Forcheimeir Effect on Transport Process with Internal Heat Source and Varying the Fluid Properties

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Abstract

The Analysis of Internal Heat source for transport processes of the physical system formulated with Forcheimeir model is studied. The extended Darcy model is used here. For Physical problem considered here, the governing measures are highly coupled. To solve them, initially similarity transformations are used to convert them into ODE and solved approximately. RK method of fifth order followed by Newton-Raphson method is employed. The graphical representations are obtained for transport processes, where the features of fluid flow are discussed. The exponential form of internal heat source enhances melting and impedes freezing. The significance of inertial parameter is highly influenced in decreasing the flow field. Besides, skin friction, heat and mass transport graphical interpretations are made for different physical parameters. A good agreement is found when interpretation of the effects of various significant parameters are analysed and the results are compared.

Keywords — *Double Diffusion, boundary layer, porous medium, Internal Heat Generation (IHG).*

I. INTRODUCTION

When free convection effects are in par with forced convection effects, mixed convection arises. Study of such flow situations finds application in many engineering processes. The Internal heat source intensifies melting and hampers freezing. Problems dealt with chemical reactions of dissociating fluids in motion releasing heat and absorbing heat cooling the surroundings gives the importance to study of heat source or sink. Many investigations have been made for natural convection for vertical plate with internal heat source. The effects of chemical reaction in presence of IHG on free convective flow were analyzed by Patil et al (2008). Frictional heating and internal heat source for a stretching sheet was studied by Vajravelu et al (1993). However, the investigations have been made for mixed convection heat and Mass transfer with internal heat generation also. Coupled transport process of natural convection of a plate with MHD and heat source or sink was investigated by Chamkha and Kaled (2001). With heat source and thermal diffusion in porous medium was studied numerically by Alam et al (2006). The exponential form of internal heat source with varying the viscosity in double diffusion problem of MHD was studied by Ferdows (2011). These researchers' studies are resulted in fluid properties of the medium treated as constants.

Due to many practical applications in engineering systems, the boundary layer flows past a vertical plate with internal heat generation continues to receive considerable attention. In literature, many works have been considered with internal heat generation for constant fluid properties and solved them using analytical or numerical procedures (see Alam et al (2004) and Mohamad (2009)). But, for the above practical application, it is required to have effect of IHG and varying the fluid properties that is not present in the recent studies and the same was analysed by Nalinakshi et al (2014). Porosity measures maximum near the wall compare to away from it in industrial appliances; Due to this varied nature non-uniformity arises in the porous medium. To emphasize on this aspect when Porosity measurements were analysed by researchers Schwartz et al (1993) and Benenati et al (1962), they found that not only porosity varies but permeability also varies. Hence the study was initiated by incorporating the variation in permeability by Chandrasekhara et al (1984, 1985) and observed there is greater influence on heat transport. Further the inertia effects and variation in permeability with the Darcy's model was analysed by Mohammadein and El-shaer (2004). In continuation with varying the porosity and permeability, Pal and Shivakumar (2006) made an attempt to vary the thermal conductivity in their study.

The main objective in this study is therefore, to understand numerically the effect of inertial terms-second order resistance on combined convective transport process past a heated plate embedded in sparsely packed porous medium for a Newtonian fluid flow, incorporating the variable porosity, permeability, thermal conductivity and solutal diffusivity in the presence of Internal heat generation (IHG) are varied. Here, the equations governing the physical system are highly coupled which are involved with non-linear partial derivatives are converted to ordinary equations analytically and solved numerically by shooting technique using Runge Kutta Fehlberg algorithm and Newton-Raphson corrector method to find the graphical representations for transport processes for different physical parameters.

