Bipolar-Valued Fuzzy Ideals of Ring and Bipolar-Valued Fuzzy Ideal Extensions in Subrings

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Abstract

In this paper, we study some properties of bipolar-valued fuzzy ideals of ring and prove some results of these. Using basic definitions, we derive the some important theorems prime ideal, subrings and commutative subrings are applied into the bipolar-valued fuzzy ideals. Also we introduce bipolar-valued fuzzy extensions of ideal in subrings and some related results are investigated.

Keywords: *Fuzzy ideals of ring, bipolar valued fuzzy subring, bipolar valued fuzzy ideal and bipolar-valued fuzzy extensions. Sub properties.*

INTRODUCTION

In this paper we present a new formulations of bipolar-valued fuzzy rings based on the notion of bipolar-valued fuzzy space. A relation between bipolar-valued fuzzy ring based on bipolar-valued fuzzy space and ordinary rings is obtained in terms of induction and correspondences. M .Z. Alma [4] introduced the concept of fuzzy ring and established the notion of fuzzy ideal and quotient ring. The author Mohammed F. Marashdeh [5] introduced the bipolar-valued fuzzy rings based on the notion of fuzzy space to the case of bipolar-valued fuzzy set. Won Kyun Jeong [12] introduced the notions of the extensions of bipolar-valued fuzzy ideals, bipolar-valued fuzzy prime ideals and bipolar-valued fuzzy commutative ideals in BCK-algebras are introduced and several properties are investigated. P. K. Sharma [6] attempt has been made to study some algebraic nature of bipolar-valued fuzzy ideals of near ring and their properties. K. Meena [3] in this we study some generalized properties of bipolar-valued L-fuzzy subrings. In this direction the concept of image and inverse image of an bipolar-valued L-fuzzy set under ring homomorphism are discussed. Further the concept of bipolar-valued L-fuzzy quotient subring and bipolar-valued L-fuzzy ideal of an bipolar-valued L-fuzzy subring are studied.

I. PRELIMINARIES

1.1 Definition [2]

A ring is a set R together with two operations on R addition and multiplication such that

1. *Addition is associative, for all $a, b, c \in R,\ a + (b + c) = (a + b) + c$.*
2. *Addition is commutative, that is $a, b \in R,\ a + b = b + a$.*
3. *R has a zero element that is, there is an element 0 in R such that, for all $a \in R,\ a + 0 = a$.*
4. *For every $a \in R$, there is an element $-a$ in R such that $a + (-a) = 0$.*
5. *Multiplication is associative, that is for all $a, b, c \in R,\ a(bc) = (ab)c$.*
6. *Multiplication is distributive over addition, that is for all $z, b, c \in R,\ (a + b)c = ac + bc$ and $a(b + c) = ab + ac$.*
7. *Multiplication is commutative, that is for all $a, b \in R,\ ab = ba$.*
8. *R has a multiplicative identity, that is there is an element 1 in R such that for all $a \in R,\ a \cdot 1 = a$.*

1.2 Definition [2]

A subset S of a ring R is a subring of R if S is closed under the addition and multiplication operation of R contain additive inverse and contains the multiplicative identity of R.

1.3 Definition [1]

Let R be a ring. A non-empty subset I of R is called a left ideal of R if

i. $a, b \in I \Rightarrow a - b \in I$

ii. $a \in I$ and $r \in R \Longrightarrow ra \in I$

If I is called a right ideal then

i. $a, b \in I \Longrightarrow a - b \in I$
ii. $a \in I$ and $r \in R \Longrightarrow ar \in I$.

I is called an ideal of R if I is both left and right ideal of R.

1.4 Definition [1]

A ring R is called commutative ring if multiplication is commutative, that is, $ab = ba$ for all $a, b \in R$.

1.5 Definition [1]

Let R be any ring and I be an ideal of R we have two well defined binary operations in R/I given by $(I + a) + (I + b) = I + (a + b)$ and $(I + a)(I + b) = I + ab$. It is easy to verify that R/I is a ring under these operations. The ring R/I is called the quotient ring of R modulo I.

