# New Bilateral Generating Function Pertaining to I-Function 

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ABSTRACT
In this paper we derive the bilateral generating relations, pertaining to the product of Saxena's I-function and the multivariable Gimel function. Further, some interesting special remarks and particular cases are given.

Keywords:I-function,H-function, multivariable Gimel-function, contour integral, bilateral generating function, multivariable H -function.
2010 Mathematics Subject Classification :33C05, 33C60

## 1. Introduction.

Let $\Delta(\zeta, \eta)$ and $\nabla(\zeta, \eta)$ stands for the $\zeta$-parameter sequence $\frac{\eta}{\zeta}, \frac{\eta+1}{\zeta}, \cdots, \frac{\eta+\zeta+1}{\zeta}$ and $1-\frac{\eta}{\zeta}, 1-\frac{\eta+1}{\zeta}, \cdots, 1-\frac{\eta+\zeta+1}{\zeta}$ respectively for an arbitrary complex number and for all integer $\zeta \geq 1$.

The I- function , introduced by Saxena [8], however the notation and complete definition is presented here in the following manner in terms of the Mellin-Barnes type integral :
$I(z)=I_{P_{i}, Q_{i} ; r^{\prime}}^{M, N}\left(\mathrm{z} \left\lvert\, \begin{array}{c|c}\left(\mathrm{a}_{j}, A_{j}\right)_{1, N},\left[\left(a_{j i}, A_{j i}\right)\right]_{N+1, P_{i} ; r^{\prime}} \\ \left(\mathrm{b}_{j}, B_{j}\right)_{1, M},\left[\left(b_{j i}, B_{j i}\right)\right]_{M+1, Q_{i} ; r^{\prime}}\end{array}\right.\right)=\frac{1}{2 \pi \omega} \int_{L} \Omega_{P_{i}, Q_{i} ; r^{\prime}}^{M, N}(s) z^{s} \mathrm{~d} s$
for all $z$ different to 0 and

$$
\begin{equation*}
\Omega_{P_{i}, Q_{i} ; r^{\prime}}^{M, N}(s)=\frac{\prod_{j=1}^{M} \Gamma\left(b_{j}-B_{j} s\right) \prod_{j=1}^{N} \Gamma\left(1-a_{j}+A_{j} s\right)}{\sum_{i=1}^{r^{\prime}} \prod_{j=N+1}^{P_{i}} \Gamma\left(a_{j i}-A_{j i} s\right) \prod_{j=M+1}^{Q_{i}} \Gamma\left(1-b_{j i}+B_{j i} s\right)} \tag{1.2}
\end{equation*}
$$

With $|\arg z|<\frac{1}{2} \pi \Omega \quad$ where $\Omega=\sum_{j=1}^{M} B_{j}+\sum_{j=1}^{N} A_{j}-\left(\sum_{j=N+1}^{P_{i}} A_{j i}+\sum_{j=M+1}^{Q_{i}} B_{j i}\right)>0, i=1, \cdots, r^{\prime}$
$M, N, P_{i}, Q_{i}$ are positive integers and verify $0 \leq N \leq P_{i}, 0 \leq M \leq Q_{i} . A_{j}, B_{j}, A_{j i}, B_{j i}$ are positive reals and $a_{j}, b_{j}$, $a_{j i}, b_{j i}$ are complex numbers. This function is more general than the H-function defined by Fox [5].

We have a generalized transcendental function called Gimel function of several complex variables.
$\beth\left(z_{1}, \cdots, z_{r}\right)=\beth_{X ; p_{i}, q_{i_{r}}, \tau_{i_{r}}: R_{r}: Y}^{U ; ;, n_{r}: V}\left(\begin{array}{c|c}\mathrm{z}_{1} & \mathbb{A} ; \mathbf{A}: \mathrm{A} \\ \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ \mathrm{z}_{r} & \mathbb{B} ; \mathbf{B}: \mathrm{B}\end{array}\right)=\frac{1}{(2 \pi \omega)^{r}} \int_{L_{1}} \cdots \int_{L_{r}} \psi\left(s_{1}, \cdots, s_{r}\right) \prod_{k=1}^{r} \theta_{k}\left(s_{k}\right) z_{k}^{s_{k}} \mathrm{~d} s_{1} \cdots \mathrm{~d} s_{r}(1.3)$
with $\omega=\sqrt{-1}$
The following quantities $\mathbb{A}, \mathbf{A}, A, \mathbb{B}, \mathbf{B}, B, X, Y, U, V, \psi\left(s_{1}, \cdots, s_{r}\right)$ and $\theta_{k}\left(s_{k}\right)(k=1, \cdots, r)$ are defined by Ayant [2] for more details.
2. Required result.

