A Study of Unified Fractional Integral Operators Involving S-Generalized Gauss's Hypergeometric, Fox's H-Function and Gimel Function

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Abstract
In this paper, we study a pair of a general class of fractional integral operators whose kernel involves the product of a Appell polynomial, Fox's H-function and S-generalized Gauss's hypergeometric function. We have given several images about the multivariable Gimel-function and generalized incomplete hypergeometric function.

Keywords : Unified fractional integral operators, Appell polynomial, generalized incomplete hypergeometric function, Gimel function, S-generalized Gauss's hypergeometric function, Fox's H-function.

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1. Introduction.

The H- function introduced by Fox [5] however the notation and complete definition is presented here in the following manner in terms of the Mellin-Barnes type integral :

$$H(z) = H_{P,Q}^{M,N}\left(z \,\middle|\, \begin{matrix} (a_j;A_j)_{1,N}, [(a_j,A_j)]_{N+1,P} \\ \cdot \\ (b_j;B_j)_{1,M}, [(b_j,B_j)]_{M+1,Q} \end{matrix} \right) = \frac{1}{2\pi\omega}\int_L \Omega_{P,Q}^{M,N}(s) z^s \mathrm{d}s \tag{1.1}$$

for all z different to 0 and

$$\Omega_{P,Q}^{M,N}(s) = \frac{\prod_{j=1}^{M}\Gamma(b_j - B_j s)\prod_{j=1}^{N}\Gamma(1-a_j+A_j s)}{\prod_{j=N+1}^{P}\Gamma(a_j - A_j s)\prod_{j=M+1}^{Q}\Gamma(1-b_j+B_j s)} \tag{1.2}$$

With $|argz| < \frac{1}{2}\pi\Omega$ where $\Omega = \sum_{j=1}^{M} B_j + \sum_{j=1}^{N} A_j - \left(\sum_{j=N+1}^{P} A_j + \sum_{j=M+1}^{Q} B_j\right) > 0$

M, N, P, Q are positive integers and verify $0 \leq N \leq P, 0 \leq M \leq Q$. A_j, B_j are positive reals and $a_j, b_j,$ are complex numbers. An empty product is equal to 1. For more details, see [5,9].

We consider a generalized transcendental function called Gimel of several complex variables.

$$\gimel(z_1,\cdots,z_r) = \gimel_{X;p_{i_r},q_{i_r},\tau_{i_r}:R_r:Y}^{U;0,n_r:V}\left(\begin{matrix} z_1 \\ \cdot \\ \cdot \\ \cdot \\ z_r \end{matrix}\,\middle|\,\begin{matrix} \mathbb{A};\mathbf{A}:A \\ \cdot \\ \cdot \\ \cdot \\ \mathbb{B};\mathbf{B}:B \end{matrix}\right) = \frac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r}\psi(s_1,\cdots,s_r)\prod_{k=1}^{r}\theta_k(s_k)z_k^{s_k}\,\mathrm{d}s_1\cdots\mathrm{d}s_r \tag{1.3}$$

with $\omega = \sqrt{-1}$

The quantities $\mathbb{A}, \mathbf{A}, A, \mathbb{B}, \mathbf{B}, B, X, Y, U, V, \psi(s_1,\cdots,s_r)$ and $\theta_k(s_k)(k=1,\cdots,r)$ are defined by Ayant [3].

The S-generalized Gauss hypergeometric function $F_p^{\alpha,\beta;\tau,\mu}(a,b;c;z)$ introduced and defined by Srivastava et al. ([8], p.350, Eq.(1.12)) is represented in the following manner :

$$F_p^{\alpha,\beta;\tau,\mu}(a,b;c;z)=\sum_{n=0}^{\infty}(a)_n\frac{B_p^{\alpha,\beta;\tau,\mu}(b+n,c-b)}{B(b,c-b)}\frac{z^n}{n!}\qquad |z|<1 \tag{1.4}$$

provided that : $Re(p)\geq 0; min[Re(\alpha),Re(\beta),Re(\tau),\Re(\mu)]>0; Re(c)>Re(b)>0$

where the S-generalized Beta function $B_p^{\alpha,\beta;\tau,\mu}(x,y)$ was introduced and defined by Srivastava et al. ([8], p.350, Eq. (1.13))

$$B_p^{\alpha,\beta;\tau,\mu}(x,y)=\int_0^1 t^{x-1}(1-t)^{y-1}{}_1F_1\left(\alpha;\beta;-\frac{p}{t^{\tau}(1-t)^{\mu}}\right)\mathrm{d}t \tag{1.5}$$

provided that : $Re(p)\geq 0; min[Re(\alpha),Re(\beta),Re(\tau),\Re(\mu)]>0; \min\{\tau,\mu\}>0$ and $(a)_n$ is the Pochhammer symbol.

