Application to Differential Transformation Method for Solving Eight Order Ordinary Differential Equations

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Abstract
The Zhou's differential transform method (ZDTM) is approximate method which construct analytical solution in the firm of polynomial which can be easily applied o many linear and non linear problems by reducing lot of computational work as compare to Tayler series for higher order linear differential equations initial value problems. In this paper the definition and operation of the one dimensional Zhou's differential transform method and investigate the particular exact solutions of eight order ordinary differential equations initial value problems by explaining concept of ZDTM obtain solution of three numerical examples for demonstration. the results are compared with exact solution with graphs. It is observed that solutions obtained from this ZDTM technique have very high degree of accuracy.

There results show that the technique introduced here is accurate & easy to apply.

Keywords *:- Ordinary differential equitations zhou's Method (DTM), Initial value problem*

I. INTRODUCTION

The Zhou's differential transformation method is a numerical method based on a Taylor expansion method beyond the treatment of ODE'S with the ZDTM in which case one should at least clearly refer to the numerous studies done with the Taylor series mehtod. It seems that the major contribution of the ZDTM is in the easy generalization of the Taylor method.

Biazar J. et al used ZDTM for Solving quadratic Riccati differential equation [1], Opanuga Used for solving numerical solution of systems of ordinary differential equation by numerical analytical method [2], Chen C. K. and S. S. Chen used ZDTM to obtained the solution of nonlinear system of differential equations [3], Zhou X. Applied ZDTM for Electrical Circuits problems [4], Ayaz F. used ZDTM to find series solution of system of differential equations [5], Duan Y.R. Liu used ZDTM for Burger's equation to obtain series solutions [6], Bert W. has applied DTM on system of linear equation and analysis of its solutions[7], Chen C.L. has applied DTM technique for steady nonlinear beat conduction problems[8], Using DTM Hassan have find out series solution and that solution compared with DTM method for linear & non linear initial value problems & proved that DTM is reliable tool to find numerical solution[9], Khaled Batiha has been used DTM to obtain the Taylor's series as solution of linear, nonlinear system of ordinary differential equations[10], Montri Thangmoon has been used to find numerical solution of ordinary differential equations[11], Edeki, A semi method for solutions of a certain class of second order ordinary differential equations [12], Gbadeyan and Agboola for Dynamic behaviour of a double Rayleigh beam-system due to uniform AriKoglu A applied DTM to obtain numerical solution of differential equations partially distributed moving load [13] Abdel Halim Hassan I. used DTM method for solving differential equations[14] Arikoglu A applied DTM to obtained numerical solution of differential equation [15], Kou B has been used to find numerical solution of the free convection problems[16].

II. BASIC DEFINITIONS & PROPERTIES OF DTM METHOD

v(t) can be expressed by Taylor's series, then v(t) can be represented as

$$v(t) = \sum_{k=0}^{\infty} \frac{(t-ti)^k}{k!} V(k)$$

v(t) is called inverse of V(k)

$$\therefore v(t) = \sum_{k=0}^{\infty} \left[\frac{(t-ti)^k}{k!}\right] V(k) = D\text{-}1\ V(k)$$

$$v(t) = \sum_{k=0}^{\infty} \left[\frac{(t-ti)^k}{k!}\right] V(k) + R\ n+1\ (t)$$

by Taylor's Series

$$V(k) = \frac{1}{k!}\left[\frac{d^k v(t)}{dt^k}\right] \quad \text{at} \quad t = t0$$

III. FUNDAMENTAL THEOREMS ON DTM

Theorem 1 :- If $n(t) = p(t) + s(t)$ then

$N(k) = P(k) + S(k)$

Theorem 2 :- If $n(t) = \propto (t)$ then $\propto p(t)$ then

$N(k) = \propto P(k)$

Theorem 3 :- If $n(t) = \frac{dp(t)}{dt}$ then

$N(k) = (k+1) P(k+1)$

Theorem 4 :- If $n(t) = \frac{d^2p(t)}{dt^2}$ then

$N(k) = (k+1)(k+2)(k+2) P(k+2)$

Theorem 5 :- If $n(t) = \frac{d^n p(t)}{dt^n}$ then

$N(k) = (k+1)(k+2)(k+3)\ldots(k+n) P(K+n)$

Theorem6 :- If $n(t) = tn$ them

$$N(K) = \sum_{l=0}^{k} S(l) P(k-l)$$

Theorem7 :- If $n(t) = tn$ them

$N(k) = \delta(k-n)$

$$\delta(k-n) = \begin{cases} 1 \text{ if } k = n \\ 0 \text{ if } k \neq n \end{cases}$$