II. MATHEMATICAL FORMULATION

Forcheimeir and internal heat source effect is analysed for laminar, two-dimensional in steady- state past a semi-infinite vertical heated plate. The boundary layer flows are interpreted for an incompressible and Newtonian fluid surrounded in a sparsely packed porous medium by varying the fluid properties. From the leading edge along the plate the x-coordinate is measured and y- axis normal to it. The temperature and concentration are uniform at the plate which is greater when compared with far away from the plate. The boundary layer theory arising with governing equations for the physical system are:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial v}{\partial y}=\vec{g}\beta_T(T-T_\infty)-\vec{g}\beta_C(C-C_\infty)+\frac{\bar{\mu}}{\rho}\frac{\partial^2 u}{\partial y^2}+\frac{\mu}{\rho}\frac{\varepsilon(y)}{k(y)}(U_o-u)+\frac{C_b\varepsilon^2(y)}{\sqrt{k(y)}}\left(U_o^2-u^2\right) \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{\partial}{\partial y}\left(\alpha(y)\frac{\partial T}{\partial y}\right)+q'''+\frac{\bar{\mu}}{\rho C_p}\left(\frac{\partial u}{\partial y}\right)^2, \tag{3}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=\frac{\partial}{\partial y}\left(\gamma(y)\frac{\partial C}{\partial y}\right) \tag{4}$$

All the physical quantities have their standard meaning. For such a physical configuration, the boundary conditions are found to be as follows:

$$u=0, v=0, T=T_W, C=C_W \text{ at } y=0, \qquad u=U_o,\ v=0,\ T=T_\infty, \mathrm{C}=\mathrm{C}_\infty \text{ as } y\to\infty \tag{5}$$

Equations (1) – (4) are solved using the following dimensionless variables f , $\theta, \phi,$ and q''' and similarity variable η with stream function ψ are introduced:

$$\eta=\left(\frac{y}{x}\right)\left(\frac{U_o x}{v}\right)^{1/2},\quad \psi=\sqrt{vU_o x}\,f(\eta),\quad \theta=\frac{T-T_\infty}{T_W-T_\infty},\quad \phi=\frac{\mathrm{C}-\mathrm{C}_\infty}{\mathrm{C}_W-C_\infty},\quad q'''=\frac{U_o(T_W-T_\infty)}{2x}e^{-\eta} \tag{6}$$

The variable permeability $k(\eta)$, the variable porosity $\varepsilon(\eta)$, variable effective thermal conductivity $\alpha(\eta)$ and the variable effective solutal diffusivity $\gamma(\eta)$ are defined as

$$\begin{aligned} k(\eta)&=k_o(1+de^{-\eta})\ ; \qquad \gamma(\eta)=\gamma_o\left[\varepsilon_o(1+d^*e^{-\eta})+\gamma^*\{1-\varepsilon_o(1+d^*e^{-\eta})\}\right]; \\ \varepsilon(\eta)&=\varepsilon_o(1+d^*e^{-\eta}); \qquad \alpha(\eta)=\alpha_o\left[\varepsilon_o(1+d^*e^{-\eta})+\sigma^*\{1-\varepsilon_o(1+d^*e^{-\eta})\}\right] \end{aligned} \tag{7}$$

$k_o, \varepsilon_o, \alpha_o$ and γ_o are fluid properties at the edge of the boundary layer respectively.

Equations (2) - (4) using Eqs. (6) and (7) are transformed to the local similarity equations as

$$f'''+\frac{1}{2}ff''+\frac{Gr}{\mathrm{Re}^2}\theta+\frac{\alpha^*(1+d^*e^{-\eta})}{\sigma\,\mathrm{Re}(1+de^{-\eta})}(1-f')+\frac{\beta^*(1+d^*e^{-\eta})^2}{(1+de^{-\eta})^{1/2}}(1-f'^2)=0 \tag{8}$$

$$\theta''=\frac{-\frac{1}{2}\Pr f\theta'-\Pr Ef''^2-\frac{1}{2}\Pr e^{-\eta}-\varepsilon_o d^*e^{-\eta}(\sigma^*-1)\theta'}{\varepsilon_o+\sigma^*(1-\varepsilon_o)+\varepsilon_o d^*e^{-\eta}(1-\sigma^*)} \tag{9}$$

$$\phi'' = \frac{-\frac{1}{2}Sc\phi' f + \varepsilon_o d^* e^{-\eta}(\gamma^* - 1)\phi'}{\varepsilon_o d^* e^{-\eta}(1-\gamma^*) + \varepsilon_o + \gamma^*(1-\varepsilon_o)} \tag{10}$$