1.6 Definition [1]

Let R and R' be rings. A function $f: R \longrightarrow R'$ is called a homomorphism if

i. $f(a + b) = f(a) + f(b)$
ii. $f(ab) = f(a)f(b)$, for all $a, b \in R$.

1.7 Definition [1]

Let R be a commutative ring. An ideal $P \neq R$ is called a prime ideal if $ab \in P \Longrightarrow$ either $a \in P$ or $b \in P$.

1.8 Definition [1]

A subset I of a commutative ring R is said to be an ideal if

1) $0 \in I$ and $a, b \in I \Longrightarrow a + b, -a \in I$ (so I is an additive subgroup)
2) $a \in I, x \in R \Longrightarrow xa \in I$.

1.9 Definition [2]

A field consists of a set F and two binary operation " $+$ " (addition) and " $\cdot$ " (multiplication), defined on R, for which the following conditions are satisfied

1) $(F, +, \cdot)$ is a ring
2) Multiplicative commutative: For any $a, b \in F, a \cdot b = b \cdot a$
3) Multiplicative identity: There exists $1 \in F$ such that $a \cdot 1 = 1 \cdot a = a$ for all $a \in F$.
4) Multiplicative inverse: If $a \in F$ and $a \neq 0$, there exists $b \in F$ such that $a \cdot b = b \cdot a = 1$.

1.10 Definition [1]

Let I be an ideal in the commutative ring R. Then quotient ring R/I is defined as follows

Set=$\{x + I / x \in R\}, 0 = I, 1 = 1 + I, -(x + I) = (-x) + I, (x + I) + (y + I) = (x + y) + I, (x + I)(y + I) = (xy) + I$.

1.11 Definition [1]

A subring of a commutative ring R is a subset S of R such that

1) $0,1 \in S$
2) $a \in S \Longrightarrow -a \in S$
3) $a, b \in S \Longrightarrow a + b \in S$
4) $a, b \in S \Longrightarrow ab \in S$.

1.12 Definition [12]

Algebra $(X,*,0)$ *of type* $(2,0)$ *is called a* BCK*-algebra if it satisfies the following conditions*

1) $((x*y)*(x*z))*(z*y)=0$
2) $(x*(x*y))*y=0$
3) $x*x=0$
4) $x*y=0, y*x=0 \Longrightarrow x=y$
5) $0*x=0$ *for all* $x,y,z \in X$.

II. FUZZY IDEAL OVER A FUZZY RING

2.1 Definition [9]

Let R *be a ring and* μ *be a fuzzy subset in* R. *Then* μ *is called a fuzzy ring of* R *if for every* $x,y \in R$ *the following conditions are satisfied*

i. $\mu(x+y) \geq \min(\mu(x),\mu(y))$
ii. $\mu(-x) \geq \mu(x)$
iii. $\mu(xy) \geq \min(\mu(x),\mu(y))$.

2.2 Definition [9]

Let R *be a ring and* μ *be a fuzzy subset in* R. *Then is called fuzzy ideal over a fuzzy ring* R *for every* $x,y \in R$ *the following conditions are satisfied*

i. $\mu(x+y) \geq \min(\mu(x),\mu(y))$
ii. $\mu(-x) \geq \mu(x)$
iii. $\mu(xy) \leq \max(\mu(x),\mu(y))$.

2.3 Definition [4]

Let R *be a ring, a fuzzy set* A *of* R *is called a fuzzy ring of* R *if*

i. $A(x-y) \geq \min(A(x),A(y))$
ii. $A(xy) \geq \min(A(x),A(y))$, *for all* $x,y \in R$.

2.4 Definition [4]

Let R *be a ring, a fuzzy ring* μ *of* R *is called a ring with operator if and only if for any* $t \in [0,1]$, μ_t *is a ring with operator of* R, *where* $\mu_t \neq \emptyset$, *where* $\mu_t = \{x \in R, \mu(x) \geq t\}$.

2.5 Definition [4]

Let μ *be a fuzzy subset of ring* R. *Then* μ *is called a fuzzy subring of* R *if for all* $x,y \in R$

1) $\mu(x-y) \geq \min(\mu(x),\mu(y))$
2) $\mu(xy) \geq \min(\mu(x),\mu(y))$.

