Eulerian Integral Associated with Product of Two Multivariable Gimel-Functions, and Special Functions of Several Variables I

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ABSTRACT
Recently, Raina and Srivastava [7] and Srivastava and Hussain [12] have provided closed-form expressions for a number of a Eulerian integral about the multivariable H-functions. The present paper is evaluated a new Eulerian integral associated with the product of two multivariable Gimel-functions defined by Ayant [1], a generalized Lauricella function , a multivariable I-function and multivariable A-function defined by Gautam and Asgar [1] with general arguments . Finally we shall give few remarks and we shall see the particular case concerning the Srivastava-Daoust polynomial [9].

Keywords: Eulerian integral, multivariable Gimel-function, generalized Lauricella function of several variables, expansion of multivariable A-function expension of multivariable I-function, generalized hypergeometric function, class of polynomials.

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1. Introduction

The well-known Eulerian Beta integral

$$\int_a^b (z-a)^{\alpha-1}(b-t)^{\beta-1}\mathrm{d}t = (b-a)^{\alpha+\beta-1}B(\alpha,\beta)(Re(\alpha)>0, Re(\beta)>0, b>a) \tag{1.1}$$

is a basic result for evaluation of numerous other potentially useful integrals involving various special functions and polynomials. The authors Raina and Srivastava [2], Saigo and Saxena [8], Srivastava and Hussain [12], Srivastava and Garg [11] and other have studied the Eulerian integral. In this paper, we evaluate a new Eulerian integral of most general characters associated with the products of two multivariable Gimel-functions defined by Ayant [2], the expansion of special functions of several variables with general arguments. The A-function of several variables is an extension of multivariable H-function defined by Srivastava and Panda [13,14].
The serie representation of the multivariable A-function is given by Gautam and Asgar [4] as

$$A[u_1,\cdots,u_v] = A^{0,\lambda:(\alpha',\beta');\cdots;(\alpha^{(v)},\beta^{(v)})}_{A,C:(M',N');\cdots;(M^{(v)},N^{(v)})}\left(\begin{array}{c} \mathrm{u}_1 \\ \cdot \\ \cdot \\ \mathrm{u}_v \end{array}\middle| \begin{array}{c} [(\mathrm{g}_j);\gamma',\cdots,\gamma^{(v)}]_{1,A}: \\ \cdots \\ [(\mathrm{f}_j);\xi',\cdots,\xi^{(v)}]_{1,C}: \end{array}\right.$$

$$\left.\begin{array}{c} (\mathrm{q}^{(1)},\eta^{(1)})_{1,M^{(1)}};\cdots;(q^{(v)},\eta^{(v)})_{1,M^{(v)}} \\ \cdots \\ \cdots \\ (\mathrm{p}^{(1)},\epsilon^{(1)})_{1,N^{(1)}};\cdots;(p^{(v)},\epsilon^{(v)})_{1,N^{(v)}} \end{array}\right) = \sum_{G_i=1}^{\alpha^{(i)}}\sum_{g_i=1}^{\infty}\phi_1\frac{\prod_{i=1}^v \phi_i u_i^{\eta_{G_i,g_i}}(-)^{\sum_{i=1}^v g_i}}{\prod_{i=1}^v \epsilon_{G_i}^{(i)} g_i!} \tag{1.2}$$

$$\phi_i = \frac{\prod_{j=1,j\neq m_i}^{\alpha^{(i)}}\Gamma\left(p_j^{(i)}-\epsilon_j^{(i)}\eta_{G_i,g_i}\right)\prod_{j=1}^{\beta^{(i)}}\Gamma\left(1-q_j^{(i)}+\eta_j^{(i)}\eta_{G_i,g_i}\right)}{\prod_{j=\alpha^{(i)}+1}^{N^{(i)}}\Gamma\left(1-p_j^{(i)}+\epsilon_j^{(i)}\eta_{G_i,g_i}\right)\prod_{j=\beta^{(i)}+1}^{M^{(i)}}\Gamma\left(q_j^{(i)}-\eta_j^{(i)}\eta_{G_i,g_i}\right)}, i=1,\cdots,v \tag{1.3}$$

and $\eta_{G_i,g_i} = \dfrac{p_{G_i}^{(i)}+g_i}{\epsilon_{G_i}^{(i)}}, i=1,\cdots,v$ (1.4)

which is valid under the following conditions : $\epsilon_{m_i}^{(i)}[p_j^{(i)}+p_i'] \neq \epsilon_j^{(i)}[p_{m_i}+g_i]$ (1.5)
and

$$u_i \neq 0, \sum_{j=1}^{A}\gamma_j^{(i)} - \sum_{j=1}^{C}\xi_j^{(i)} + \sum_{j=1}^{M^{(i)}}\eta_j^{(i)} - \sum_{j=1}^{N^{(i)}}\epsilon_j^{(i)} < 0, i = 1, \cdots, v \tag{1.6}$$

Here $\lambda, A, C, \alpha_i, \beta_i, M_i, N_i \in \mathbb{N}^*; i = 1, \cdots, v; f_j, g_j, p_j^{(i)}, q_j^{(i)}, \gamma_j^{(i)}, \xi_j^{(i)}, \eta_j^{(i)}, \epsilon_j^{(i)} \in \mathbb{C}$