Theorem 8:- If $n(t) = e^{\lambda t}$ then

$N(k) = \frac{\lambda^k}{k!}$

Theorem 9:- If $n(t) = (1+t)n$ then

$N(k) = \frac{M(n-1)..(n-k+1)}{k!}$

Theorem 10:- if $n(t) = (1+t)n$ then

$N(k) = \frac{w^k}{ki} \sin\left(\frac{\pi k}{2} + \propto\right)$

Theorem 11:- if $n(t) = \cos(wt + \propto)$ then

$N(k) = \frac{w^k}{ki} \cos\left(\frac{\pi k}{2} + \propto\right)$

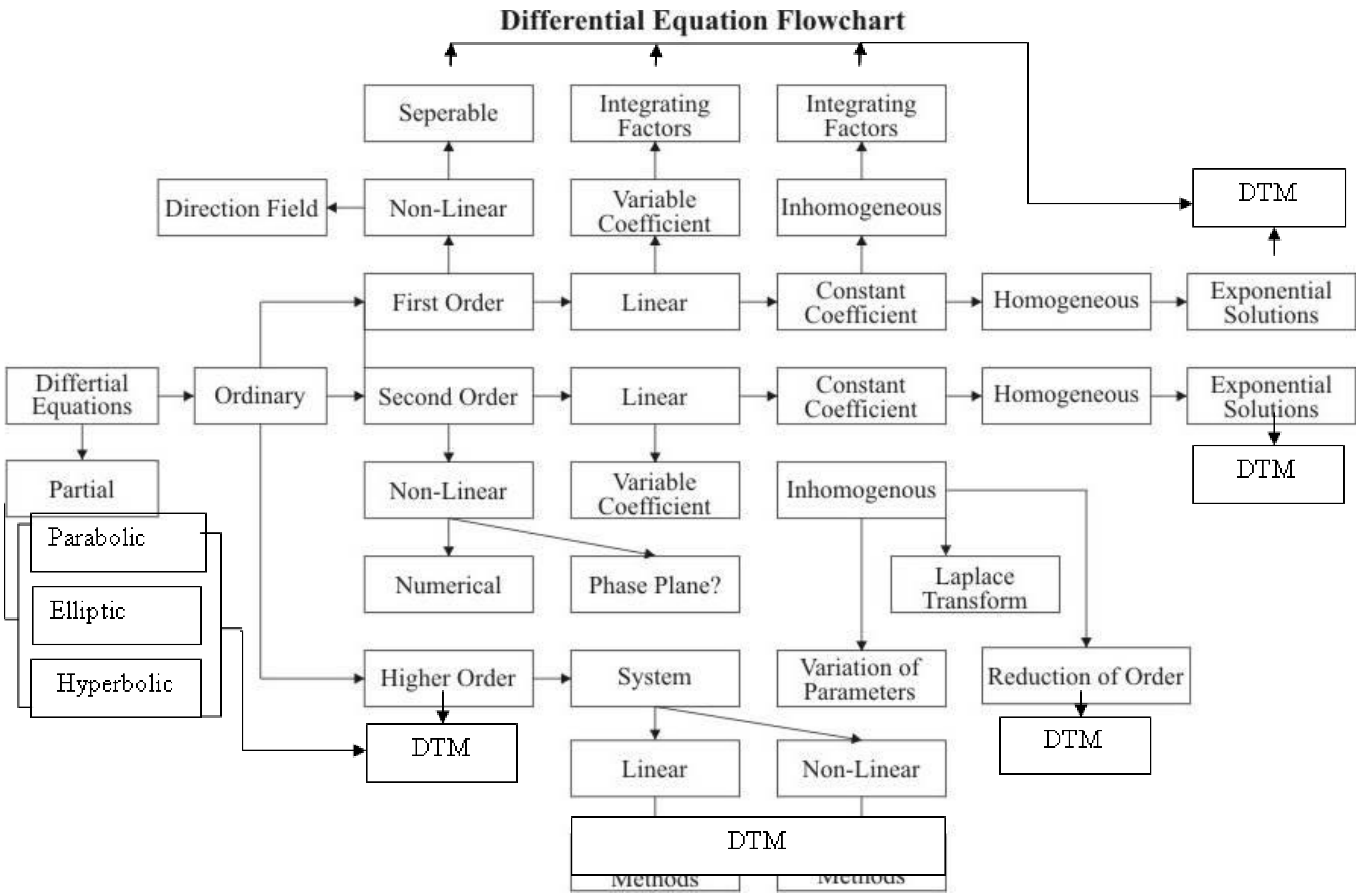
Theorem 12 :- if $P(k) = D[p(t)]$

$S(k) = D[s(t)]$' & C1, C2

Are independent of t, k then

$D[c1, p(t) + c2\ s(t)] = c1, P(k) + c2\ S(k)$

IV. FLOWCHART OF ZDTM METHOD



V. EXPERIMENTATION OF ZDTM RESULTS

Example : 1

Solve non homogeneous linear differential equation $y^{viii} - y^{iv} = 0$

$y(0) = y^{i}(0) = y^{ii}(0) = 0,\ y^{iii}(0) = 78,\ y^{iv}(0) = 0,\ y^{v}(0) = 0,\ y^{vi}(0) = 0$

$y^{vii}(0) = 78,$

→ Exact Solution is