2.6 Proposition

If $\{A_i\}_{i\in I}$ *are fuzzy (left, right) ideals of* R, *then* $\bigcap_{i\in I} A_i$ *is (left, right) ideal of* R

Proof: If A is a fuzzy ring in R. Let $\{A_i\}_{i\in I}$ be a family of fuzzy ideal over a fuzzy ring R, let $A=\bigcap_{i\in I} A_i$ then for all $x,y \in R$

$$A(x+y) = \inf_{i\in I} A_i(x+y)$$

$$\geq \inf_{i\in I} \min(A_i(x),A_i(y))$$

$$\geq \min[\inf_{i\in A} A_i(x), \inf_{i\in A} A_i(y)]$$

$\geq \min(A(x), A(y))$.

$A(-x) = \inf_{i\in I} A_i(-x) \geq \inf_{i\in I} A_i(x) \geq A(x)$.

$A(xy) = \inf_{i\in I} A_i(xy) \geq \inf_{i\in I}[\max(A_i(x), A_i(y))]$

$\geq \max[\inf_{i\in I} A_i(x), \inf_{i\in I} A_i(y)]$

$\geq \max(A(x), A(y))$. Hence A is a fuzzy ideal in R.

2.7 Proposition [4]

If $\{A_i\}_{i\in I}$ are fuzzy (left, right) ideal of R, then $\cup_{i\in I} A_i$ is a (left, right) ideal of R.

Proof: If A is a fuzzy ring R. Let $\{A_i\}_{i\in I}$ be a family of fuzzy ideal over a fuzzy ring R, let $A = \cup_{i\in I} A_i$ then for all $x, y \in R$

$A(x+y) = \sup_{i\in I} A_i(x+y) \geq \sup_{i\in I} \min(A_i(x), A_i(y))$

$\geq \min[\sup_{i\in I} A_i(x), \sup_{i\in I} A_i(y)]$

$\geq \min(A(x), A(y))$.

$A(-x) = \sup_{i\in I} A_i(-x) \geq \sup_{i\in I} A_i(x) \geq A(x)$.

$A(xy) = \sup_{i\in I} A_i(xy) \geq \sup_{i\in I} \max(A_i(x), A_i(y))$

$\geq \max[\sup_{i\in I} A_i(x), \sup_{i\in I} A_i(y)]$

$\geq \max(A(x), A(y))$.

2.8 Proposition [9]

Let R_1, R_2 be ring $f: R_1 \longrightarrow R_2$ be a ring homomorphism. A be a fuzzy ring on R_1 and B be a fuzzy ring on R_2. Then $f(A)$ is a fuzzy ring on R_2 and $f^{-1}(B)$ is a fuzzy ring on R_1.

Proof: Assume that B is an fuzzy ring on R_2 and let $x, y \in R$.

Then $f^{-1}(B)(x-y) = B(f(x-y)) = B(f(x) - f(y))$

$\geq \min[B(f(x)), B(f(y))]$

$= \min[f^{-1}(B(x)), f^{-1}(B(y))]$.

$f^{-1}(B)(xy) = B(f(xy)) = B(f(x)f(y))$

$\geq B[\min(f(x), f(y))] = \min[f^{-1}(B(x)), f^{-1}(B(y))]$.

Therefore $f^{-1}(B)$ is a fuzzy ring of R_1.

Assume that A is a fuzzy ring of R_1, we have to prove that $f(A)$ is a fuzzy ring on R_2 and let $x, y \in R$,

Then $f(A)(x-y) = A(f^{-1}(x-y)) = A(f^{-1}(x) - f^{-1}(y))$

$\geq \min[A(f^{-1}(x)), A(f^{-1}(y))] = \min[f(A(x)), f(A(y))]$.

$f(A)(xy) = A(f^{-1}(xy)) = A[f^{-1}(x)f^{-1}(y)]$

$\geq \min[A(f^{-1}(x)), A(f^{-1}(y))] = \min[f(A(x)), f(A(y))]$.

Therefore $f(A)$ is a fuzzy ring on R_2.

2.9 Definition [11]

Let A be a non-constant fuzzy left (right) ideal of R. Then A is called a fuzzy maximal left (right) ideal of R if for any fuzzy left (right) ideal B of R . $A \subset B \Longrightarrow A_0 = B_0$.