## Lemma.

We have the following formula
$\sum_{\eta=0}^{\infty} \frac{(\sigma)_{\eta}}{\eta!} I_{P_{i}+\zeta, Q_{i}+\zeta ; r^{\prime}}^{M, N+\zeta}\left(\mathrm{z} \left\lvert\, \begin{array}{c}\left(\mathrm{a}_{j}, A_{j}\right)_{1, N},(\nabla(\zeta,-\eta), 1)_{1, \zeta},\left[\left(a_{j i}, A_{j i}\right)\right]_{N+1, P_{i} ; r^{\prime}} \\ \left(\mathrm{b}_{j}, B_{j}\right)_{1, Q_{i}},(\nabla(\zeta,-\eta), 0)_{1, \zeta}\end{array}\right.\right) t^{\eta}$
$=(1-t)^{-\sigma} I_{P_{i}+\zeta, Q_{i}+\zeta ; r^{\prime}}^{M, N+\zeta}\left(\mathrm{z}\left(\frac{t}{t-1}\right)^{\zeta} \left\lvert\, \begin{array}{c}\left(\mathrm{a}_{j}, A_{j}\right)_{1, N},(\nabla(\zeta, \sigma), 1)_{1, \zeta},\left[\left(a_{j i}, A_{j i}\right)\right]_{N+1, P_{i} ; r^{\prime}} \\ \left(\mathrm{b}_{j}, B_{j}\right)_{1, Q_{i}},(\nabla(\zeta, \sigma), 0)_{1, \zeta}\end{array}\right.\right)$

To prove the above formula, we replace the I-function by it Mellin Barnes integral contour, we use the bilateral generating relation and then interpret the resulting Mellin-Barnes integral contour to get the required result.

## 3. Main result.

In this section, we see the bilateral generating function formula pertaining to the product of Saxena's I-function and the multivariable Gimel function. We shall note
$\mathbf{A}_{1}=\left[\left(\mathrm{a}_{r j} ; \alpha_{r j}^{(1)}, \cdots, \alpha_{r j}^{(r)}, 0 ; A_{r j}\right)_{1, n_{r}}\right],\left[\tau_{i_{r}}\left(a_{r j i_{r}} ; \alpha_{r j i_{r}}^{(1)}, \cdots, \alpha_{r j i_{r}}^{(r)}, 0 ; A_{r j i_{r}}\right)_{\mathfrak{n}+1, p_{i_{r}}}\right]$
$\mathbf{B}_{1}=\left[\tau_{i_{r}}\left(b_{r j i_{r}} ; \beta_{r j i_{r}}^{(1)}, \cdots, \beta_{r j i_{r}}^{(r)}, 0 ; B_{r j i_{r}}\right)_{1, q_{i_{r}}}\right]$

About the following quantities $\mathbb{A}, A, \mathbb{B}$ and $B$, see Ayant [2] for more informations. We have the first formula

## Theorem

$\left.\sum_{\eta=0}^{\infty} \frac{(\sigma)_{\eta}}{\eta!} I_{P_{i}+\zeta, Q_{i}+\zeta ; r^{\prime}}^{M, N+\zeta} \mathrm{z} \left\lvert\, \begin{array}{c}\left(\mathrm{a}_{j}, A_{j}\right)_{1, N},(\nabla(\zeta,-\eta), 1)_{1, \zeta},\left[\left(a_{j i}, A_{j i}\right)\right]_{N+1, P_{i} ; r^{\prime}} \\ \left(\mathrm{b}_{j}, B_{j}\right)_{1, Q},(\nabla(\zeta,-\eta), 0)_{1, \zeta}\end{array}\right.\right)$
$\mathcal{I}_{X ; p_{i_{r}}+\zeta, q_{i_{r}}, \tau_{i_{r}}: R_{r}: Y}^{U ; 0, n_{r}+\zeta: V}\left(\begin{array}{c|c}\mathrm{z}_{1} & \mathbb{A} ;(\nabla(\zeta, \sigma+\eta), 1, \cdots, 1 ; 1)_{1, \zeta}, \mathbf{A}: A \\ \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ \mathrm{z}_{r} & \mathbb{B} ; \mathbf{B}: \mathrm{B}\end{array}\right) \frac{z^{\eta}}{\eta!}=\left(1-\frac{z}{\zeta}\right)^{-\sigma}$