$Y = 78 \sinh x - 78 \sin x\ 13x^3$

By DTM

$(k+1)(k+2)(k+3)(k+4)(k+5)(k+6)(k+7)(k+8)\, U(k+8)$

$= (k+1)(k+2)(k+3)(k+4)\, U(k+4)$

Put $k = 0, 1, 2, 3, 4, 5\ldots\ldots$

$U(0) = 0$

$U(1) = 0$

$U(2) = 0$

$U(3) = 78$

$U(4) = 0$

$U(5) = 0$

$U(6) = 13$

$U(7) = \frac{13}{140}$

$U(8) = 0$

$U(9) = 0$

$U(10) = 0$

$U(11) = 0$

$U(12) = 0$

$U(13) = 0$

$U(14) = 0$

$U(15) = 0$

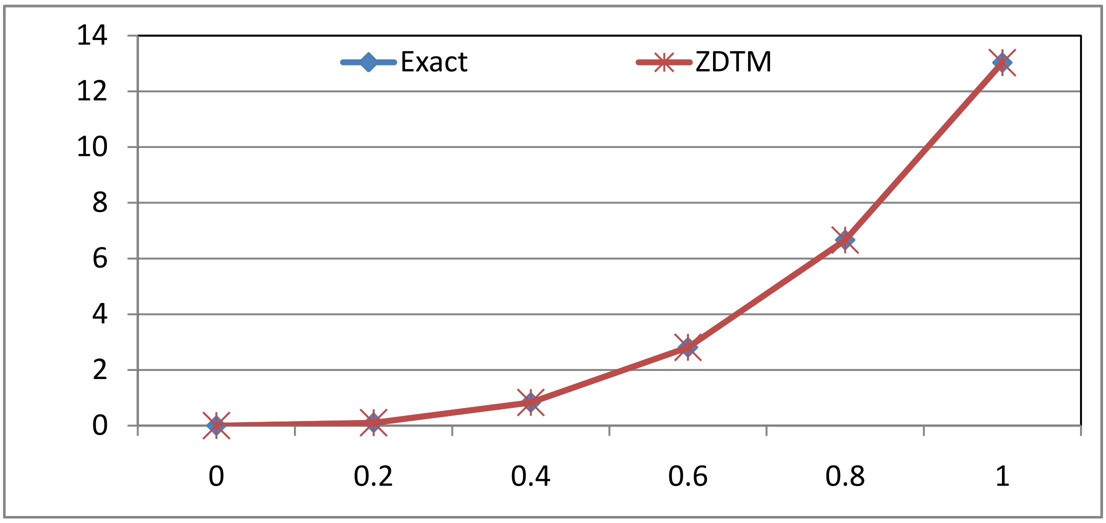
$U(16) = 0$

$$u(+) = 78t^3 + \frac{13}{140} t^3 + \left(\frac{13}{140} \times \frac{1}{8.9.10.11}\right) t^{13} + \left(\frac{13}{8.9.10.11.12.13.14.15} \times \frac{1}{140}\right) t^{15} + \ldots.$$

Table 1 : Numerical Result for example 2

t	Exact	D.T.M.	DTM Error
0	0.00000	0.00000	0.00000
0.2	0.104000	0.104000	0.00000
0.4	0.832050	0.832050	0.00000
0.6	2.808866	2.808866	0.00000
0.8	6.662491	6.662491	0.00000
1.0	13.030956	13.030956	0.00000

Fig.