2.10 Proposition

If A is a fuzzy ideal of R, then

i. $A(0) \geq A(x)$ *and* $A(-x) = A(x), \forall x \in R$
ii. *If R has multiplicative identity 1, then* $A(1) \leq A(x), \forall x \in R$.
iii. *For* $x, y \in R, A(x-y) = A(0) \Longrightarrow A(x) = A(y)$.

Proof: i) we have for any $x \in R, A(0) = A(x-x) \geq \min\big(A(x), A(x)\big) = A(x)$

And $A(-x) = A(0-x) \geq \min\big(A(0), A(-x)\big) = A(x)$.

ii) If R has multiplicative identity 1, then $A(1) = A(xx^{-1}) \leq \max\big(A(x), A(x^{-1})\big) = A(x) \forall x \in R$.

iii) Let $x, y \in R$ be such that $A(x-y) = A(0)$. Then

$A(x) = A(x-y+y) \geq \min\big(A(x-y), A(y)\big) = A(y)$. Similarly $A(y) \geq A(x)$ and so $A(x) = A(y)$.

2.11 Theorem [11]

Let I be a prime ideal of R and α a prime element in L. Let P be the fuzzy subset of R defined by $P(x) = \begin{cases} 1 & if \quad x \in I \\ \alpha & otherwise \end{cases}$ *Then P is a fuzzy prime ideal.*

Proof: By corollary " Let I be any ideal of R and let $\alpha \leq \beta \neq 0$ be elements in L. Then fuzzy subset A defined by $A(x) = \begin{cases} \beta & if \quad x \in I \\ \alpha & otherwise \end{cases}$ is a fuzzy ideal" from this P is clearly a non-constant fuzzy ideal. Let A and B be any fuzzy ideals and let $A \leq P, B \leq P$. Then there exist x, y in R, such that $A(x) \nleq P(x), B(x) \nleq P(x)$. This implies that $P(x) = \alpha = P(y)$ and hence $x \notin I$ and $y \notin I$. Since I is prime, there exists an element r in R such that $xry \in I$. Now we have $A(x) \nleq \alpha$ and $B(ry) \nleq \alpha$ (0 otherwise $B(y) \nleq \alpha$) and since α is prime, $A(x) \wedge B(ry) \nleq \alpha$ and hence $(A \circ B)(xry) \nleq \alpha = P(xry)$ so that $A \circ B \nleq P$. Hence P is fuzzy prime ideal.

III. MAIN RESULTS

3.1 Definition

Let X be a nonempty set. A bipolar-valued fuzzy set A in X is defined as an object of the form $A = \big\{\big(x, A^+(x), A^-(x)\big)/x \in X\big\}$, *where* $A^+ : X \longrightarrow [0,1]$ *and* $A^- : X \longrightarrow [-1,0]$.

3.2 Definition

Let R be a ring. An bipolar-valued fuzzy set $A = \big\{\big(x, \mu_A^+(x), \mu_A^-(x)\big)/x \in X\big\}$ *of R is said to be bipolar-valued fuzzy ring of R if*

i. $\mu_A^+(x+y) \geq \min\big(\mu_A^+(x), \mu_A^+(y)\big)$,
ii. $\mu_A^-(x+y) \leq \max\big(\mu_A^-(x), \mu_A^-(y)\big)$,
iii. $\mu_A^+(-x) \geq \mu_A^+(x)$ *and* $\mu_A^-(-x) \leq \mu_A^-(x)$,
iv. $\mu_A^+(xy) \geq \min\big(\mu_A^+(x), \mu_A^+(y)\big)$ *and* $\mu_A^-(xy) \leq \max\big(\mu_A^-(x), \mu_A^-(y)\big)$.

3.3 Definition

An bipolar-valued fuzzy set $A = \big\{\big(x, \mu_A^+(x), \mu_A^-(x)\big)/x \in X\big\}$ *of a ring R is said to be an bipolar-valued left ideal if*

i. $\mu_A^+(x-y) \geq \min(\mu_A^+(x), \mu_A^+(y)$,
ii. $\mu_A^+(xy) \geq \mu_A^+(x)$,

iii. $\mu_A^-(x-y) \le \max(\mu_A^-(x), \mu_A^-(y))$,
iv. $\mu_A^-(xy) \le \mu_A^-(x), \forall x, y \in R$.