The generalized incomplete hypergeometric function introduced and defined by Srivastava et al. ([7], p.675, Eq. (4.1)) is represented in the following manner :

$${}_p\gamma_q\left(\mathrm{z}\;\middle|\;\begin{matrix}(\mathrm{E}_p;\sigma)\\ .\\ \mathrm{F}_q\end{matrix}\right)={}_p\gamma_q\left(\mathrm{z}\;\middle|\;\begin{matrix}(\mathrm{e}_1;\sigma)_n,(e_2)_n\cdots(e_p)_n\\ .\\ (\mathrm{f}_1)_n,\cdots,(f_q)_n\end{matrix}\right)$$

$$=\sum_{n=0}^{\infty}\frac{(E_q;\sigma)}{(F_q;0)_n}\frac{z^n}{n!}=\sum_{n=0}^{\infty}\frac{(e_1;\sigma)_n(e_2)_n\cdots(e)_n}{(f_1)_n\cdots(f_q)_n}\frac{z^n}{n!} \tag{1.6}$$

where the incomplete Pochhammer symbols are defined as follows :

$$(a;\sigma)_n=\frac{\gamma(a+n,\sigma)}{\Gamma(a)}\qquad a,n\in\mathbb{C};x\geq 0. \tag{1.7}$$

and the incomplete gamma function $\gamma(s,x)$ is

$$\gamma(s,x)=\int_0^x t^{s-1}e^{-t}\mathrm{d}t\qquad Re(s)>0;x\geq 0 \tag{1.8}$$

provided that the above infinite series is absolutely convergent.

The Appell polynomial introduced and defined by [1] is represented in the following manner :

$$A(z)=\sum_{k=0}^{n}a_{n-k}\frac{z^k}{k!}\qquad n=0,1,2,\cdots \tag{1.9}$$

where a_{n-k} is the complex coefficient with $a_0\neq 0$

2. Fractional integral operators

We study two unified fractional integral operators involving the Appell polynomial, Fox's H-function and S-generalized Gauss hypergeometric function having general arguments

$$I_x^{\upsilon,\lambda}\left[A_n,H,F_p\right]f(t)=\int_0^x A_n\left[z_1\left(\frac{t}{x}\right)^{\upsilon_1}\left(1-\frac{t}{x}\right)^{\lambda_1}\right]H_{P,Q}^{M,N}\left(\mathrm{z}_2\left(\tfrac{t}{x}\right)^{\upsilon_2}\left(1-\tfrac{t}{x}\right)^{\lambda_2}\;\middle|\;\begin{matrix}(\mathrm{a}_j,A_j)_{1,P}\\ .\\ (\mathrm{b}_j,B_j)_{1,Q}\end{matrix}\right)$$

$$F_p^{\alpha,\beta;\tau,\mu}\left[a,b;c;z_3\left(\frac{t}{x}\right)^{\upsilon_3}\left(1-\frac{t}{x}\right)^{\lambda_3}\right]f(t)\mathrm{d}t \tag{2.1}$$

where, the operators are defined for $f(t)\in\Delta,\Delta$ denote the class of functions f(t) for which

$$f(t)=\left[\begin{array}{l} 0|t|^{\zeta}\}.\ .\ .\ \max(|t|)\to 0 \\ . \\ 0\left[|t|^{w_1}e^{-w_2|t|}\right]\ldots\min(|t|)\to\infty \end{array}\right. \tag{2.2 }$$

provided that

$$\min_{1\leqslant j\leqslant M} Re\left[\upsilon+\upsilon_2\frac{b_j}{B_j}+\zeta+1,\lambda+\lambda_2\frac{b_j}{B_j}+1\right]>0 \text{ and } min\{\upsilon_1,\upsilon_3,\lambda_1,\lambda_3\}\geq 0$$