Example 2 **Numerical Examples :-**

$y^{viii} - y^{vi} = 2\sin x$ Boundary conditions

$y(0) = y^{i}(0) = y^{ii}(0) = y^{iii}(0) = 0$

$y^{iv}(0) = 0,\ y^{v}(0) = 2,\ y^{vi}(0) = 0,\ y^{vii}(0) = 0$

Exact soi^{n} is given my

$y = -2x + \sinh x + \sin x$

(K+1) (K+2) (K+3) (K+4) (K+5) (K+6) (K+7) (K+8) U(8)

$-$ (k +1) (k+2) (k+3) (k+4) U (6) $2\,\frac{(1)^k}{k1}\sin\left(\frac{\pi k}{2}\right)$

Put K = 0, 1, 2, 3, 4, 5, 6……

$8!\,U(8) - 6!\,U(6) = 2\,\frac{(1)^0}{0!}\sin 0$

$8!\,U(8) = 6!\,U(6) = 0$

$= 6! \times 2$

$U(8) = \frac{1}{28}$

K=1 2,3,4,5,6,7,8,9 U(9) – 7! U(7) = 2

$9!\,U(9) = 2$

$U(9) = \frac{2}{9!}$

K=2 3,4,5,6,7,8,9,10, U(10) - 3,4,5,6,7,8 U(8) = 0

7,8,9,10, U(10) = 3,4,5,6, 2 x 1

= 2

$$U(10) = \frac{2}{8.9.10} = \frac{12}{4x9x10} = \frac{1}{360}$$

K=3 4, 5, 6, 7, 8, 9, 10, 11 U(11) – 4,5,6,7,8,9, U(9) = $\frac{-2}{3!}$

$$-\frac{1}{3} = -\frac{1}{3}$$

4, 5, 6, 7, 8, 9, 10, 11 U(11) = 0

U (11) = 0

K=4 5, 6, 7, 8, 9, 10, 11, 12 U(12) – 5,6,7,8,9,10 U(10) = 0

5, 6, 7, 8, 9, 10, 11, 12 U(12) – 5,6,7,8,9,10 U(10) = 0

$$U(12) - \frac{1}{6,8,9,10,11}$$

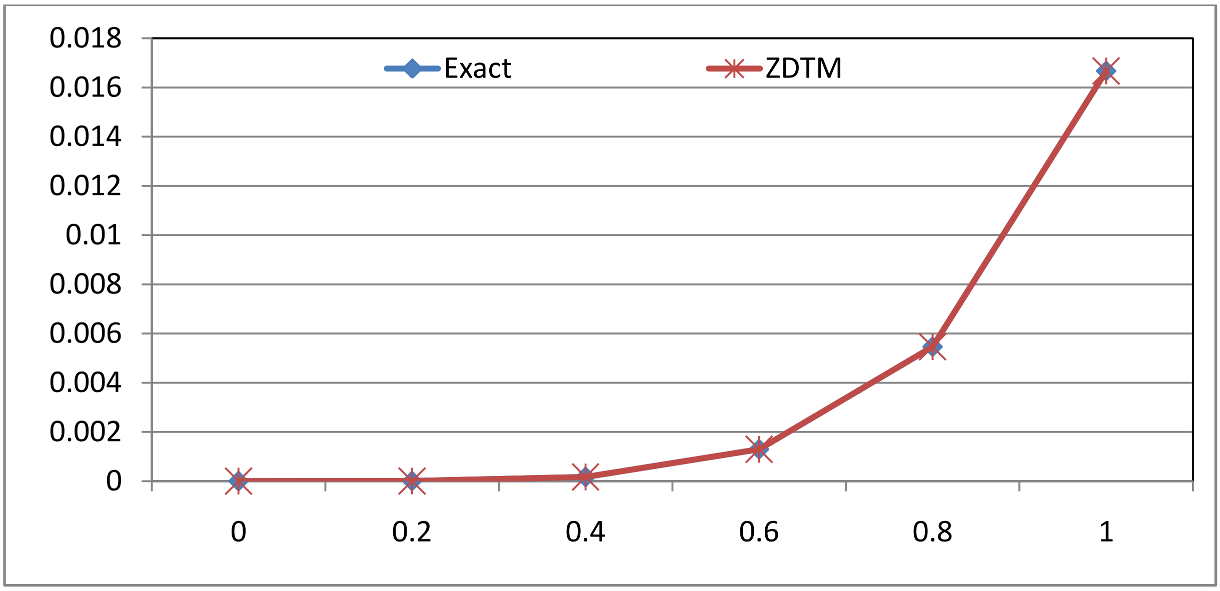
u(+)= $U(0) + tU(1) + t^2U(2) + t^3U(3) + t^4U(4) + t^5U(5) + t^6U(6) + t^7U(7) + t^8U(8) + t^9U(9) + t^{10}U(10) + t^{11}U(11) + t^{12}U(12) + \ldots\ldots\ldots\ldots\ldots$