3.4 Definition

An bipolar-valued fuzzy set $A = (\mu_A^+, \mu_A^-)$ of a ring R is said to be an bipolar-valued fuzzy right ideal if

i. $\mu_A^+(x-y) \ge \min(\mu_A^+(x), \mu_A^+(y))$,
ii. $\mu_A^+(xy) \ge \mu_A^+(y)$,
iii. $\mu_A^-(x-y) \le \max(\mu_A^-(x), \mu_A^-(y))$,
iv. $\mu_A^-(xy) \le \mu_A^-(y), \forall x, y \in R$.

3.5 Definition

An bipolar-valued fuzzy set $A = (\mu_A^+, \mu_A^-)$ of a ring R is said to be an bipolar-valued fuzzy ideal if

i. $\mu_A^+(x-y) \ge \min(\mu_A^+(x), \mu_A^+(y))$,
ii. $\mu_A^+(xy) \ge \min(\mu_A^+(x), \mu_A^+(y))$,
iii. $\mu_A^-(x-y) \le \max(\mu_A^-(x), \mu_A^-(y))$,
iv. $\mu_A^-(xy) \le \max(\mu_A^-(x), \mu_A^-(y))$.

3.6 Theorem

Let A and B be two bipolar-valued fuzzy ideals of R. Then $A \cap B$ is also an bipolar-valued fuzzy ideal of R.

Proof: Let $A \cap B = \{(x, \mu_A^+(x) \wedge \mu_B^+(x), \mu_A^-(x) \vee \mu_B^-(x))/x \in X\}$. For any $x, y \in R$, we have that

$$(\mu_A^+ \wedge \mu_B^+)(x-y) = \mu_A^+(x-y) \wedge \mu_B^+(x-y)$$
$$\ge (\mu_A^+(x) \wedge \mu_A^+(y)) \wedge (\mu_B^+(x) \wedge \mu_B^+(y))$$
$$\ge (\mu_A^+ \wedge \mu_B^+)(x) \wedge (\mu_A^+ \wedge \mu_B^+)(y).$$
$$(\mu_A^- \vee \mu_B^-)(x-y) = \mu_A^-(x-y) \vee \mu_B^-(x-y)$$
$$\le (\mu_A^-(x) \vee \mu_A^-(y)) \vee (\mu_B^-(x) \vee \mu_B^-(y))$$
$$\le (\mu_A^- \vee \mu_B^-)(x) \vee (\mu_A^- \vee \mu_B^-)(y).$$

And $(\mu_A^+ \wedge \mu_B^+)(xy) = \mu_A^+(xy) \wedge \mu_B^+(xy)$
$$\ge (\mu_A^+(x) \wedge \mu_A^+(y)) \wedge (\mu_B^+(x) \wedge \mu_B^+(y))$$
$$\ge (\mu_A^+ \wedge \mu_B^+)(x) \wedge (\mu_A^+ \wedge \mu_B^+)(y).$$
$$(\mu_A^- \vee \mu_B^-)(xy) = \mu_A^-(xy) \vee \mu_B^-(xy)$$
$$\le (\mu_A^-(x) \vee \mu_A^-(y)) \vee (\mu_B^-(x) \vee \mu_B^-(y)) \quad \le (\mu_A^- \vee \mu_B^-)(x) \vee (\mu_A^- \vee \mu_B^-)(y).$$

Hence $A \cap B$ is a bipolar-valued fuzzy ideal of R.

3.7 Theorem

If $A = \{(x, \mu_A^+(x), \mu_A^-(x))/x \in X\}$ be bipolar-valued fuzzy ideal of R. Then

i. $\mu_A^+(0) \ge \mu_A^+(x)$ *and* $\mu_A^-(0) \le \mu_A^-(x)$,
ii. $\mu_A^+(-x) = \mu_A^+(x)$ *and* $\mu_A^-(-x) = \mu_A^-(x), \forall x \in R$,
iii. *If R is ring with unity 1, then* $\mu_A^+(1) \le \mu_A^+(x)$ *and* $\mu_A^-(1) \ge \mu_A^-(x), \forall x \in R$.