The transformed boundary condition takes in the form

$$\begin{aligned} &f = 0, \quad f' = 0, \quad \theta = 1, \quad \phi = 1 \quad \text{at} \quad \eta = 0, \\ &f' = 1, \quad \theta = 0, \quad \phi = 0 \quad \text{as} \quad \eta \to \infty \end{aligned} \tag{11}$$

The rate of heat and mass transfer expressions skin friction τ , Nusselt number Nu and the Sherwood number Sh are very important to be computed due to many practical applications

$$\tau = -f''(0)/\sqrt{\mathrm{Re}}, \quad Nu = -\sqrt{\mathrm{Re}}\,\theta'(0) \quad \text{and } Sh = -\sqrt{\mathrm{Re}}\,\phi'(0). \tag{12}$$

III. METHOD OF SOLUTION

The governing equations arising pertaining to the physical system considered are coupled and nonlinear equations which are solved by reducing into first order ordinary differential equations (14). A numerical technique called shooting technique which involves RK-Fehlberg scheme and Newton-Raphson method for the accuracy of 10^{-6} is applied to obtain the distributions and physical interpretations of various parameters.

$$\begin{aligned}
&f = f_1, \quad \frac{df_1}{d\eta} = f_2, \quad \frac{df_2}{d\eta} = f_3, \\
&\frac{df_3}{d\eta} = -\frac{1}{2} f_1 f_3 - \frac{Gr}{\mathrm{Re}^2}(f_4 - Nf_6) - \frac{\alpha^*}{\sigma \mathrm{Re}}\left(\frac{1+d^* e^{-\eta}}{1+de^{-\eta}}\right)(1-f_2) - \frac{\beta^*(1+d^* e^{-\eta})}{(1+de^{-\eta})^{1/2}}(1\text{-}\, f_2^2) \\
&\theta = f_4, \quad \frac{df_4}{d\eta} = f_5 \\
&\frac{df_5}{d\eta} = -\frac{(1/2)\Pr f_1 f_5 + \Pr E f_3^2 + \varepsilon_o d^* e^{-\eta}(\sigma^* - 1) f_5}{\varepsilon_o + \sigma^*(1-\varepsilon_o) + \varepsilon_o d^* e^{-\eta}(1-\sigma^*)} \\
&\phi = f_6, \frac{df_6}{d\eta} = f_7 \\
&\frac{df_7}{d\eta} = -\frac{(1/2) Sc f_1 f_7 + \varepsilon_o d^* e^{-\eta}(\gamma^* - 1) f_7}{\varepsilon_o + \gamma^*(1-\varepsilon_o) + \varepsilon_o d^* e^{-\eta}(1-\gamma^*)}.
\end{aligned} \tag{13}$$

Boundary conditions (11) are now transformed to,

$$\begin{aligned} &f_1(0) = 0, \; f_2(0) = 0, \; f_3(0) = P, \; f_4(0) = 1, \; f_5(0) = Q, \; f_6(0) = 1, \; f_7(0) = R, \\ &f_2(\infty) = 1, \; f_3(\infty) = 1, \; f_4(\infty) = 0, \; f_6(\infty) = 0. \end{aligned} \tag{14}$$

The initiation of the Numerical technique in solving the system of I order ODEs with boundary conditions, led the understanding that seven initial conditions are needed, but only two initial conditions on velocity, one on temperature and one on concentration are available. The unknown conditions are not prescribed, which are determined by using the four ending boundary condition given in equations (14). A suitable finite value of η_∞ need to be guessed depending on the convergence in the shooting technique. The accuracy of initial conditions is checked at the terminal point, and the process can be stopped when the converged results are obtained. Since RK method of fifth order is used the results are more accurate.