The multivariable I-function defined by Prathima et al. [4] is defined in term of multiple Mellin-Barnes type integral :

$$\bar{I}(z_1,\cdots,z_u) = I_{P,Q:P_1,Q_1;\cdots;P_u,Q_u}^{0,N:M_1,N_1;\cdots;M_u,N_u}\left(\begin{array}{c|c} \mathrm{z}_1 & (\mathrm{a}_j;\alpha_j^{(1)},\cdots,\alpha_j^{(u)};A_j)_{1,P}: \\ \cdot & \\ \cdot & \\ \cdot & \\ \mathrm{z}_u & (\mathrm{b}_j;\beta_j^{(1)},\cdots,\beta_j^{(u)};B_j)_{1,Q}: \end{array}\right.$$

$$\left.\begin{array}{c} (\mathrm{c}_j^{(1)},\gamma_j^{(1)};C_j^{(1)})_{1,N_1},(c_j^{(1)},\gamma_j^{(1)};C_j^{(1)})_{N_1+1,P_1};\cdots;(c_j^{(r)},\gamma_j^{(r)};C_j^{(u)})_{1,N_u},(c_j^{(r)},\gamma_j^{(r)};C_j^{(u)})_{N_u+1,P_u} \\ (\mathrm{d}_j^{(1)},\delta_j^{(1)};1)_{1,M_1},(d_j^{(1)},\delta_j^{(1)};D_1)_{M_1+1,Q_1};\cdots;(d_j^{(r)},\delta_j^{(u)};1)_{1,M_u},(d_j^{(u)},\delta_j^{(u)};D_u)_{M_u+1,Q_u} \end{array}\right) \tag{1.7}$$

$$= \frac{1}{(2\pi\omega)^u}\int_{L_1}\cdots\int_{L_u}\phi(t_1,\cdots,t_u)\prod_{i=1}^{u}\theta_i'(u_i)z_i^{u_i}\mathrm{d}t_1\cdots\mathrm{d}t_u \tag{1.8}$$

where $\phi'(t_1,\cdots,t_u), \theta_i'(t_i), i = 1,\cdots,u$ are given by :

$$\phi'(t_1,\cdots,t_u) = \frac{\prod_{j=1}^{N}\Gamma^{A_j}\left(1-a_j+\sum_{i=1}^{u}\alpha_j^{(i)}t_j\right)}{\prod_{j=N+1}^{P}\Gamma^{A_j}\left(a_j-\sum_{i=1}^{u}\alpha_j^{(i)}t_j\right)\prod_{j=1}^{Q}\Gamma^{B_j}\left(1-b_j+\sum_{i=1}^{u}\beta_j^{(i)}t_j\right)} \tag{1.9}$$

$$\phi_i'(t_i) = \frac{\prod_{j=1}^{N_i}\Gamma^{C_j^{(i)}}\left(1-c_j^{(i)}+\gamma_j^{(i)}t_i\right)\prod_{j=1}^{M_i}\Gamma\left(d_j^{(i)}-\delta_j^{(i)}t_i\right)}{\prod_{j=N_i+1}^{P_i}\Gamma^{C_j^{(i)}}\left(c_j^{(i)}-\gamma_j^{(i)}t_i\right)\prod_{j=M_i+1}^{Q_i}\Gamma^{D_j^{(i)}}\left(1-d_j^{(i)}+\delta_j^{(i)}t_i\right)} \tag{1.10}$$

For more details, see Prathima et al. [4].

We can obtain the series representation and behaviour for small values for the function $\bar{I}(z_1,\cdots,z_r)$ defined and represented by (1.16). The series representation may be given as follows : The poles aer simple.

which is valid under the following conditions :

$$\delta_i^{(h)}[d_i^{(j)}+r] \neq \delta_i^{(j)}[d_i^{(h)}+\mu] \text{ for } j \neq h, j,h = 1,\cdots,M_i, r,\mu = 0,1,2,\cdots$$

$$U_i = \sum_{j=1}^{P}A_j\alpha_j^{(i)} - \sum_{j=1}^{Q}B_j\beta_j^{(i)} + \sum_{j=1}^{P_i}C_j^{(i)}\gamma_j^{(i)} - \sum_{j=M_i+1}^{Q_i}D_j^{(i)}\delta_j^{(i)} \leqslant 0, i = 1,\cdots,r \text{ and } z_i \neq 0$$

and if all the poles of (1.16) are simple ,then the integral (1.16) can be evaluated with the help of the Residue theorem to give

$$\bar{I}(z_1,\cdots,z_u)=\sum_{H_i=1}^{M_i}\sum_{h_i=1}^{\infty}\phi_1'\frac{\prod_{i=1}^{u}\phi_i' z_i^{\eta_{h_i,H_i}}(-)^{\sum_{i=1}^{u}g_i}}{\prod_{i=1}^{u}\delta_{h^{(i)}}^{(i)}\prod_{i=1}^{u}h_i!} \quad (1.12)$$

where ϕ_1 and ϕ_i are defined by

$$\phi_1'=\frac{\prod_{j=1}^{N}\Gamma^{A_j}\left(1-aj+\sum_{i=1}^{u}\alpha_j^{(i)}\eta_{H_i,h_i}\right)}{\prod_{j=N+1}^{P}\Gamma^{A_j}\left(a_j-\sum_{i=1}^{u}\alpha_j^{(i)}\eta_{H_i,h_i}\right)\prod_{j=1}^{Q}\Gamma^{B_j}\left(1-bj+\sum_{i=1}^{u}\beta_j^{(i)}\eta_{H_i,h_i}\right)} \quad (1.13)$$

and

$$\phi_i'=\frac{\prod_{j=1}^{N_i}\Gamma^{C_j^{(i)}}\left(1-c_j^{(i)}+\gamma_j^{(i)}\eta_{H_i,h_i}\right)\prod_{j=1}^{M_i}\Gamma\left(d_j^{(i)}-\delta_j^{(i)}\eta_{H_i,h_i}\right)}{\prod_{j=N_i+1}^{P_i}\Gamma^{C_j^{(i)}}\left(c_j^{(i)}-\gamma_j^{(i)}\eta_{H_i,h_i}\right)\prod_{j=M_i+1}^{Q_i}\Gamma^{(D_j^{(i)}}\left(1-d_j^{(i)}+\delta_j^{(i)}\eta_{H_i,h_i}\right)},i=1,\cdots,u \quad (1.14)$$

where $\eta_{H_i,h_i}=\dfrac{d_{H^{(i)}}^{(i)}+h_i}{\delta_{h^{(i)}}^{(i)}},i=1,\cdots,u$

We consider a generalized transcendental function called Gimel function of several complex variables.

$$\gimel(z_1,\cdots,z_r)=\gimel_{X;p_{i_r},q_{i_r},\tau_{i_r}:R_r:Y}^{U;0,n_r:V}\left(\begin{array}{c|c} z_1 & \mathbb{A};\mathbf{A}:\ A\\ \cdot & \cdot\\ \cdot & \cdot\\ \cdot & \cdot\\ z_r & \mathbb{B};\mathbf{B}:\ B\end{array}\right)=\frac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r}\psi(s_1,\cdots,s_r)\prod_{k=1}^{r}\theta_k(s_k)z_k^{s_k}\,\mathrm{d}s_1\cdots\mathrm{d}s_r \quad (1.15)$$

with $\omega=\sqrt{-1}$

The following quantities $\mathbb{A},\mathbf{A},A,\mathbb{B},\mathbf{B},B,X,Y,U,V,\psi(s_1,\cdots,s_r)$ and $\theta_k(s_k)(k=1,\cdots,r)$ are defined by Ayant [2].