Proof: i) We have that for every $x \in R, \mu_A^+(0) = \mu_A^+(x-x) \geq \min(\mu_A^+(x), \mu_A^+(x)) = \mu_A^+(x)$ and $\mu_A^-(0) = \mu_A^+(x-x) \leq \max(\mu_A^-(x), \mu_A^-(x)) = \mu_A^-(x)$. Therefore $\mu_A^+(0) \geq \mu_A^+(x)$ and $\mu_A^-(0) \leq \mu_A^-(x)$.

ii) By using (i), we get $\mu_A^+(-x) = \mu_A^+(0-x) \geq \min(\mu_A^+(0), \mu_A^+(x)) = \mu_A^+(x)$

$\mu_A^-(-x) = \mu_A^-(0-x) \leq \max(\mu_A^-(0), \mu_A^-(x)) = \mu_A^-(x)$.

iii) If R is a ring with unity 1, we get $\mu_A^+(1) = \mu_A^+(xx^{-1}) \leq \max(\mu_A^+(x), \mu_A^+(x^{-1}))$

$= \max(\mu_A^+(x), \mu_A^+(x)) = \mu_A^+(x)$ and $\mu_A^-(1) = \mu_A^-(xx^{-1}) \geq \min(\mu_A^-(x), \mu_A^-(x^{-1})) = \mu_A^-(x)$. Therefore $\mu_A^+(1) \leq \mu_A^+(x)$ and $\mu_A^-(1) \geq \mu_A^-(x)$.

3.8 Definition [13]

Let X be a set and let $A \in X$ is a bipolar-valued fuzzy subring. Then A is said to have the sub property if for each $Y \in P(x)$, there exists $y_0 \in Y$ such that

$$A(y_0) = \left(\bigvee_{x \in Y} \mu_A^+(x), \bigwedge_{x \in Y} \mu_A^-(x)\right),$$

Where $P(Y)$ denoted the proper set of X.

3.9 Definition [13]

Let X be a set, let $A \in X$ is a bipolar-valued fuzzy subring and let $(\lambda, \mu) \in I \times I$ with $\lambda + \mu \leq 1$

1) *The set $A^{(\lambda,\mu)} = \{x \in X: \mu_A^+(x) \geq \lambda, \mu_A^-(x) \leq \mu\}$ is called a (λ, μ)-level subset of A*
2) *The set $A^{(\lambda,\mu)} = \{x \in X: \mu_A^+(x) > \lambda, \mu_A^-(x) < \mu\}$ is called a strong (λ, μ)-level s-1ubset of A.*

3.10 Theorem

If $A = (A^+, A^-)$ is a bipolar-valued fuzzy subring of a ring R, then $H = \{x \in R / A^+(x) = 1, A^-(x) = -1\}$ is either empty or is a subring.

Proof: If no element satisfies this condition, then H is empty. If x and y in H then $A^+(xy^1) \geq \min(A^+(x), A^+(y^{-1})) = \min(1,1) = 1$. Therefore $A^+(xy^{-1}) = 1$. And $A^-(xy^{-1}) \leq \max(A^-(x), A^-(y^{-1})) = \max(-1,-1) = -1$ Therefore $A^-(xy^{-1}) = -1$. That is $xy^{-1} \in H$. Hence H is a subring of R. Hence H is either empty or is a subring of R.

3.11 Theorem

If $A = (A^+, A^-)$ is a bipolar-valued fuzzy subring of R, then $H = \{x \in R / A^+(x) = A^+(e), A^-(x) = A^-(e)\}$ is a subring of R