$$J_x^{\upsilon,\lambda}\left[A_n,H,F_p\right]f(t)=x^{\upsilon}\int_x^{\infty}A_n\left[z_1\left(\frac{x}{t}\right)^{\upsilon_1}\left(1-\frac{x}{t}\right)^{\lambda_1}\right]H_{P,Q}^{M,N}\left(\ z_2\left(\frac{x}{t}\right)^{\upsilon_2}\left(1-\frac{x}{t}\right)^{\lambda_2}\ \middle|\ \begin{array}{c}(\mathrm{a}_j,A_j)_{1,P}\\ .\\ (\mathrm{b}_j,B_j)_{1,Q}\end{array}\right)$$

$$F_p^{\alpha,\beta;\tau,\mu}\left[a,b;c;z_3\left(\frac{x}{t}\right)^{\upsilon_3}\left(1-\frac{x}{t}\right)^{\lambda_3}\right]f(t)\mathrm{d}t \tag{2.3}$$

provided $Re(w_2)>0$ or $Re(w_2)=0$ and $\min_{1\leqslant j\leqslant M} Re\left[\upsilon-w_1+\upsilon_2\frac{b_j}{B_j}\right]>0$; $\min_{1\leqslant j\leqslant M} Re\left[\lambda+\lambda_2\frac{b_j}{B_j}+1\right]>0,$

$min\{\upsilon_1,\upsilon_3,\lambda_1,\lambda_3\}\geq 0.$

3. Images formulae.

In the theorem 1 , we shall use the following notations

$$\mathbf{A}_1=\ [(\mathrm{a}_{rj};\alpha_{rj}^{(1)},\cdots,\alpha_{rj}^{(r)},0,0,0;A_{rj})_{1,n_r}],[\tau_{i_r}(a_{rji_r};\alpha_{rji_r}^{(1)},\cdots,\alpha_{rji_r}^{(r)},0,0,0;A_{rji_r})_{\mathfrak{n}+1,p_{i_r}}]: \tag{3.1}$$

$$A_1=(-\rho-\upsilon-\upsilon_1k;\upsilon^{(1)},\cdots,\upsilon^{(r)},\upsilon_2,\upsilon_3,0;1),(-\lambda-\lambda_1k;\lambda^{(1)},\cdots,\lambda^{(r)},\lambda_2,\lambda_3,0;1);$$

$$(1-b;\underbrace{0,\cdots,0}_{r},0,1,\tau;1),\mathbf{A}_1:A;(a_j;A_j;1)_{1,P} \tag{3.2}$$

$$\mathbf{B}_1=\ [\tau_{i_r}(b_{rji_r};\beta_{rji_r}^{(1)},\cdots,\beta_{rji_r}^{(r)},0,0,0;B_{rji_r})_{1,q_{i_r}}] \tag{3.3}$$

$$B_1=\mathbf{B}_1,(-1-\rho-\lambda-\upsilon-(\lambda+\upsilon_1)k;\upsilon^{(1)}+\lambda^{(1)},\cdots,\upsilon^{(r)}+\lambda^{(r)},0,0,0;1),(1-c;\underbrace{0,\cdots,0}_{r},0,1,\tau+\mu;1):$$

$$B;(b_j;B_j;1)_{1,Q} \tag{3.4}$$

We have the following results

Theorem 1.

$$I_x^{\upsilon,\lambda}\left[A_n,H,F_p\right]\beth\left(\begin{array}{c}\mathrm{z}^{(1)}t^{\upsilon^{(1)}}(x-t)^{\lambda^{(1)}}\\ .\\ .\\ \mathrm{z}^{(r)}t^{\upsilon^{(r)}}(x-t)^{\lambda^{(r)}}\end{array}\right)=\frac{\Gamma(\beta)x^{\rho}}{\Gamma(a)\Gamma(\alpha)B(b,c-b)}\sum_{k=0}^{n}\frac{a_{n-k}}{k!}z_1^k$$

