The Fundamental Theorem of Arithmetic and Goldbach Conjecture

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Abstract

If the fundamental theorem of arithmetic is called the product fundamental theorem of arithmetic ,then we give a conjecture in this paper. That is, if Goldbach conjecture is correct,then it is called the sum fundamental theorem of arithmetic.The sum fundamental theorem of arithmetic and the product fundamental theorem of arithmeticare collectively called the fundamental theorem of arithmetic.

Key word—*Prime number, the fundamental theorem of arithmetic, the sum fundamental theorem of arithmetic, the product fundamental theorem of arithmetic.*

I. INTRODUCTION

It is well known to all that there is a fundamental theorem in number theory — the fundamental theorem of arithmetic. It plays a basic function and a key role. The number theory is so rich and colorful from it. This theorem as follows.

For positive integer N>1,it is always factored into the form uniquely

$$N= p_1^{\alpha_1}p_2^{\alpha_2}\dots p_n^{\alpha_n}.$$

Where $p_1<p_2<\cdots<p_n$;$p_1, p_2, \cdots, p_n$are primes;$\alpha_1,\alpha_2,\cdots,\alpha_n$ are positive integers.(unique)

In fact it seems to be called the productfundamental theorem of arithmetic more accurately.Because computational result is product of prime number. Now that there is the product fundamental theorem of arithmetic, of course there should be its dual— the sum fundamental theorem of arithmetic.What is the sum fundamental theorem of arithmetic?

We reveal the form of the sum fundamental theorem of arithmetic in this paper. It is Goldbach conjecture if Goldbach conjecture is correct.

II. THEPRODUCTFUNDAMENTAL THEOREM OF ARITHMETIC ANDTHE SUM FUNDAMENTAL THEOREM OF ARITHMETIC

I walked in the campus one day.It was very quiet.

I thought the following question suddenly.

The fundamental theorem of arithmetic is the form

$$N= p_1^{\alpha_1}p_2^{\alpha_2}\cdots p_n^{\alpha_n}. \qquad (1)$$

Where $p_1, p_2,\cdots,p_n$are primes;$\alpha_1,\alpha_2,\cdots,\alpha_n$ are positive integers.(I omitted the unique.)

Since $p_1^{\alpha_1}\times p_2^{\alpha_2}\times\cdots\times p_n^{\alpha_n}$. Therefore,if it is called “The productfundamental theorem of arithmetic”, it seems to be more precise.

So I thought what is the sumfundamental theorem of arithmetic?

Obvious it has the form

$$N= q_1^{\beta_1}+q_2^{\beta_2} + q_3^{\beta_3}+ \cdots +q_m^{\beta_m}. \qquad (2)$$

Where $q_1,q_2,\cdots,q_m$are primes;$\beta_1,\beta_2,\cdots,\beta_m$ are positive integers.

The (2) is called “The sumfundamental theorem of arithmetic”.

So we have the dualrelation

$$N= p_1^{\alpha_1}p_2^{\alpha_2}\cdots p_n^{\alpha_n}\leftrightarrow q_1^{\beta_1}+q_2^{\beta_2} + q_3^{\beta_3}+ \cdots +q_m^{\beta_m}= N. \qquad (3)$$

(3) is a duality.How wonderful!

I came back to study room at once,and began to think this problem.

Goldbach conjecture is

i.Every positive even number N, $N\geq 6$,then

$$N= q_1 + q_2,\ q_1\leq q_2. \qquad (4)$$

Where q_1,q_2 are primes.

ii.Every positive odd number N, $N\geq 9$,then

$$N= q_1 + q_2 + q_3, q_1\leq q_2\leq q_3.$$

(For sufficient large N,I.M.Vinogradov proved it is correct,So it becomes theorem.)

Therefore,we can have

The product fundamental theorem of arithmetic for all N,N> 1,then

$$N= p_1^{\alpha_1} p_2^{\alpha_2} \cdots p_n^{\alpha_n}.$$

Where $p_1 \le p_2 \le \cdots \le p_n$; $p_1, p_2, \cdots p_n$ are primes, $\alpha_1, \alpha_2, \cdots, \alpha_n$ *are positive integers*.

$$\updownarrow$$

The sum fundamental theorem of arithmetic

i.For every even positive number N,N≥ 6,then

$$N= q_1 + q_2,$$

Where q_1, q_2 areprimes;$q_1 \le q_2$.

(Nowit is conjecture.)

ii.For every odd positive number N,N≥ 9,then

$N = q_1 + q_2 + q_3$,

Where q_1, q_2, q_3 are primes; $q_1 \le q_2 \le q_3$.

(Now it is theorem for sufficient large N.)

Where I omitted the uniqueness.

In a word,we have

$$\text{The fundamental theorem of arithmetic} \Leftrightarrow \begin{cases} \text{The product fundamental theorem of arithmetic.} \\ \text{The sum fundamental theorem of arithmetic.} \end{cases}$$

We hope that the sum fundamental theorem of arithmetic will appear. Because the beautiful things are dual.

III. CONCLUSION

All in all,if Goldbach conjecture were true, it will become the sum fundamental theorem of arithmetic.So we have

$$\text{The fundamental theorem of arithmetic} \Leftrightarrow \begin{cases} \text{The product fundamental theorem of arithmetic.} \\ \text{The sum fundamental theorem of arithmetic.} \end{cases}$$

Thus, the product fundamental theorem of arithmetic$\overset{dual}{\Longleftrightarrow}$the sum fundamental theorem of arithmetic.

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