IV. RESULTS AND DISCUSSIONS

The paper highlights the Numerical results for the effects of second order resistance and variable fluid properties with IHG for the study of combined flow past a heated plate placed vertically. From the Table 1 and 2, it is observed that for different values of non-dimensional pertinent parameters of the physical problem considered, the velocity, temperatures and concentration values are tabulated for far away from the plate. We observe that the variable permeability dominates more when compared to uniform permeability due to Forcheimeir drag and internal heat generation (IHG).

Fig 1 depicts the variations of velocity profiles for different Forcheimeir drag β^* for varying the fluid properties case. High resistance of the porous medium to the fluid flow leads to positive correlation in the velocity profile. The boundary layer thickness diminishes with increase β^* . Fig 2 depicts the temperature profile for β^* values which shows rate of cooling is much faster for higher values of Forcheimeir drag in both the cases. The exponential term of internal heat source gives the better cooling.

The profiles from Fig 3, reveals that upsurge in the value of N, escalates velocity profile, whereas negatively correlates with boundary layer. Energy and solutal profiles negatively correlates with rise in N within the boundary layer as shown in Fig 4 and 5 and gives the observation that cooling is faster for huge number of N.

Mixed convective flow behaviour is observed by the value of Ri in Figs 6, 7 and 8, significance in the boundary layer is high with the rise of Gr/Re^2 . When the Richardson number takes the value 2.0 the effect of VP is more prominent as observed in velocity profiles and boundary layer decreases. Since free stream is directed upward, the temperature change currents induce the average speed to rise when heated plate is cooled. Illustrations of energy and solutal distributions, where the parameter Gr/Re^2 enhances the surface heat and thickness of the boundary layer lowers leading to increase in wall temperature.

Value of $\alpha^*/\sigma\mathrm{Re}$ is varied for velocity profiles and found varied fluid properties are less when compared to uniform as shown in Fig 9. This is due to high viscous forces arising because of low value of local permeability parameter and Reynolds number. For various values of Prandtl number, velocity profiles are illustrated in Fig. 10. The boundary layer is more significant in the center. Fig 11 depicts the variation of Nusselt number for different values of Gr/Re^2, σ^* and Prandtl number Pr. Higher the Prandtl number Pr and σ^* the Nusselt number increases and slowly uniforms as Gr/Re^2 rises, due to heat difference present in ratio of Gr/Re^2. Also the dependence of Nu on diffusivity ratio is weak at least for the range shown. Mass transfer behaviour is observed in Fig 12 for various values of Gr/Re^2 and Schmidt number Sc. with fixed Sc. the mass transfer coefficient remains same after a slight increase initially for both UP and VP case. Increase in value of Sc leads to higher mass transfer coefficient.

V. CONCLUSIONS

In this article, a computational numerical analysis for Forcheimeir effect with internal heat source is discussed for transport processes and the following observations are made

- The graph depicts that rate of cooling is faster with Forcheimeir drag by varying the fluid properties with internal heat.
- Combined flow is more prominent due to drag and internal heat generation.
- Prandtl number enhances the temperature. Schmidt number leads to higher mass transfer coefficient.

This shows that the better transport processes results, with the drag and variability of fluid properties with internal heat source.

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I. TABLE I

Results for $f''(0), -\theta'(0)$ and $-\phi'(0)$ for Uniform Permeability (UP) and Variable Permeability (VP) Cases with IHG for Various Fixed Parameters.