$$\beth_{X;p_{i_r}+3,q_{i_r}+2,\tau_{i_r}:R_r:Y;1,1;3,1}^{U;0,n_r+3:V;1,1;1,2}\left(\begin{array}{c|c} \mathrm{z}^{(1)}x^{\upsilon^{(1)}+\lambda^{(1)}} & \mathbb{A};\mathrm{A}_1;(1-a;1;1),(1;1;1),(1-c+b;\mu;1),(\beta;1;1) \\ \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ \mathrm{z}^{(r)}x^{\upsilon^{(r)}+\lambda^{(r)}} & \cdot \\ \mathrm{z}_2 & \mathbb{B};\ \mathrm{B}_1;(0;1;1);(\alpha;1;1) \\ \mathrm{z}_3 & \\ \mathrm{p}^{-1} & \end{array}\right) \tag{3.5}$$

provided that

1)$f(t)\in\Delta$

2) $\lambda^{(i)},\upsilon^{(i)}>0, i=1,\cdots,r, Re(p)>0$

3) $\min\limits_{1\leqslant j\leqslant M} Re\left[\upsilon+\upsilon_2\frac{b_j}{B_j}+\zeta+1,\lambda+\lambda_2\frac{b_j}{B_j}+1\right]>0$ and $min\{\upsilon_1,\upsilon_3,\lambda_1,\lambda_3\}\geq 0$ and

4) $\left|arg(z^{(i)}t^{\upsilon^{(i)}}(x-t)^{\lambda^{(i)}})\right|<\frac{1}{2}A_i^{(k)}\pi$ where $A_i^{(k)}$ is defined by Ayant [3].

Proof

To prove the theorem 1, first we express the I-operator involved in its left hand side in the integral form with the help of (2.1). Then we express Appell polynomial in terms of series with the help of (1.9). Now we interchange the order of series and t-integral, express both the Fox H-function, multivariable Gimel-function and S-Generalized Gauss's Hypergeometric Function in terms of Mellin-Barnes type integrals contour with the help of (1.1), (1.3) and (1.4) respectively. Then we interchange the order of s_i(i = 1, 2, ..., r + 3)-integral and t-integral, (which is permissible under the condition stated). Finally, on evaluating the t-integral and reinterpreting the result thus obtained in terms of multivariable Gimel-function, we easily arrive at the required result after algebraic manipulations.

Let

$$A_2=(1+\rho-\upsilon-\upsilon_1 k;\upsilon^{(1)},\cdots,\upsilon^{(r)},\upsilon_2,\upsilon_3,0;1),(-\lambda-\lambda_1 k;\lambda^{(1)},\cdots,\lambda^{(r)},\lambda_2,\lambda_3,0;1),$$

$$(1-b;\underbrace{0,\cdots,0}_{r},0,1,\tau;1),\mathbf{A}_1\text{: A};(a_j;A_j;1)_{1,P} \tag{3.6}$$

$$B_2=\mathbf{B}_1,(-1-\rho-\lambda-\upsilon-(\lambda+\upsilon_1)k;\upsilon^{(1)}+\lambda^{(1)},\cdots,\upsilon^{(r)}+\lambda^{(r)},0,0,0;1),(1-c;\underbrace{0,\cdots,0}_{r},0,1,\tau+\mu;1):$$

$$B;(b_j;B_j;1)_{1,Q} \tag{3.7}$$

The others quantities are used in the theorem 1.

Theorem 2

$$J_x^{\upsilon,\lambda}\left[A_n,H,F_p\right]\left(\begin{array}{c} \mathrm{z}^{(1)}t^{\upsilon^{(1)}}(x-t)^{\lambda^{(1)}} \\ \cdot \\ \cdot \\ \mathrm{z}^{(r)}t^{\upsilon^{(r)}}(x-t)^{\lambda^{(r)}} \end{array}\right)=\frac{\Gamma(\beta)x^{\rho}}{\Gamma(a)\Gamma(\alpha)B(b,c-b)}\sum_{k=0}^{n}\frac{a_{n-k}}{k!}z_1^k$$