$$= \frac{2}{5!}t^5 + \frac{1}{28}t^8 + \frac{2}{9!}t^9 + \frac{1}{360}t^{10} + \frac{1}{6,8,9,10,11} + t^{12} + \ldots..$$

Table 2 : Numerical Result for example 2

t	Exact	D.T.M.	DTM Error
0	0.00000	0.00000	0.00000
0.2	5.33334e-06	5.33334e-06	0.00000
0.4	0.000170	0.000170	0.00000
0.6	0.001296	0.001296	0.00000
0.8	0.005462	0.005462	0.00000
1.0	0.0166721	0.0166721	0.00000

Fig.



Q.3 Solve $y^{viii} + y^{vi} = 2 \sinh x$

$y^{iv} + y^{11} = 2 \sin hx$ $\quad$ $y(0) = y^{1}(0) = y^{II}(0)\ y^{III}(0), y^{IV}(0) = 0$

$y^{v}(0) = \frac{2}{51}, y^{vi}(0) = 0, y^{vii}(0) = 0$

Extract Solution is

$y = \sin x + \sinh x - 2x$

By DTM

$(K+1)(K+2)(K+3)(K+4)(K+5)(K+6)(K+7)(K+8)\,U(K+8)$

$+ (K+1)(K+2)(K+3)(K+4)\,U(K+4)$

$= \left[\frac{(1)^k}{k!} + \frac{(-1)^k}{k!}\right]$

Put k = 0 $\quad 8!\,U(8) + 4!\,U(4) = 2$

$U(8) = \frac{2}{8!}$

K= 19! U(9) + s! U(5) = 0

$9!\,U(9) = -5!$

$U(9) = \frac{-5!\ x\ 2}{9!}$

$= \frac{-1}{9x8x7x3} = -1$

K=2 3,4,5,6,7,8,9,10, U(10) + $\underline{3,4,5,6,\ U(6)}$ =1

0

$U(10) = \frac{1}{34,5,6,7,8,9,10}$

K=3 $\quad$ 4,5,6,7,8,9,10,11 U(11) + $\frac{4,5,6,7U(7)}{0}$ =0

U(11) =0

K= 4 $\quad$ 5,6,7,8,9,10,11 U(12) + 5,6,7,8, x $\frac{2}{8!} = \frac{2}{4!}$

5, 6,7,8,9,10,11,12 U(12) + 5,6,7,8 x $\frac{2}{8!}$ $\quad$ x $\frac{2}{4!}$

5,6,7,8,9,10,11,12 (12) + $\frac{2}{4!}$

$= \frac{1}{12} - \frac{1}{12} = 0$

U (12) = 0

K = 5 $\quad$ 6,7,8,9,10,11,12,13U (13) + 6,7,8,9U (9) = 0

∴ 6,7,8,9,10,11,12,13 U(13) + 2 x -1 =0

6,7,8,9,10,11,12,13 U(13) = 2

$U(13) = \frac{2}{6,7,8,9,10,11,12,13}$

K=6 $\quad$ 7,8,9,10,11,12,13,14 U(14) + 7,8,9,10 U(10) = $\frac{2}{6!}$

7,8,9,10,11,12,13,14 U (14) x $\frac{1}{3,5,6,7,}$ = $\frac{2}{6!}$ = $\frac{1}{6x5x4x3}$