N	σ^*	Gr/Re^2	$\alpha^*/\sigma Re$	β^*	Uniform Permeability (UP)			Variable Permeability (VP)		
					$f''(0)$	$-\theta'(0)$	$-\phi'(0)$	$f''(0)$	$-\theta'(0)$	$-\phi'(0)$
0	2	0.0	0.1	0.1	0.446543	0.293567	0.298751	0.435670	0.325750	0.328590
				0.5	0.771564	0.382654	0.388657	0.675800	0.400580	0.400780
		0.2	0.1	0.1	0.856509	0.567800	0.574200	0.534500	0.541456	0.545672
				0.5	0.998750	0.587650	0.593450	0.778500	0.561578	0.567652
		2.0	0.1	0.1	1.396500	0.889950	0.905500	1.378900	0.881132	0.901256
				0.5	1.654550	0.987540	0.993540	2.004500	0.980023	0.988976
1	2	0.0	0.1	0.1	0.465650	0.324565	0.329550	0.426789	0.300670	0.300470
				0.5	0.778550	0.425500	0.429650	0.777850	0.456700	0.456100
		0.2	0.1	0.1	0.824535	0.567430	0.572340	0.813453	0.498540	0.497530
				0.5	0.984565	0.651789	0.641456	0.945675	0.685933	0.683915
		2.0	0.1	0.1	1.521784	0.689450	0.694550	1.531562	0.785933	0.782134
				0.5	1.734256	0.789450	0.795550	1.739874	0.876540	0.874786
5	2	0.2	0.1	0.1	1.036789	0.675431	0.650034	0.995432	0.657875	0.651053
			0.5	0.1	1.456782	0.681432	0.670134	1.405673	0.677892	0.670001
10	2	0.2	0.1	0.1	1.558978	0.743451	0.731342	1.554322	0.734561	0.729324
			0.5	0.1	1.778956	0.781322	0.779432	1.740345	0.775641	0.767891

I. TABLE 2

Values of Rate of Heat Transfer (Nu) and Mass Transfer (Sh) $(-\theta'(0)$ and $-\phi'(0))$ for Uniform and Variable Permeability Cases.

Gr/Re^2	Pr = 0.71		Pr =3.0		Pr = 7.0		Sc = 0.22		Sc = 0.44		Sc = 0.60	
	$-\theta'(0)$		$-\theta'(0)$		$-\theta'(0)$		$-\phi'(0)$		$-\phi'(0)$		$-\phi'(0)$	
	UP	VP	UP	VP	UP	VP	UP	VP	UP	VP	UP	VP
0.0	0.4276	0.4357	0.4598	0.4956	0.5465	0.5567	0.4316	0.4457	0.4798	0.5156	0.5565	0.5767
0.2	0.4356	0.4516	0.4867	0.5234	0.5612	0.5834	0.4356	0.4516	0.4867	0.5234	0.5612	0.5834
2.0	0.4465	0.4968	0.5367	0.5714	0.5814	0.6356	0.4465	0.4968	0.5367	0.5714	0.5814	0.6356