$$\beth_{X;p_{i_r}+3,q_{i_r}+2,\tau_{i_r}:R_r:Y;1,1;3,1}^{U;0,n_r+3:V;1,1;1,2}\left(\begin{array}{c|c} \mathrm{z}^{(1)}x^{\upsilon^{(1)}+\lambda^{(1)}} & \mathbb{A};\mathrm{A}_2;(1-a;1;1),(1;1;1),(1-c+b;\mu;1),(\beta;1;1) \\ \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ \mathrm{z}^{(r)}x^{\upsilon^{(r)}+\lambda^{(r)}} & \cdot \\ \mathrm{z}_2 & \\ \mathrm{z}_3 & \mathbb{B};\ \mathrm{B}_2;(0;1;1);(\alpha;1;1) \\ \mathrm{p}^{-1} & \end{array}\right) \quad (3.8)$$

provided

1)$f(t) \in \Delta$

2) $\lambda^{(i)}, \upsilon^{(i)} > 0, i = 1, \cdots, r, Re(p) > 0$

3) $Re(w_2) > 0$ or $Re(w_2) = 0$ and $\min\limits_{1 \leqslant j \leqslant M} Re\left[\upsilon - w_1 + \upsilon_2 \frac{b_j}{B_j}\right] > 0$; $\min\limits_{1 \leqslant j \leqslant M} Re\left[\lambda + \lambda_2 \frac{b_j}{B_j} + 1\right] > 0$,

$min\{\upsilon_1, \upsilon_3, \lambda_1, \lambda_3\} \geq 0$.

4) $\left|arg(z^{(i)} t^{\upsilon^{(i)}} (x-t)^{\lambda^{(i)}})\right| < \frac{1}{2} A_i^{(k)} \pi$

To prove (3.10), we use the similar methods that (3.7).

Theorem 3. (see Bansal and Jain [4])

$$I_x^{\upsilon,\lambda}\left[A_n, H, F_p\right]_p\gamma_q\left(\begin{array}{c|c} \mathrm{z}_4\left(\frac{t}{x}\right)^{\upsilon_4}\left(1-\frac{t}{x}\right)^{\lambda_4} & \begin{array}{c}(\mathrm{E}_p;\sigma)\\ \cdot \\ \mathrm{F}_q\end{array}\end{array}\right) = \frac{\Gamma(\beta)}{\Gamma(a)\Gamma(\alpha)B(b,c-b)}$$

$$\sum_{i,k=0}^{\infty} \frac{a_{n-k}(e_1;\sigma)_i(e_2)_i \cdots (e)_i}{(f_1)_i \cdots (f_i)_i} \frac{z_1^k z_4^i x^{\rho}}{k! i!}$$

$$H_{3,2:P,Q;1,1;3,1}^{0,3:M,N;1,1;1,2}\left(\begin{array}{c|c} & \mathrm{C}{:}(\mathrm{a}_j;A_j;1)_{1,P};(1-a;1;1);(1,1),(1-c+b;\mu;1),(\beta;1;1) \\ \mathrm{z}_2 & \cdot \\ \mathrm{z}_3 & \cdot \\ \mathrm{p}^{-1} & \cdot \\ & \mathrm{D}{:}\ (\mathrm{b}_j;B_j;1)_{1,Q};(0;1;1);(\alpha;1;1) \end{array}\right) \quad (3.9)$$

where

$$C = (-\rho - \upsilon - \upsilon_1 k - \upsilon_4 i; \upsilon_2, \upsilon_3, 0; 1), (-\lambda - \lambda_1 k - \lambda_4 i, \lambda_2, \lambda_3, 0; 1), (1-b; 0, 1, \tau; 1) \quad (3.10)$$

$$D = (-1 - \rho - \upsilon - \lambda - (\upsilon_1 + \lambda_1)k - (\upsilon_4 + \lambda_4)i; \upsilon_2 + \lambda_2, \upsilon_3 + \lambda_3, 0; 1), (1-c; 0, 1, \tau + \mu; 1) \quad (3.11)$$

provided that

1)$f(t) \in \Delta$

2) $\min\limits_{1 \leqslant j \leqslant M} Re\left[\upsilon + \upsilon_2 \frac{b_j}{B_j} + \zeta + 1, \lambda + \lambda_2 \frac{b_j}{B_j} + 1\right] > 0$ and $min\{\upsilon_1, \upsilon_3, \lambda_1, \lambda_3\} \geq 0$, $Re(p) > 0$