U (14) = 0

U= $\quad$ U(0) + U(1)t + U(2) t^2 + U(3) t^3 + U(W) t^4

+ U(5) t^5 + U(6) t^6 + U(7) t^7 + U(8)t^8

+ U(12) t^{12} + U(13) t^{13} + U(14) t^{14}

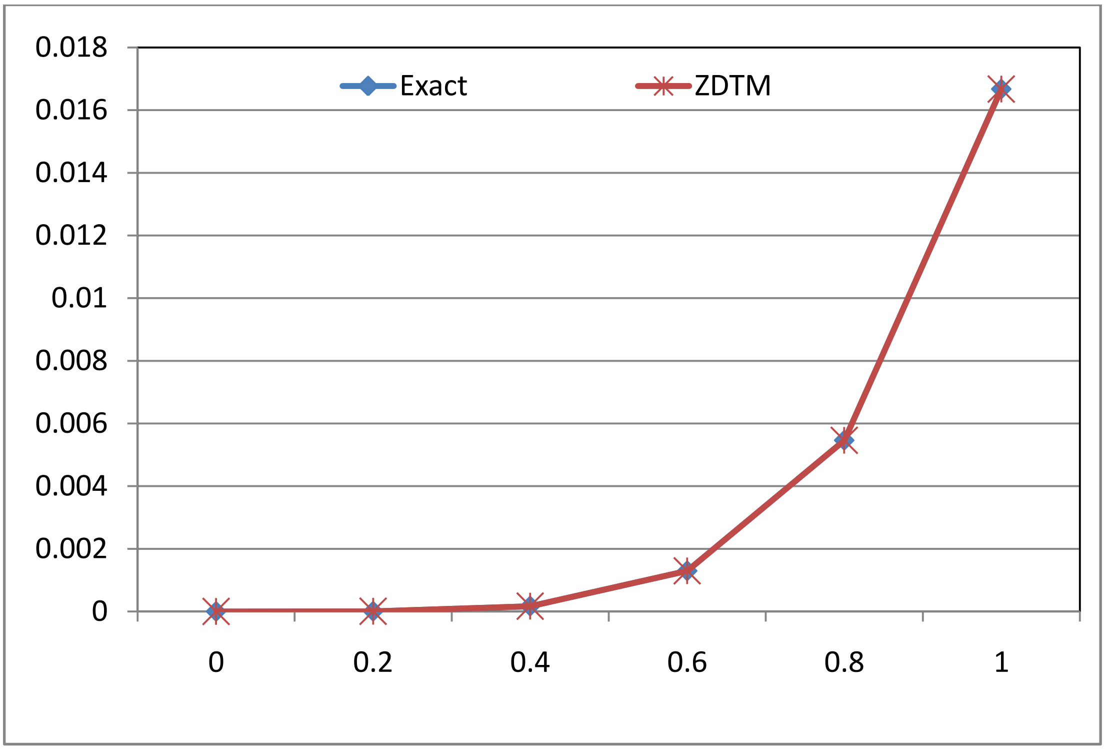
U= $\quad$ $0 + \frac{2}{5!}t^5 + \frac{2}{8!}t^8 + (\frac{-1}{9x8x7x3})\, t^9 + \frac{1}{3,4,5,6,7,8,9,10}\, t^{10} +$

$U + 0 + \frac{2}{6,7,8,9,10,11,12,13}\, t^3 + 0 + \ldots\ldots$

Table 3 : Numerical Result for example 3

t	Exact	D.T.M.	DTM Error
0	0.00000	0.00000	0.00000
0.2	5.33334e-06	5.33334e-06	0.00000
0.4	0.000170	0.000170	0.00000
0.6	0.001296	0.001296	0.00000
0.8	0.005462	0.005462	0.00000
1.0	0.0166721	0.0166721	0.00000

Fig.



VI. VALIDATION AND COMPARISON

The ZDTM has promissing approach for many application in various field of science. The drawback of ZDTM is solution is truncated series which does not exhibit the behavious of given problem but gives good approximation or turn solution in small region multistep ZDTM accelerates accuracy of ZDTM The fact is ZDTM is applicable to many other nonlinear models which is reliable than existing other methods.

VIII. CONCLUSION

In this work ZDTM applied for initial value problems of ordinary differential equation for eight order ordinary differential equations it reduces the computational difficulties of other traditional methods like Laplase Trans method, Exact solutions by complimentary functions and particular integral method etc. ZDTM is reliable method that needs less work and dose not require any kind of strong assumptions by increasing the order of approximate more accuracy can be obtained graphical comparison show that ZDTM is powerful method.

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