Proof: Here $H = \{x \in R / A^+(x) = A^+(e), A^-(x) = A^-(e)\}$, By theorem " Let $A = (A^+, A^-)$ be a bipolar-valued fuzzy subring of a ring R. Then $A^+(-x) = A^+(x)$ and $A^-(-x) = A^-(x), A^+(x) \leq A^+(e)$ and $A^-(x) \geq A^-(e)$ for all x in R and identity $e \in R$." We have $A^+(x^{-1}) = A^+(x) = A^+(e)$ and $A^-(x^{-1}) = A^-(x) = A^-(e)$. Therefore $x^{-1} \in H$. Now $A^+(xy^{-1}) \geq \min(A^+(x), A^+(y^{-1})) = \min(A^+(e), A^+(e)) = A^+(e)$ and $A^+(e) = A^+((xy^{-1})(xy^{-1})^{-1}) \geq \min(A^+(xy^{-1}), A^+(xy^{-1})) = A^+(xy^{-1})$. Hence $A^+(e) = A^+(xy^{-1})$. Also $A^-(e) = A^-(xy^{-1})$. Hence $A^+(e) = A^+(xy^{-1})$ and $A^-(e) = A^-(xy^{-1})$, therefore $xy^{-1} \in H$. Hence H is a subring of R.

3.12 Definition

Let R be a ring. An bipolar-valued fuzzy set $A = \{(x, \mu_A^+(x), \mu_A^-(x)) / x \in R\}$ of R is said to be bipolar-valued fuzzy subring of R if

i. $\mu_A^+(x-y) \geq \min(\mu_A^+(x), \mu_A^+(y))$,
ii. $\mu_A^+(xy) \geq \min(\mu_A^+(x), \mu_A^+(y))$,
iii. $\mu_A^-(x-y) \leq \max(\mu_A^-(x), \mu_A^-(y))$,

iv. $\mu_A^-(xy) \leq \max(\mu_A^-(x), \mu_A^-(y))$.

3.13 Definition

An bipolar-valued fuzzy left ideal $A = (A^+, A^-)$ is called an bipolar-valued fuzzy left k-ideal of a subring S if for all $x, y, z \in S, x + y = z$ implies

1) $A^+(x) \geq \min(A^+(y), A^+(z))$
2) $A^-(x) \leq \max(A^-(y), A^-(z))$.

3.15 Definition

An bipolar-valued fuzzy ideal $A = (A^+, A^-)$ of a subring S is called a bipolar-valued fuzzy completely prime ideal of S if

i. $A^+(xy) = \max(A^+(x), A^+(y))$,

ii. $A^-(xy) = \min(A^-(x), A^-(y)), \forall x, y \in S$.

3.16 Definition

Let S be a subring $A = (A^+, A^-)$ be an bipolar-valued fuzzy subset of a set S and $x \in S$. The bipolar-valued fuzzy subset $(x, A) = ((x, A^+), (x, A^-))$ where $(x, A^+): S \longrightarrow [0,1]$ and $(x, A^-): S \longrightarrow [0,1]$ defined by $(x, A^+)(y) = A^+(xy)$ and $(x, A^-)(y) = A^-(xy)$ is called the bipolar-valued fuzzy extension of $A = (A^+, A^-)$ by x

3.17 Example

Let $S = Z_0^+$ let $A = (A^+, A^-)$ be an bipolar-valued fuzzy subset of S, defined as follows

$A^+(0) = 1, A^-(0) = 0, A^+(n) = \begin{cases} 0.5 & if\ n\ is\ even \\ 0.3 & if\ n\ is\ odd \end{cases}$ and $A^-(n) = \begin{cases} 0.5 & if\ n\ is\ even \\ 0.5 & if\ n\ is\ odd \end{cases}$ Then the bipolar-valued fuzzy extension of A is given by $(2, A^+) = \begin{cases} 1 & if\ n = 0 \\ 0.5 & if\ n \neq 0 \end{cases}$ and $(2, A^-) = \begin{cases} 0 & if\ n = 0 \\ 0.5 & if\ n \neq 0 \end{cases}$

3.18 Definition

Let A and B be two bipolar-valued fuzzy sets of a set X Then the following expressions holds

i. $A \subset B$ *if and only if* $(x, A) \leq (x, B)$

ii. $(x, A^c) = (x, A)^c$

iii. $(x, A \cap B) = (x, A) \cap (x, B)$

iv. $(x, A \cup B) = (x, A) \cup (x, B)$

v. $(x, A)^c_{\beta_\alpha} = (x, A^c_{\beta_\alpha})$.