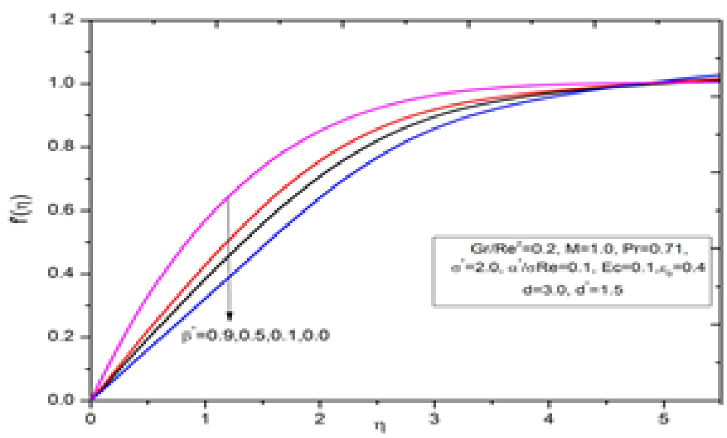

Fig 1 Velocity profiles of different values of β^* for VP case

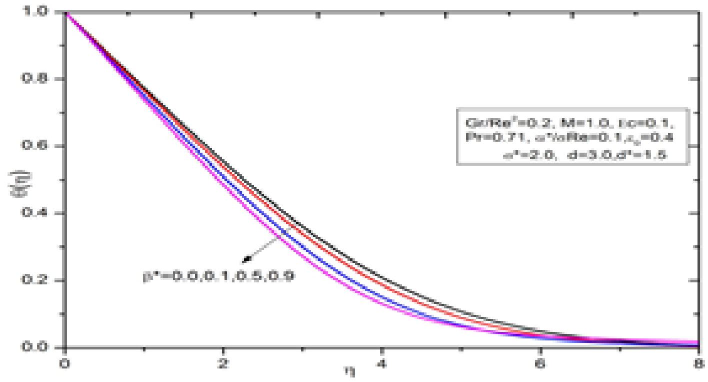

Fig 2 Temperature profiles for different values of β^* for VP case

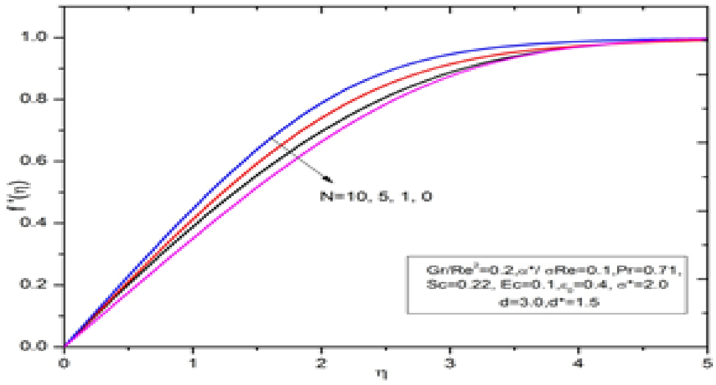

Fig 3 Velocity profiles for different values of N for VP case

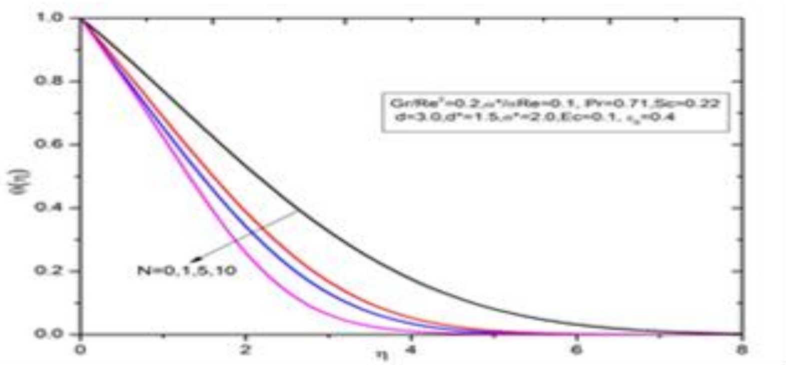

Fig 4 Temperature distributions for various values of buoyancy ratio N for VP case

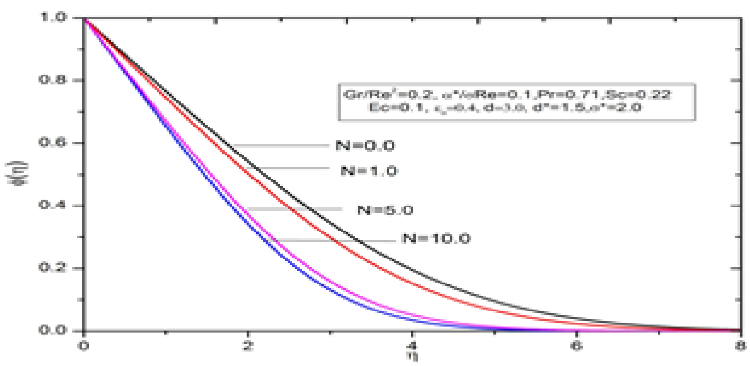

Fig 5 Concentration distribution for various values of buoyancy ratio N for VP case