Following the lines of Braaksma ([3] p. 278), we may establish the the asymptotic expansion in the following convenient form :

$\aleph(z_1,\cdots,z_r)=0(\,|z_1|^{\alpha_1},\cdots,|z_r|^{\alpha_r}\,),max(\,|z_1|,\cdots,|z_r|\,)\rightarrow 0$

$\aleph(z_1,\cdots,z_r)=0(\,|z_1|^{\beta_1},\cdots,|z_r|^{\beta_r}\,),min(\,|z_1|,\cdots,|z_r|\,)\rightarrow\infty$ where $i=1,\cdots,r:$

$\alpha_i=\min\limits_{1\leqslant j\leqslant m^{(i)}}Re\left[D_j^{(i)}\left(\dfrac{d_j^{(i)}}{\delta_j^{(i)}}\right)\right]$ and $\beta_i=\max\limits_{1\leqslant j\leqslant n^{(i)}}Re\left[C_j^{(i)}\left(\dfrac{c_j^{(i)}-1}{\gamma_j^{(i)}}\right)\right]$

2. Integral representation of generalized Lauricella function of several variables

In order to evaluate a number of integrals of modified multivariable H-function, we first establish the formula

$$\int_a^b(t-a)^{\alpha-1}(b-t)^{\beta-1}\prod_{j=1}^{k}(f_jt+g_j)^{\sigma_j}\mathrm{d}t=(b-a)^{\alpha+\beta-1}B(\alpha,\beta)\prod_{j=1}^{k}(af_j+g_j)^{\sigma_j}$$

$$F^{1:1,\cdots,1;1\cdots,1}_{1:0,\cdots,0;0,\cdots,0}\left(\begin{array}{c}(\alpha:h_1,\cdots,h_l,1,\cdots,1):(\lambda_1:1),\cdots,(\lambda_l:1);(-\sigma_1:1),\cdots,(-\sigma_k:1)\\ \cdots\\ (\alpha+\beta:h_1,\cdots,h_l,1,\cdots,1):-,\cdots,-;-,\cdots,-\end{array}\right.$$

$$\left.;\tau_1(b-a)^{h_1},\cdots,\tau_l(b-a)^{h_l},-\frac{(b-a)f_1}{af_1+g_1},\cdots,-\frac{(b-a)f_k}{af_k+g_k}\right) \tag{2.1}$$

where $a,b\in\mathbb{R}(a<b),\alpha,\beta,f_i,g_i,\sigma_i,\tau_j,h_j\in\mathbb{C},\lambda_j\in\mathbb{R}^+(i=1,\cdots,k;j=1,\cdots,l)$

$$min(Re(\alpha),Re(\beta))>0,\ \max_{1\leqslant j\leqslant l}\left\{\left|\tau_j(b-a)^{h_j}\right|\right\}<1,\ \max_{1\leqslant j\leqslant k}\left\{\left|\frac{(b-a)f_i}{af_i+g_i}\right|\right\}<1,$$

and $F^{1:1,\cdots,1;1\cdots,1}_{1:0,\cdots,0;0,\cdots,0}$ is a particular case of the generalized Lauricella function introduced by Srivastava-Daoust [10, page 454] given by :

$$F^{1:1,\cdots,1;1\cdots,1}_{1:0,\cdots,0;0,\cdots,0}\left(\begin{array}{c}(\alpha:h_1,\cdots,h_l,1,\cdots,1):(\lambda_1:1),\cdots,(\lambda_l:1);(-\sigma_1:1),\cdots,(-\sigma_k:1)\\ \cdots\\ (\alpha+\beta:h_1,\cdots,h_l,1,\cdots,1):-,\cdots,-;-,\cdots,-\end{array}\right.$$

$$\left.;\tau_1(b-a)^{h_1},\cdots,\tau_l(b-a)^{h_l},-\frac{(b-a)f_1}{af_1+g_1},\cdots,-\frac{(b-a)f_k}{af_k+g_k}\right)=\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\prod_{j=1}^{l}\Gamma(\lambda_j)\prod_{j=1}^{k}\Gamma(-\sigma_j)}$$

$$\frac{1}{(2\pi\omega)^{l+k}}\int_{L_1''}\cdots\int_{L_{l+k}''}\frac{\Gamma\left(\alpha+\sum_{j=1}^{l}h_js_j+\sum_{j=1}^{k}s_{l+j}\right)}{\Gamma\left(\alpha+\beta+\sum_{j=1}^{l}h_js_j+\sum_{j=1}^{k}s_{l+j}\right)}\prod_{j=1}^{l}\Gamma(\lambda_j+s_j)\prod_{j=1}^{k}\Gamma(-\sigma_j+s_{l+j})$$

$$\prod_{j=1}^{l+k}\Gamma(-s_j)z_1^{s_1}\cdots z_l^{s_l}z_{l+1}^{s_{l+1}}\cdots,z_{l+k}^{s_{l+k}}\,\mathrm{d}s_1\cdots\mathrm{d}s_{l+k} \tag{2.2}$$

Here the contour $L_j''s$ are defined by $L_j''=L''_{w\zeta_j\infty}(Re(\zeta_j)=v_j'')$ starting at the point $v_j''-\omega\infty$ and terminating at the point $v_j''+\omega\infty$ with $v_j''\in\mathbb{R}(j=1,\cdots,l)$ and each of the remaining contour $L''_{l+1},\cdots,L''_{l+k}$ run from $-\omega\infty$ to $\omega\infty$

(2.1) can be easily established by expanding $\prod_{j=1}^{l}\left[1-\tau_j(t-a)^{h_i}\right]^{-\lambda_j}$ by means of the formula :

$$(1-z)^{-\alpha}=\sum_{r=0}^{\infty}\frac{(\alpha)_r}{r!}z^r(|z|<1) \tag{2.3}$$

integrating term by term with the help of the integral given by Saigo and Saxena [8, page 93, eq.(3.2)] and applying the definition of the generalized Lauricella function [10, page 454].