provided that
$\left|\arg z_{i}\right|<\frac{1}{2} A_{i}^{(k)} \pi, A_{i}^{(k)}$ is defined by Ayant [2], $\sigma \in \mathbb{C}, \zeta \in \mathbb{N}, \zeta \geq 1$ and $|\arg z|<\frac{1}{2} \pi \Omega$ where
$\Omega=\sum_{j=1}^{M} B_{j}+\sum_{j=1}^{N} A_{j}-\left(\sum_{j=N+1}^{P_{i}} A_{j i}+\sum_{j=M+1}^{Q_{i}} B_{j i}\right)>0, i=1, \cdots, r^{\prime}$.
Proof
To prove the theorem, express the multivariable Gimel-function occurring left-hand side of (3.5) with the help of (1.3) and then interchange the order of summation and integration we find (say J)
$J=\frac{1}{(2 \pi \omega)^{r}} \int_{L_{1}} \cdots \int_{L_{r}} \psi\left(s_{1}, \cdots, s_{r}\right) \prod_{k=1}^{r} \theta_{k}\left(s_{k}\right) z_{k}^{s_{k}} \prod_{j=1}^{\zeta} \Gamma\left(\Delta(\zeta, \sigma)+\sum_{i=1}^{r} s_{i}\right)$
$\sum_{\eta=0}^{\infty} \frac{\left(\sigma+\zeta \sum_{i=1}^{r} s_{i}\right)_{\eta}}{\eta!}\left(\frac{z}{\zeta}\right)^{\eta} I_{P_{i}+\zeta, Q_{i}+\zeta ; r^{\prime}}^{M, N+\zeta}\left(\mathrm{z} \left\lvert\, \begin{array}{c}\left(\mathrm{a}_{j}, A_{j}\right)_{1, N},(\nabla(\zeta,-\eta), 1)_{1, \zeta},\left[\left(a_{j i}, A_{j i}\right)\right]_{N+1, P_{i} ; r^{\prime}} \\ \left(\mathrm{b}_{j}, B_{j}\right)_{1, Q},(\nabla(\zeta,-\eta), 0)_{1, \zeta}\end{array}\right.\right)$
$z_{1}^{s_{1}} \cdots z_{r}^{s_{r}} \mathrm{~d} s_{1} \cdots \mathrm{~d} s_{r}$
Now, we use the lemma. The new bilateral generating function pertaining to I-function and then replace the I-function of one variable by its Mellin-Barnes integral contour given by (1.1) and interpret the resulting multiple Mellin-Barnes integrals contour in terms of multiple Gimel-function with the help of (1.3). After several algebraic manipulations, we obtain the desired result.

## 4. Particular cases.

In this section, we shall see only one case. The I-function reduces to H -function, we obtained

## Corollary . 1

$\sum_{\eta=0}^{\infty} \frac{(\sigma)_{\eta}}{\eta!} H_{P+\zeta, Q+\zeta}^{M, N+\zeta}\left(\mathrm{z} \left\lvert\, \begin{array}{c}\left(\mathrm{a}_{j}, A_{j}\right)_{1, N},(\nabla(\zeta,-\eta), 1)_{1, \zeta},\left[\left(a_{j}, A_{j}\right)\right]_{N+1, P} \\ \left(\mathrm{~b}_{j}, B_{j}\right)_{1, Q},(\nabla(\zeta,-\eta), 0)_{1, \zeta}\end{array}\right.\right)$


provided that
$\left|\arg z_{i}\right|<\frac{1}{2} A_{i}^{(k)} \pi, \quad \sigma \in \mathbb{C}, \zeta \in \mathbb{N}, \zeta \geq 1$ and $|\arg z|<\frac{1}{2} \pi \Omega \quad$ where
$\Omega=\sum_{j=1}^{M} B_{j}+\sum_{j=1}^{N} A_{j}-\left(\sum_{j=N+1}^{P} A_{j}+\sum_{j=M+1}^{Q} B_{j}\right)>0$.
Now the multivariable Gimel-function reduces to multivariable H-function defined by Srivastava and Panda [9,10], we have

Corollary 2. ( see Chaurasia and Meghwal [4])
$\left.\sum_{\eta=0}^{\infty} \frac{(\sigma)_{\eta}}{\eta!} I_{P_{i}+\zeta, Q_{i}+\zeta ; r^{\prime}}^{M, N+\zeta} \mathrm{z}^{\mathrm{z}} \begin{array}{c|c}\left(\mathrm{a}_{j}, A_{j}\right)_{1, N},(\nabla(\zeta,-\eta), 1)_{1, \zeta},\left[\left(a_{j i}, A_{j i}\right)\right]_{N+1, P_{i} ; r^{\prime}} \\ \left(\mathrm{b}_{j}, B_{j}\right)_{1, Q}, \dot{(\nabla(\zeta,-\eta), 0)_{1, \zeta}}\end{array}\right)$
$H_{p_{r}+\zeta, q_{r}: Y}^{0, n_{r}+\zeta: V}\left(\begin{array}{c|c}\mathrm{z}_{1} & (\nabla(\zeta, \sigma+\eta), 1, \cdots, 1)_{1, \zeta}, A_{1}: A_{1}^{\prime} \\ \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \mathrm{B}_{1}: B_{1}^{\prime}\end{array}\right) \frac{z^{\eta}}{\eta!}=\left(1-\frac{z}{\zeta}\right)^{-\sigma}$