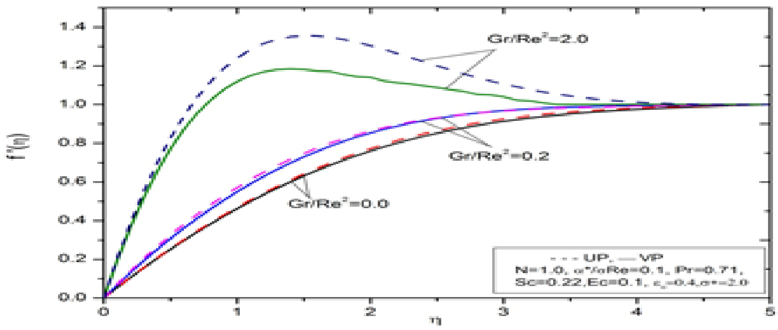

Fig 6 Velocity distributions for various values of Gr/Re^2 for both UP and VP cases

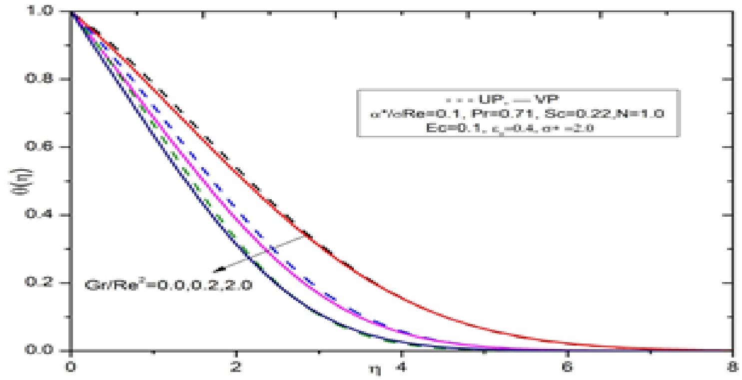

Fig 7 Temperature profiles for various values of Gr/Re^2 for both UP and VP cases

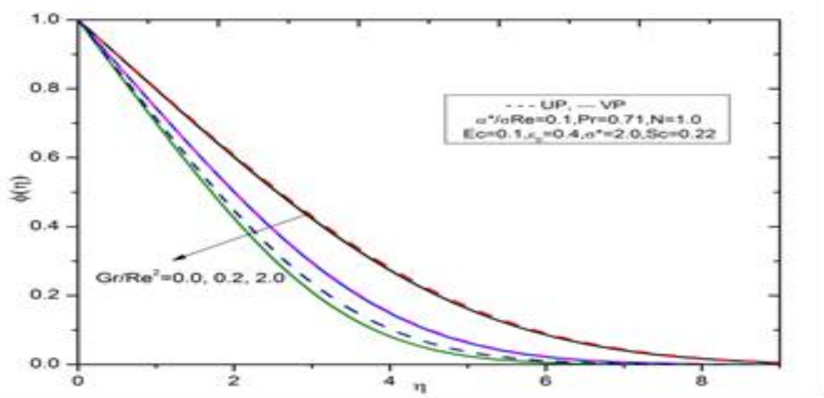

Fig 8 Concentration profiles for various values of Gr/Re^2 for both UP and VP cases

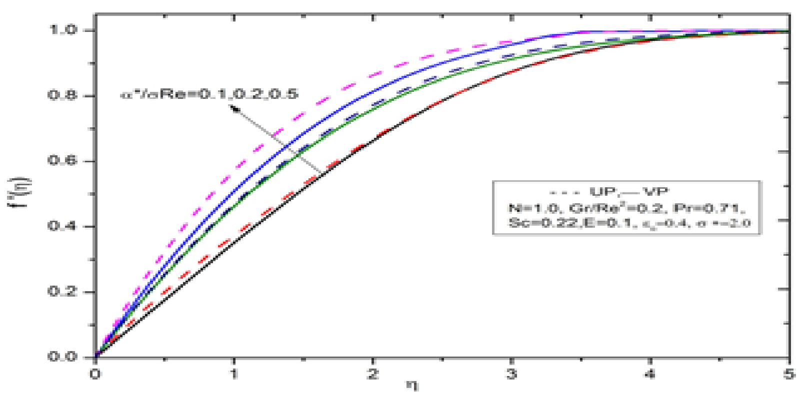

Fig 9 Velocity profiles for different values of $\alpha^*/\sigma\mathrm{Re}$ for both UP and VP cases