3. Notations

We shall note :

$$\theta_i=\prod_{j=1}^{l}\left[1-\tau_j(t-a)^{h_i}\right]^{-\zeta_j^{(i)}},\zeta_j^{(i)}>0(i=1,\cdots,r),\ \theta_i'=\prod_{j=1}^{l}\left[1-\tau_j(t-a)^{h_i}\right]^{-\zeta_j'^{(i)}},\zeta_j'^{(i)}>0(i=1,\cdots,s)$$

$$\theta_i'' = \prod_{j=1}^{l} \left[1-\tau_j(t-a)^{h_i}\right]^{-\zeta_j''^{(i)}}, \zeta_j''^{(i)} > 0 (i=1,\cdots,u)$$

$$\theta_i''' = \prod_{j=1}^{l} \left[1-\tau_j(t-a)^{h_i}\right]^{-\zeta_j'''^{(i)}}, \zeta_j'''^{(i)} > 0 (i=1,\cdots,v)$$

$$U = 0,n_2;0,n_3;\cdots;0,n_{r-1};0,n_2';0,n_3';\cdots;0,n_{s-1}';\underbrace{0,0;\cdots;0,0}_{l+k} \tag{3.1}$$

$$V = m^{(1)},n^{(1)};m^{(2)},n^{(2)};\cdots;m^{(r)},n^{(r)};m'^{(1)},n'^{(1)};m'^{(2)},n'^{(2)};\cdots;m'^{(s)},n'^{(s)};$$

$$\underbrace{1,0;\cdots;1,0}_{k},\underbrace{1,0;\cdots;1,0}_{l} \tag{3.2}$$

$$X = p_{i_2},q_{i_2},\tau_{i_2};R_2;\cdots;p_{i_{r-1}},q_{i_{r-1}},\tau_{i_{r-1}}:R_{r-1};\underbrace{0;\cdots;0}_{s+l+k} \tag{3.3}$$

$$Y = p'_{i'^{(1)}},q'_{i'^{(1)}},\tau'_{i'^{(1)}};R'^{(1)};\cdots;p'_{i^{(s)}},q'_{i'^{(s)}};\tau'_{i'^{(s)}};R'^{(s)};\underbrace{0,1;\cdots;0,1}_{l},\underbrace{0,1;\cdots;0,1}_{k} \tag{3.4}$$

$$\mathbb{A} = [(a_{2j};\alpha_{2j}^{(1)},\alpha_{2j}^{(2)};A_{2j})]_{1,n_2},[\tau_{i_2}(a_{2ji_2};\alpha_{2ji_2}^{(1)},\alpha_{2ji_2}^{(2)};A_{2ji_2})]_{n_2+1,p_{i_2}};[(a_{3j};\alpha_{3j}^{(1)},\alpha_{3j}^{(2)},\alpha_{3j}^{(3)};A_{3j})]_{1,n_3},$$

$$[\tau_{i_3}(a_{3ji_3};\alpha_{3ji_3}^{(1)},\alpha_{3ji_3}^{(2)},\alpha_{3ji_3}^{(3)};A_{3ji_3})]_{n_3+1,p_{i_3}};\cdots;[(\mathrm{a}_{(r-1)j};\alpha_{(r-1)j}^{(1)},\cdots,\alpha_{(r-1)j}^{(r-1)};A_{(r-1)j]1,n_{r-1}},$$

$$[\tau_{i_{r-1}}(a_{(r-1)ji_{r-1}};\alpha_{(r-1)ji_{r-1}}^{(1)},\cdots,\alpha_{(s-1)ji_{r-1}}^{(r-1)};A_{(r-1)ji_{r-1}})]_{n_{r-1}+1,p_{i_{r-1}}};$$

$$(a_{2j}';\alpha_{2j}'^{(1)},\alpha_{2j}'^{(2)};A_{2j}')]_{1,n_2'},[\tau_{i_2'}'(a_{2ji_2'}';\alpha_{2ji_2'}'^{(1)},\alpha_{2ji_2'}'^{(2)};A_{2ji_2'}')]_{n_2'+1,p_{i_2'}'};[(a_{3j}';\alpha_{3j}'^{(1)},\alpha_{3j}'^{(2)},\alpha_{3j}'^{(3)};A_{3j}')]_{1,n_3'},$$

$$[\tau_{i_3'}'(a_{3ji_3'}';\alpha_{3ji_3'}'^{(1)},\alpha_{3ji_3'}'^{(2)},\alpha_{3ji_3'}'^{(3)};A_{3ji_3'}')]_{n_3'+1,p_{i_3'}'};\cdots;[(\mathrm{a'}_{(s-1)j};\alpha_{(s-1)j}'^{(1)},\cdots,\alpha_{(s-1)j}'^{(s-1)};A_{(s-1)j}')]_{1,n_{s-1}'},$$

$$,[\tau_{i_{s-1}'}'(a_{(s-1)ji_{s-1}'}';\alpha_{(s-1)ji_{s-1}'}'^{(1)},\cdots,\alpha_{(s-1)ji_{s-1}'}'^{(s-1)};A_{(s-1)ji_{s-1}'}')]_{n_{s-1}'+1,p_{i_{s-1}'}'};$$

$$\mathbf{A} = [(\mathrm{a}_{rj};\alpha_{rj}^{(1)},\cdots,\alpha_{rj}^{(r)},\underbrace{0,\cdots,0}_{s},\underbrace{0,\cdots,0}_{l},\underbrace{0,\cdots,0}_{k};A_{rj})]_{1,n_r},$$