$\left|\arg z_{i}\right|<\frac{1}{2} A_{i}^{(k)} \pi, A_{i}^{(k)}$ is defined by Srivastava and Panda [9,10], $\sigma \in \mathbb{C}, \zeta \in \mathbb{N}, \zeta \geq 1$ and $|\arg z|<\frac{1}{2} \pi \Omega$ where $\Omega=\sum_{j=1}^{M} B_{j}+\sum_{j=1}^{N} A_{j}-\left(\sum_{j=N+1}^{P_{i}} A_{j i}+\sum_{j=M+1}^{Q_{i}} B_{j i}\right)>0 . i=1, \cdots, r^{\prime}$

The quantities $A_{1}, A_{1}^{\prime}, B_{1}, B_{1}^{\prime}$ are defined by Srivastava and Panda [9,10], for more details, see Chaurasia and Meghwal [4]. Now the I-function reduces in Fox's H-function, we get

Corollary 3. ( see Chaurasia and Meghwal [4])
$\sum_{\eta=0}^{\infty} \frac{(\sigma)_{\eta}}{\eta!} H_{P+\zeta, Q+\zeta}^{M, N+\zeta}\left(\mathrm{z} \left\lvert\, \begin{array}{c}\left(\mathrm{a}_{j}, A_{j}\right)_{1, N},(\nabla(\zeta,-\eta), 1)_{1, \zeta},\left[\left(a_{j}, A_{j}\right)\right]_{N+1, P} \\ \left(\mathrm{~b}_{j}, B_{j}\right)_{1, Q}, \dot{(\nabla(\zeta,-\eta), 0)_{1, \zeta}}\end{array}\right.\right)$
$H_{p_{r}+\zeta, q_{r}: Y}^{0, n_{r}+\zeta V}\left(\begin{array}{c|c}\mathrm{z}_{1} \\ \cdot & (\nabla(\zeta, \sigma+\eta), 1, \cdots, 1)_{1, \zeta}, A_{1} A_{1}^{\prime} \\ \cdot & \vdots \\ \mathrm{z}_{r} & \mathrm{~B}_{1} \vdots B_{1}^{\prime}\end{array}\right) \frac{z^{\eta}}{\eta!}=\left(1-\frac{z}{\zeta}\right)^{-\sigma}$
$\left.H_{p_{r}+\zeta, q_{r}: Y ; P, Q}^{0, n_{r}+\zeta: V ; M, N}\left(\left.\begin{array}{c}\mathrm{z}_{1}\left(1-\frac{z}{\zeta}\right)^{-\zeta} \\ \cdot \\ \cdot \\ \cdot \\ \mathrm{z}_{r}\left(1-\frac{z}{\zeta}\right)^{-\zeta} \\ \mathrm{z}\left(1-\frac{z}{\zeta}\right)^{-\zeta}\end{array} \right\rvert\, \nabla(\zeta, \sigma+\eta), 1, \cdots, 1\right)_{1, \zeta}, A_{1}: A_{1}^{\prime} ;\left(a_{j}, A_{j}\right)_{1, P}\right) \cdot$.
provided
$\left|\arg z_{i}\right|<\frac{1}{2} A_{i}^{(k)} \pi, A_{i}^{(k)}$ is defined by Srivastava and Panda [9,10], $\sigma \in \mathbb{C}, \zeta \in \mathbb{N}, \zeta \geq 1$ and $|\arg z|<\frac{1}{2} \pi \Omega \quad$ where
$\Omega=\sum_{j=1}^{M} B_{j}+\sum_{j=1}^{N} A_{j}-\left(\sum_{j=N+1}^{P} A_{j}+\sum_{j=M+1}^{Q} B_{j}\right)>0$
We have used the same notation that the above corollary. For more informations, see Chaurasia and Kumawat [3].

## Remarks

We obtain the same bilateral generating function concerning the multivariable Aleph- function defined by Ayant [1], the multivariable I-function defined by Prathima et al. [7], the multivariable I-function defined by Prasad [6] and the multivariable H-function defined by Srivastava and Panda [9,10], see Chaurasia and Kumawat [3] for more details about the multivariable H -function.

## 4. Conclusion.

Firstly, the I-function presented in this document is quite nature. Therefore, on specializing the parameters of this function, we obtain various other results as its special cases. Secondly, by specializing the various parameters as well as variables in the generalized multivariable Gimel-function, we obtain a large number of bilateral generating functions involving remarkably wide variety of useful functions ( or product of such functions) which are expressible in terms of E, F, G, H, I, Aleph-function of one and several variables and simpler special functions of one and several variables. Hence the formulae derived in this paper are most general in character and may prove to be useful in several interesting cases appearing in literature of Pure and Applied Mathematics and Mathematical Physics.

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