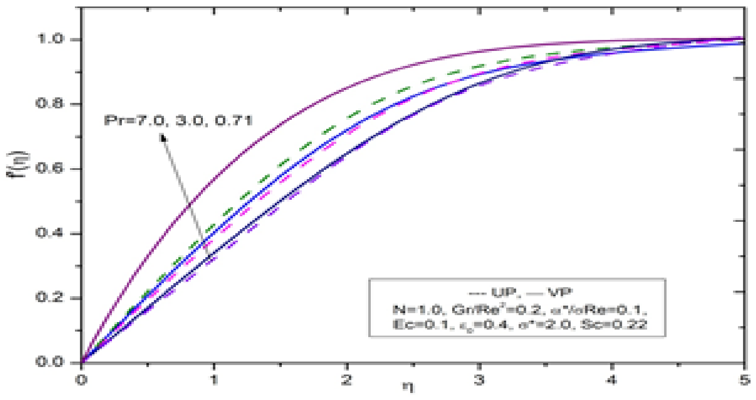

Fig 10 Velocity variations for different values of Pr for UP and VP cases

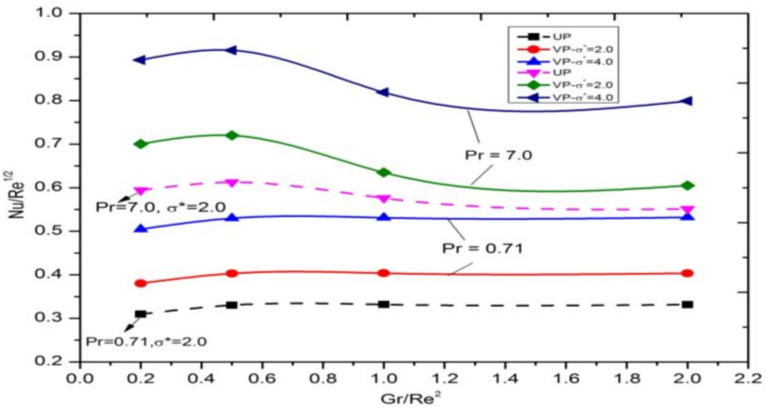

Fig 11 Nusselt number with $\theta'(0)$ for different values of Pr for UP and VP cases

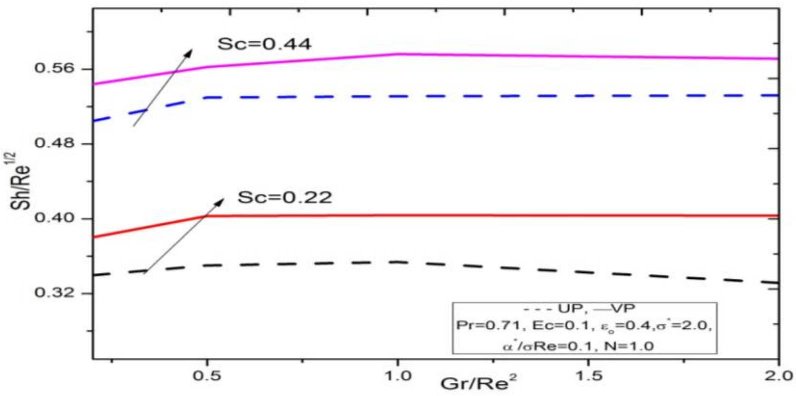

Fig 12 Variations in local Sherwood number with Gr/Re^2 for various values of Sc for UP and VP cases

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