3.19 Proposition

Let $A = (A^+, A^-)$ be a bipolar-valued fuzzy ideal of a commutative subring S and $x \in S$. Then (x, A) is an BVFI of S

Proof: Obviously (x, A) is an bipolar-valued fuzzy subsets of S. Let $y, z \in S$, then $(x, A^+) = (y + z) = A^+(x(y + z)) = A^+(xy + xz) \geq \min(A^+(xy), A^+(xz)) = \min\{(x, A^+)(y), (x, A^+)(z)\}$. Again $(x, A^-)(y + z) = A^-(x(y + z)) = A^-(xy + xz) \leq \max(A^-(xy), A^-(xz)) = \min\{(x, A^-)(y), (x, A^-)(z)\}$. Also $(x, A)(yz) = A(xyz) = A(xy) = (x, A)(y)$ and $(x, A) = (x, A)(z)$. Hence (x, A) is an bipolar-valued fuzzy ideal of S.

3.20 Proposition

Let $A = (A^+, A^-)$ be an bipolar-valued fuzzy k –ideal of a commutative subring S and $x \in S$. Then (x, A) is an bipolar-valued fuzzy k-ideal of S.

Proof: Clearly (x, A) is a bipolar-valued fuzzy ideal of S. Since A is an bipolar-valued fuzzy k-ideal of $S, A^{+}(xy) \geq \min\left(A^{+}(xy + xz)\right), A^{+}(xz))$ for all $y, z \in S$. Then $(x, A^{+})(y) \geq \min\{(x, A^{+})(y + z), (x, A^{+})(z)\}$ again $A^{-}(xy) \leq \max\{A^{-}(xy + xz), A^{-}(xz)\}, \forall y, z \in S$. Then $(x, A^{-})(y) \leq \max\{(x, A^{-})(y + z), (x, A^{-})(z)\}$. Thus (x, A) is a bipolar-valued fuzzy k-ideal of S. Here also the converse may not be true.

IV. CONCLUSION

In this paper we present the concept of fuzzy subgroupoid to the bipolar-valued fuzzy subgroupoid and bipolar-valued fuzzy normal subgroup we investigate some results related to the topic

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REFERENCES

[1] Arumugam S, Group and rings Fields Ideals
[2] David Helbert, Rings and Subrings, Journal of mathematics analysis and application, 1-10(1888).
[3] K. Meena and K. V. Thomas, Intuitionistic L-fuzzy rings, Global journal of science fountier research mathematics and decision sciences Vol 12, 2012 17-31.
[4] M. Z. Alam, Fuzzy rings and Anti fuzzy rings with operations, Journal of Mathematics, Vol.11 (4), 48-54 (2015)
[5] Mohammad Yamin and Poonam Kumar Sharma, Intuitionstic fuzzy rings with operators, vol.2 1860-1866 (2018)
[6] P. K. Sharma, t-Intuitionstic fuzzy subringd, international journal of Math. Sci . vol 11 (3-4) 265-275(2012)
[7] P. Narasimhn Swamy, K. Vijay kumar, Bipolar fuzzy near algebra over bipolar fuzzy ideal, Advance in Fuzzy mathematics vol 12 No 4 2017, pp. 945-955.
[8] P. Uma Maheswari, K. Arjunan and R. Mangayarkarasi, Note on bipolar-valued fuzzy subring of a ring, International journal of applied mathematical science vol 9 1 89-97 (2016)
[9] Sourian Sebastian, Mercy K. Jacob, V. M. Mary, On fuzzy ideals and rings, J. Comp and Math. Sci. Vol.3 (1), 115-120 (2012)
[10] Thirumagal M.R and Murugadas. P, Intuitionstic fuzzy ideal extensions in semiring, International journal of applied mathematical science vol 117(6), 197-207(2017)
[11] V. M Swamy and K. L. Swamy, Fuzzy prime ideal of rings , Journal of Mathematical analysis and application 134, 94-103 (1988)
[12] Won Kyun Jeong, Bipolar-valued fuzzy ideal extension of BCK-algebra, Journal of the chungcheong mathematical society vol.26, no.1, 2012, 17-27.
[13] YOUNG SIM AHN, KUL HUR and DAE SIG KIM, The Lattice of intuitionistic fuzzy ideal of a ring, J. Appl. Math and computing Vol 19 (2005) No 1-2. Pp. 551-572.