. However if we want to obtain the area of an object using Calculus we will get definite integral results using Vedic maths. We won't be able to verify the Vedic intermediate steps using Greek Model of Science but any result bound operation is possible.

It's a fallacy that Newton invented Calculus . Calculus was there in many cultures. Not only that I think the maths was far more developed in the ancient world than one might think and that is reflected with the perfect precision on architecture like in the Temples of Egypt or the Kailasha Temple. One cannot achieve that level of precision if they are not advanced in the field of mathematics. And not just Calculus, forms of trigonometry, parrallel lines theorem, most of them were there. It's another think that the modern model of Science is expanded from the Greek Science . Had it been done on the Vedic Science we could have established various things as Ramanujan started to do. But Indians are such sell-outs that they will accept anything western without even looking into it's own culture. The idea that Vedic Maths is limited just to petty multiplications and divisions is the most preposterous thing. We may look at the precise circles of Mahabalipuram. Such reflection is only possible if one knows the value of PI. PI was thought to be an European invention until the golden ratio was found in both the Pyramids and in the structures of Ancient India. Also remember the modern science evolved after the DARK AGE OF EUROPE when the rest of the world was already enlightened. The renessaince began using Greek revival knowledge and that's why the scientists of that time were termed heretics. That's why we say classical buildings coming up during that period. But such periods already existed elsewhere in the world. The west believes the quadratic formula was invented by Europeans when Shreedarcharya invented it way before."

Hence, we may conclude that Indians had the knowledge of Calculus much more before western countries. The dark period of Indians started just after Bhaskaracharya due to the invasion of foreigners. Their main attitude was to destroy the knowledge and culture of golden past of Indians and then rule over India.

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