$$[\tau_{i_r}(a_{rji_r};\alpha_{rji_r}^{(1)},\cdots,\alpha_{rji_r}^{(r)},\underbrace{0,\cdots,0}_{s},\underbrace{0,\cdots,0}_{l},\underbrace{0,\cdots,0}_{k};A_{rji_r})]_{1,n_r},$$

$$[(\mathrm{a}_{sj}';\underbrace{0,\cdots,0}_{r},\alpha_{sj}'^{(1)},\cdots,\alpha_{sj}'^{(s)},\underbrace{0,\cdots,0}_{l},\underbrace{0,\cdots,0}_{k};A_{sj}')]_{1,n_s'},$$

$$[\tau_{i_s'}'(a_{sji_s'}';\underbrace{0,\cdots,0}_{r},\alpha_{sji_s'}'^{(1)},\cdots,\alpha_{sji_s'}'^{(s)},\underbrace{0,\cdots,0}_{l},\underbrace{0,\cdots,0}_{k};A_{sji_s'}')]_{n_s'+1,p_{i_s'}'}:$$

$$\mathrm{A} = [(\mathrm{c}_j^{(1)},\gamma_j^{(1)};C_j^{(1)})]_{1,n^{(1)}},[\tau_{i^{(1)}}(c_{ji^{(1)}}^{(1)},\gamma_{ji^{(1)}}^{(1)};C_{ji^{(1)}}^{(1)})]_{n^{(1)}+1,p_i^{(1)}};\cdots;$$

$$[(c_j^{(r)},\gamma_j^{(r)};C_j^{(r)})]_{1,m^{(r)}},[\tau_{i^{(r)}}(c_{ji^{(r)}}^{(r)},\gamma_{ji^{(r)}}^{(r)};C_{ji^{(r)}}^{(r)})]_{m^{(r)}+1,p_i^{(r)}},$$

$$[(\text{c'}_j^{(1)},\gamma_j'^{(1)};C_j'^{(1)})]_{1,n'^{(1)}},[\tau_{i'^{(1)}}(c_{ji^{(1)}}'^{(1)},\gamma_{ji^{(1)}}'^{(1)};C_{ji^{(1)}}'^{(1)})]_{n'^{(1)}+1,p_i'^{(1)}};\cdots;$$

$$[(c_j'^{(s)},\gamma_j'^{(s)};C_j'^{(s)})]_{1,m'^{(s)}},[\tau'_{i'^{(s)}}(c_{ji'^{(s)}}'^{(s)},\gamma_{ji'^{(s)}}'^{(s)};C_{ji'^{(s)}}'^{(s)})]_{m'^{(s)}+1,p_i'^{(s)}},\underbrace{(1,0;1),\cdots,(1,0;1)}_{l},\underbrace{(1,0;1),\cdots,(1,0;1)}_{k} \quad (3.5)$$

$$A_1=(1-\alpha-\sum_{i=1}^{u}\eta_{H_i,h_i}a_i-\sum_{i=1}^{v}\eta_{G_i,g_i}a_i';\mu_1,\cdots,\mu_r,\mu_1',\cdots,\mu_s',h_1,\cdots,h_l,\underbrace{1,\cdots,1}_{k};1)$$

$$(1-\beta-\sum_{i=1}^{u}\eta_{H_i,h_i}b_i-\sum_{i=1}^{v}\eta_{G_i,g_i}b_i';\rho_1,\cdots,\rho_r,\rho_1',\cdots,\rho_s',\underbrace{0,\cdots,0}_{l},\underbrace{0,\cdots,0}_{l};1),$$

$$[1-\lambda_j-\sum_{i=1}^{u}\eta_{H_i,h_i}\zeta_j''^{(i)}-\sum_{i=1}^{v}\eta_{G_i,g_i}\zeta_j'''^{(i)};\zeta_j^{(1)},\cdots,\zeta_j^{(r)},\zeta_j'^{(1)}\cdots,\zeta_j'^{(s)},\underbrace{0,\cdots,1,\cdots,0}_{\substack{l\\ \text{j}}},\underbrace{0\cdots,0}_{k};1]_l,$$

$$[1+\sigma_j-\sum_{i=1}^{u}\eta_{H_i,h_i}\lambda_j''^{(i)}-\sum_{i=1}^{v}\eta_{G_i,g_i}\lambda_j'''^{(i)};\lambda_j^{(1)},\cdots,\lambda_j^{(r)},\lambda_j'^{(1)}\cdots,\lambda_j'^{(s)},\underbrace{0,\cdots,0}_{l},\underbrace{0\cdots,1,\cdots,0}_{\substack{k\\ \text{j}}};1]_{1,k} \quad (3.6)$$

$$\mathbb{B}=[(b_{2j};\beta_{2j}^{(1)},\beta_{2j}^{(2)};B_{2j})]_{1,m_2},[\tau_{i_2}(b_{2ji_2};\beta_{2ji_2}^{(1)},\beta_{2ji_2}^{(2)};B_{2ji_2})]_{m_2+1,q_{i_2}};[(b_{3j};\beta_{3j}^{(1)},\beta_{3j}^{(2)},\beta_{3j}^{(3)};B_{3j})]_{1,m_3},$$

$$[\tau_{i_3}(b_{3ji_3};\beta_{3ji_3}^{(1)},\beta_{3ji_3}^{(2)},\beta_{3ji_3}^{(3)};B_{3ji_3})]_{m_3+1,q_{i_3}};\cdots;[(\text{b}_{(r-1)j};\beta_{(r-1)j}^{(1)},\cdots,\beta_{(r-1)j}^{(r-1)};B_{(r-1)j})]_{1,m_{r-1}},$$

$$;\ (b_{2j}';\beta_{2j}'^{(1)},\beta_{2j}'^{(2)};B_{2j}')]_{1,m_2'},[\tau_{i_2'}'(b_{2ji_2'}';\beta_{2ji_2'}'^{(1)},\beta_{2ji_2'}'^{(2)};B_{2ji_2'}')]_{m_2'+1,q_{i_2'}'};[(b_{3j}';\beta_{3j}'^{(1)},\beta_{3j}'^{(2)},\beta_{3j}'^{(3)};B_{3j}')]_{1,m_3'},$$