Theorem 4 (see Bansal and Jain [4])

$$J_x^{\upsilon,\lambda}\left[A_n,H,F_p\right]{}_p\gamma_q\left(\begin{array}{c} z_4\left(\frac{t}{x}\right)^{\upsilon_4}\left(1-\frac{t}{x}\right)^{\lambda_4}\end{array}\middle|\begin{array}{c}(\mathrm{E}_p;\sigma)\\ \cdot\\ \mathrm{F}_q\end{array}\right)=\frac{\Gamma(\beta)}{\Gamma(a)\Gamma(\alpha)B(b,c-b)}$$

$$\sum_{i,k=0}^{\infty}\frac{a_{n-k}(e_1;\sigma)_i(e_2)_i\cdots(e)_i}{(f_1)_i\cdots(f_i)_i}\frac{z_1^kz_4^ix^{\rho-\upsilon_4 i}}{k!i!}$$

$$H_{3,2:P,Q;1,1;3,1}^{0,3:M,N;1,1;1,2}\left(\begin{array}{c} \mathrm{z}_2\\ \mathrm{z}_3\\ \mathrm{p}^{-1}\end{array}\middle|\begin{array}{c}\mathrm{C}:(\mathrm{a}_j;A_j;1)_{1,P};(1-a;1;1);(1;1;1),(1-c+b;\mu;1),(\beta;1;1)\\ \cdot\\ \cdot\\ \cdot\\ \mathrm{D}:\ (\mathrm{b}_j;B_j;1)_{1,Q};(0;1;1);(\alpha;1;1)\end{array}\right) \tag{3.12}$$

where

$$C'=(\rho+1-\upsilon-\upsilon_1k-\upsilon_4i;\upsilon_2,\upsilon_3,0;1),(-\lambda-\lambda_1k-\lambda_4i,\lambda_2,\lambda_3,0;1),(1-b;0,1,\tau;1) \tag{3.13}$$

$$D'=(\rho-\upsilon-\lambda-(\upsilon_1+\lambda_1)k-(\upsilon_4+\lambda_4)i;\upsilon_2+\lambda_2,\upsilon_3+\lambda_3,0;1),(1-c;0,1,\tau+\mu;1) \tag{3.14}$$

provided that

1) $f(t)\in\Delta$

2) $Re(w_2)>0$ or $Re(w_2)=0$ and $\min\limits_{1\leqslant j\leqslant M}Re\left[\upsilon-w_1+\upsilon_2\frac{b_j}{B_j}\right]>0$; $\min\limits_{1\leqslant j\leqslant M}Re\left[\lambda+\lambda_2\frac{b_j}{B_j}+1\right]>0,$

$min\{\upsilon_1,\upsilon_3,\lambda_1,\lambda_3\}\geq 0$, $Re(p)>0$.

To prove the theorems 3 and 4, first we express the I-operator involved in its left hand side in the integral form with the help of (2.1). Then we express Appell polynomial and generalized incomplete hypergeometric function in terms of series with the help of (1.9) and (1,6) respectively. Now we interchange the order of series and t-integral, express both the Fox H-function and S-Generalized Gauss's Hypergeometric Function in terms of Mellin-Barnes type contour integrals with the help of (1.1) and (1.4) respectively. Then we interchange the order of s_i-triple integrals and t-integral, (which is permissible under the condition stated). Finally, on evaluating the t-integral and reinterpreting the result thus obtained in terms of H-function of three variables, we easily arrive at the required result after algebraic manipulations.

Remarks
We obtain the same formulae concerning the multivariable Aleph- function defined by Ayant [2], the multivariable I-function defined by Prathima et al. [7], the multivariable I-function defined by Prasad [6] and the multivariable H-function defined by Srivastava and Panda [10,11], see Bansal and Jain [4] for more details about the multivariable H-function

4. Conclusion.

Firstly, the pair of fractional integral operators presented in this document are quite nature. Therefore, on specializing the parameters of these functions involving in this paper, we obtain various other results as its special cases. Secondly, by specializing the various parameters as well as variables in the generalized multivariable Gimel-function , we obtain a large number of formulae involving remarkably wide variety of useful functions or product of such functions) which are expressible in terms of E, F, G, H, I, Aleph-function of one and several variables and simpler special functions of one and several variables. Hence the formulae derived in this paper are most general in character and may prove to be useful in several interesting cases appearing in literature of Pure and Applied Mathematics and Mathematical Physics.

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