$$[\tau_{i_3'}'(b_{3ji_3'}';\beta_{3ji_3'}'^{(1)},\beta_{3ji_3'}'^{(2)},\beta_{3ji_3'}'^{(3)};b_{3ji_3'}')]_{m_3'+1,q_{i_3'}'};\cdots;[(\text{a'}_{(s-1)j};\alpha_{(s-1)j}'^{(1)},\cdots,\alpha_{(s-1)j}'^{(s-1)};A_{(s-1)j}')]_{1,n_{s-1}'},$$

$$[\tau_{i_{s-1}'}'(b_{(s-1)ji_{s-1}'}';\beta_{(s-1)ji_{s-1}'}'^{(1)},\cdots,\beta_{(s-1)ji_{s-1}'}'^{(s-1)};B_{(s-1)ji_{s-1}'}')]_{m_{s-1}'+1,q_{i_{s-1}'}'};$$

$$\mathbf{B}=[\tau_{i_r}(b_{rji_r};\beta_{rji_r}^{(1)},\cdots,\beta_{rji_r}^{(r)},\underbrace{0,\cdots,0}_{s},\underbrace{0,\cdots,0}_{l},\underbrace{0,\cdots,0}_{k};B_{rji_r})]_{m_r+1,q_r},$$

$$[\tau_{i_s'}'(b_{sji_s'};\underbrace{0,\cdots,0}_{r},\beta_{sji_s'}'^{(1)},\cdots,\beta_{sji_s'}'^{(s)},\underbrace{0,\cdots,0}_{l},\underbrace{0,\cdots,0}_{k};B_{sji_s'}')]_{m_s'+1,q_{i_s'}'}:$$

$$\text{B}=[(\text{d}_j^{(1)},\delta_j^{(1)};D_j^{(1)})]_{1,m^{(1)}},[\tau_{i^{(1)}}(d_{ji^{(1)}}^{(1)},\delta_{ji^{(1)}}^{(1)};D_{ji^{(1)}}^{(1)})]_{m^{(1)}+1,q_i^{(1)}};\cdots;$$

$$[(d_j^{(r)},\delta_j^{(r)};D_j^{(r)})_{1,m^{(r)}}],[\tau_{i^{(r)}}(d_{ji^{(r)}}^{(r)},\delta_{ji^{(r)}}^{(r)};D_{ji^{(r)}}^{(r)})]_{m^{(r)}+1,q_i^{(r)}},$$

$$[(\mathrm{d'}_j^{(1)},\delta_j'^{(1)};D_j'^{(1)})]_{1,m'^{(1)}},[\tau'_{i'^{(1)}}(d'^{(1)}_{ji'^{(1)}},\delta'^{(1)}_{ji'^{(1)}};D'^{(1)}_{ji'^{(1)}})]_{m'^{(1)}+1,q_i'^{(1)}};\cdots;$$

$$[(d_j'^{(s)},\delta_j'^{(s)};D_j'^{(s)})]_{1,m'^{(s)}},[\tau'_{i'^{(s)}}(d'^{(s)}_{ji'^{(s)}},\delta'^{(s)}_{ji'^{(s)}};D'^{(s)}_{ji'^{(s)}})]_{m'^{(s)}+1,q_i'^{(s)}},\underbrace{(0,1;1),\cdots,(0,1;1)}_{l},\underbrace{(0,1;1),\cdots,(0;1,1)}_{k} \tag{3.7}$$

$$B_1=(1-\alpha-\beta-\sum_{i=1}^{u}\eta_{H_i,h_i}(a_i+b_i)-\sum_{i=1}^{v}(a_i'+b_i')\eta_{G_i,g_i};\mu_1+\rho_1,\cdots,\mu_r+\rho_r,\mu_1'+\rho_1',\cdots,\mu_r'+\rho_r',$$

$$h_1,\cdots,h_l,\underbrace{1,\cdots,1}_{k};1),$$

$$[1-\lambda_j-\sum_{i=1}^{u}\eta_{H_i,h_i}\zeta_j''^{(i)}-\sum_{i=1}^{v}\zeta_j'''^{(i)}\eta_{G_i,g_i};\zeta_j^{(1)},\cdots,\zeta_j^{(r)},\zeta_j'^{(1)}\cdots,\zeta_j'^{(s)},\underbrace{0,\cdots,0}_{l},\underbrace{0,\cdots,0}_{k};1]_{1,l} \tag{3.8}$$

$$[1+\sigma_j-\sum_{i=1}^{u}\eta_{H_i,h_i}\lambda_j''^{(i)}-\sum_{i=1}^{v}\lambda_j'''^{(i)}\eta_{G_i,g_i};\lambda_j^{(1)},\cdots,\lambda_j^{(r)},\lambda_j'^{(1)}\cdots,\lambda_j'^{(s)},\underbrace{0,\cdots,0}_{l},\underbrace{0,\cdots,0}_{k};1]_{1,k} \tag{3.9}$$

$$P_1=(b-a)^{\alpha+\beta-1}\left\{\prod_{j=1}^{h}(af_j+g_j)^{\sigma_j}\right\} \tag{3.10}$$

$$B_{u,v}=(b-a)^{\sum_{i=1}^{v}(a_i'+b_i')\eta G_i,g_i+\sum_{i=1}^{u}(a_i+b_i)\eta_{H_i,h_i}}\left\{\prod_{j=1}^{h}(af_j+g_j)^{-\sum_{i=1}^{v}\lambda_i'''\eta_{G_i,g_i}-\sum_{i=1}^{u}\lambda_i''\eta_{H_i,h_i}}\right\}_{G_v} \tag{3.11}$$

4. Main integral

Theorem

$$\int_a^b(t-a)^{\alpha-1}(b-t)^{\beta-1}\prod_{j=1}^{l}\left[1-\tau_j(t-a)^{h_i}\right]^{-\lambda_j}\prod_{j=1}^{k}(f_jt+g_j)^{\sigma_j}$$

$$\bar{I}\begin{pmatrix} \mathrm{z}_1''\theta_1''(t-a)^{a_1}(b-t)^{b_1}\prod_{j=1}^{k}(f_jt+g_j)^{-\lambda_j''^{(1)}}\\ \cdot\\ \cdot\\ \cdot\\ \mathrm{z}_u''\theta_u''(t-a)^{a_u}(b-t)^{b_u}\prod_{j=1}^{k}(f_jt+g_j)^{-\lambda_j''^{(u)}}\end{pmatrix}$$

$$A\begin{pmatrix} \mathrm{z}_1'''\theta_1'''(t-a)^{a_1'}(b-t)^{b_1'}\prod_{j=1}^{k}(f_jt+g_j)^{-\lambda_j'''^{(1)}}\\ \cdot\\ \cdot\\ \cdot\\ \mathrm{z}_v'''\theta_v'''(t-a)^{a_v'}(b-t)^{b_v'}\prod_{j=1}^{k}(f_jt+g_j)^{-\lambda_j'''^{(v)}}\end{pmatrix}$$

$$\gimel\begin{pmatrix} z_1\theta_1(t-a)^{\mu_1}(b-t)^{\rho_1}\prod_{j=1}^k(f_jt+g_j)^{-\lambda_j^{(1)}} \\ \cdot \\ \cdot \\ \cdot \\ z_r\theta_r(t-a)^{\mu_r}(b-t)^{\rho_r}\prod_{j=1}^k(f_jt+g_j)^{-\lambda_j^{(r)}} \end{pmatrix}$$

$$\gimel\begin{pmatrix} z'_1\theta'_1(t-a)^{\mu'_1}(b-t)^{\rho'_1}\prod_{j=1}^k(f_jt+g_j)^{-\lambda_j'^{(1)}} \\ \cdot \\ \cdot \\ \cdot \\ z'_s\theta'_s(t-a)^{\mu'_s}(b-t)^{\rho'_s}\prod_{j=1}^k(f_jt+g_j)^{-\lambda_j'^{(s)}} \end{pmatrix}\mathrm{d}t$$

$$= P_1 \sum_{G_i=1}^{\alpha^{(i)}}\sum_{g_i=1}^{\infty}\phi_1\frac{\prod_{i=1}^v\phi_i u_i^{\eta_{G_i,g_i}}(-)^{\sum_{i=1}^v g_i}}{\prod_{i=1}^v\epsilon_{G_i}^{(i)}g_i!}\sum_{H_i=1}^{M_i}\sum_{h_i=1}^{\infty}\phi'_1\frac{\prod_{i=1}^u\phi'_i z_i^{\eta_{h_i,H_i}}(-)^{\sum_{i=1}^u h_i}}{\prod_{i=1}^u\delta_{h^{(i)}}^{(i)}\prod_{i=1}^u h_i!}B_{u,v}$$

$$\gimel^{U;0,n_r+n'_s+4:V}_{X;p_{i_r}+p'_{i_s}+4:q_{i_r}+q'_{i'_s}+4,\tau_{i_r},\tau_{i'_s}:R_r,R_{s'}:Y}\left(\begin{array}{c|c} \frac{z_1(b-a)^{\mu_1+\rho_1}}{\prod_{j=1}^k(af_j+g_j)^{\lambda_j^{(1)}}} & \mathbb{A};\mathrm{A}_1,\mathbf{A}:A \\ \cdots & \cdot \\ \cdots & \cdot \\ \frac{z_r(b-a)^{\mu_r+\rho_r}}{\prod_{j=1}^k(af_j+g_j)^{\lambda_j^{(r)}}} & \cdot \\ \frac{z'_1(b-a)^{\mu'_1+\rho'_1}}{\prod_{j=1}^k(af_j+g_j)^{\lambda_j'^{(1)}}} & \cdot \\ \cdots & \cdot \\ \cdots & \cdot \\ \frac{z'_s(b-a)^{\mu'_s+\rho'_s}}{\prod_{j=1}^k(af_j+g_j)^{\lambda_j'^{(s)}}} & \cdot \\ \tau_1(b-a)^{h_1} & \cdot \\ \cdots & \cdot \\ \cdots & \cdot \\ \tau_l(b-a)^{h_l} & \cdot \\ \frac{(b-a)f_1}{af_1+g_1} & \cdot \\ \cdots & \mathbb{B};\mathbf{B},\mathrm{B}_1:B \\ \cdots & \\ \frac{(b-a)f_k}{af_k+g_k} & \end{array}\right) \tag{4.1}$$

We obtain a gimel-function of $(r+s+l+k)$-variables.

Provided that

(A) $a,b\in\mathbb{R}(a<b);\mu_i,\mu'_u,\rho_i,\rho'_u,\lambda_j^{(i)},\lambda_j'^{(u)},h_v\in\mathbb{R}^+,f_i,g_j,\tau_v,\sigma_j,\lambda_v\in\mathbb{C}\ (i=1,\cdots,r;j=1,\cdots;k;$

$u=1,\cdots,s;v=1,\cdots,l),a_i,b_i,\lambda_j''^{(i)},\zeta_j''^{(i)}\in\mathbb{R}^+,(i=1,\cdots,u;j=1,\cdots,k)$

$a'_i,b'_i,\lambda_j'''^{(i)},\zeta_j'''^{(i)}\in\mathbb{R}^+,(i=1,\cdots,v;j=1,\cdots,k)$

(B) $\max_{1\leqslant j\leqslant k}\left\{\left|\frac{(b-a)f_i}{af_i+g_i}\right|\right\}<1,\ \max_{1\leqslant j\leqslant l}\left\{\left|\tau_j(b-a)^{h_j}\right|\right\}<1$

(C) $Re\left(\alpha+\sum_{i=1}^{v}a'_i\eta_{G_i,g_i}+\sum_{i=1}^{u}a_i\eta_{H_i,h_i}\right)+\sum_{i=1}^{r}\mu_i\min_{1\leqslant j\leqslant m^{(i)}}Re\left[D_j^{(i)}\left(\frac{d_j^{(i)}}{\delta_j^{(i)}}\right)\right]+\sum_{i=1}^{s}\mu'_i\min_{1\leqslant j\leqslant m'^{(i)}}Re\left[D'^{(i)}_j\left(\frac{d'^{(i)}_j}{\delta'^{(i)}_j}\right)\right]>0$

(C) $Re\left(\beta+\sum_{i=1}^{v}b_i\eta_{G_i,g_i}+\sum_{i=1}^{u}b'_i\eta_{H_i,h_i}\right)+\sum_{i=1}^{r}\rho_i\min_{1\leqslant j\leqslant m^{(i)}}Re\left[D_j^{(i)}\left(\frac{d_j^{(i)}}{\delta_j^{(i)}}\right)\right]+\sum_{i=1}^{s}\rho'_i\min_{1\leqslant j\leqslant m'^{(i)}}Re\left[D'^{(i)}_j\left(\frac{d'^{(i)}_j}{\delta'^{(i)}_j}\right)\right]>0$

(D) $Re\left(\alpha+\sum_{i=1}^{v}\eta_{G_i,g_i}a'_i+\sum_{i=1}^{u}\eta_{H_i,h_i}a_i+\sum_{i=1}^{r}\mu_is_i+\sum_{i=1}^{s}t_i\mu'_i\right)>0$

$$Re\left(\beta+\sum_{i=1}^{v}\eta_{G_i,g_i}b'_i+\sum_{i=1}^{u}\eta_{H_i,h_i}b_i+\sum_{i=1}^{r}v_is_i+\sum_{i=1}^{s}t_i\rho'_i\right)>0$$

$$Re\left(\lambda_j+\sum_{i=1}^{v}\eta_{G_i,g_i}{}_i\lambda'''^{(i)}_j+\sum_{i=1}^{u}\eta_{H_i,h_i}\lambda''^{(i)}_j+\sum_{i=1}^{r}s_i\zeta_j^{(i)}+\sum_{i=1}^{s}t_i\zeta'^{(i)}_j\right)>0(j=1,\cdots,l);$$

$$Re\left(-\sigma_j+\sum_{i=1}^{v}\eta_{G_i,g_i}\lambda'''^{(i)}+\sum_{i=1}^{u}\eta_{H_i,h_i}\lambda''^{(i)}_j+\sum_{i=1}^{r}s_i\lambda_j^{(i)}+\sum_{i=1}^{s}t_i\lambda'^{(i)}_j\right)>0(j=1,\cdots,k);$$

(E) Let $\frac{1}{2}\pi B_i^{(k)}=\frac{1}{2}\pi A_i^{(k)}-\mu_i-\rho_i-\sum_{l=1}^{k}\lambda_j^{(i)}-\sum_{l=1}^{l}\zeta_j^{(i)}>0\quad(i=1,\cdots,r)$

and $\frac{1}{2}\pi B'^{(k)}_i=\frac{1}{2}\pi A'^{(k)}_i-\mu'_i-\rho'_i-\sum_{l=1}^{k}\lambda'^{(i)}_j-\sum_{l=1}^{l}\zeta'^{(i)}_j>0\quad(i=1,\cdots,s)$

(F) $\left|arg\left(z_i\prod_{j=1}^{l}\left[1-\tau_j(t-a)^{h_i}\right]^{-\zeta_j^{(i)}}\prod_{j=1}^{k}(f_jt+g_j)^{-\lambda_j^{(i)}}\right)\right|<\frac{1}{2}B_i^{(k)}\pi\quad(a\leqslant t\leqslant b;i=1,\cdots,r)$

$$\left|arg\left(z'_i\prod_{j=1}^{l}\left[1-\tau'_j(t-a)^{h'_i}\right]^{-\zeta'^{(i)}_j}\prod_{j=1}^{k}(f_jt+g_j)^{-\lambda'^{(i)}_j}\right)\right|<\frac{1}{2}B'^{(k)}_i\pi\quad(a\leqslant t\leqslant b;i=1,\cdots,s)$$

Proof

To prove (4.1), first, we express in serie the class of multivariable A-function defined by Gautam et al. [4] and the multivariable I-function defined by Prathima et al. [6] with the help of (1.2) and (1.12) respectively, we interchange the order of summations and t-integral (which is permissible under the conditions stated). Expressing the Gimel-functions of r-variables and s-variables in terms of Mellin-Barnes type integrals contour with the help of (1.10) respectively and interchange the order of integrations which is justifiable due to absolute convergence of the integral involved in the process . Now collect the power of $\left[1-\tau_j(t-a)^{h_i}\right]$ with $(i=1,\cdots,r;j=1,\cdots,l)$ and collect the power of (f_jt+g_j) with $j=1,\cdots,k$. Use the equations (2.1) and (2.2) and express the result in Mellin-Barnes integrals contour. Interpreting the $(r+s+k+l)$ dimensional Mellin-Barnes integrals in multivariable Gimel-function defined , we obtain the equation (4.1).

Remark 1:

By the following similar procedure, the results of this document can be extented to product of any finite number of multivariable Gimel-functions defined by Ayant [2], multivariable I-function defined by Prathima et al. [6] and multivariable A-function defined by Gautam et al. [4] .
We have similar integrals concerning other multivariables special functions.
By the following similar procedure, we can to obtain a product of any finite number of multivariable Gimel-functions class of polynomials [9] and class of Srivastava and Garg polynomials [11].

Remark 2:
We obtain the same formulae concerning the multivariable Aleph- function defined by Ayant [1], the multivariable I-function defined by Prathima et al. [6], the multivariable I-function defined by Prasad [5] and the multivariable H-function defined by Srivastava and Panda [13,14].

5. Conclusion.

The importance of our all the results lies in their manifold generality. Firstly, in view of the Eulerian integrals with general arguments utilized in this study, we can obtain a large variety of single Eulerian finites integrals. Secondly by specializing the various parameters as well as variables in the generalized multivariable Gimel-function, we get a several formulae involving remarkably wide variety of useful functions (or product of such functions) which are expressible in terms of E, F, G, H, I, Aleph-function of one and several variables and simpler special functions of one and several variables. Thirdly, by specialising the various parameters as well as variables of class of the multivariable $\bar{I}$ function multivariable A-function, we can get a large number of special function of one and several variables. Hence the formulae derived in this paper are most general in character and may prove to be useful in several interesting cases appearing in literature of Pure and Applied Mathematics and Mathematical Physics.

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