Some General Characterization Coloring of Cubic Fuzzy Graphs

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Abstract

In this paper to generalize the coloring of cubic fuzzy graphs. We discuss the concept of two types coloring namely vertex coloring and edge coloring of cubic fuzzy graphs. We determine some result on strong and weak coloring of odd and even vertices of cubic fuzzy graphs.

Keywords - *Fuzzy graph, Coloring of fuzzy graph, Coloring of Cubic Fuzzy Graph.*

I. INRODUCTION

Let G= (V, E) be a graph such that V is the vertex set and E is the edge set. In graph theory graph coloring is one of the most important problems of combinatorial optimizations. Many problem of practical interest can be modeled as coloring problems. Two types of coloring namely vertex coloring and edge coloring are usually associated with any graph edge coloring is a function which assigns colors to the edges so that incident edges receive different colors. A proper coloring of a graph G is a function from the set of vertices of a graph to a set of colors such that any two adjacent vertices colored with the same color is called color class. In this paper we focus on K- coloring of a cubic fuzzy graph by taking fuzzy set of vertices and fuzzy set of edges. Also discuss the odd and even vertices coloring of cubic fuzzy graph.

II. PRELIMINARIES

Definition. 2.1 (Fuzzy Graph)

A fuzzy graph G ($,\mu$) is a pair of function σ: V→ [0,1] and μ : v×v → [0,1]

such that [0,1] . σ (u, v)≤ $\sigma(u) \wedge \sigma(v)$ for all u, v in V

Definition. 2.2 (Coloring of fuzzy graph)

A family, Γ { $\gamma 1, \gamma 2, \gamma 3 \ldots \ldots \ldots \ldots \gamma k$ } of fuzzy sets on a set V is called k- fuzzy coloring of G = (V ,σ,μ) if a) $\vee \Gamma = \sigma$ b) $\gamma i \wedge \gamma j = 0$ c) For every strong edge (x , y) of G min { $\gamma i\ (x),\ \gamma i\ (y)$} = 0 ($1 \leq i \leq k$)

Definition : 2.3 (Vertex and Edge coloring of fuzzy graph)

A graph is said to be vertex (Edge) K-colored if i it admits a vertex (Edge) coloring the minimum value of K for which G is vertex (Edge) K- colored is called vertex (Edge) chromatic number of G and is denoted by $\chi_{\mathbf{v}}$ (G) and $\chi_{\mathbf{E}}$ (G)

Definition :2.4 (Strong and Weak coloring fuzzy Graph)

A graph is said to be strong and weak vertex (Edge) K- colored if it admits an strong and weak vertex (Edge) K- coloring the maximum and minimum value of k for which G is edge K colored is called chromatic number of G and is denoted by $\chi_{\mathbf{s}}$(G) and $\chi_{\mathbf{w}}$ (G)

Definition :2.5 (coloring of regular fuzzy graph)

Let G: (σ,μ) be a fuzzy graph on G : (V. E) if d_G (V) = k for all v$\in V$. That is if each vertx has same degree k ,then G is said to be a regular fuzzy graph of degree k

Definition :2.6 (coloring of cubic fuzzy graph)

Let G: (σ,μ) be a fuzzy graph on G : (V. E) if d_G (V) =3 for all v$\in V$
That is if each vertex has same degree ,then G is said to be a cubic fuzzy graph of degree 3.

III. MAIN RESULTS

Theorem 3.1

If the coloring of a fuzzy cubic graph is p. Where n is the number of vertices G

Proof:

Let G be a cubic fuzzy graph G= (V, E) since d (v_1) = d (v_2) = d (v_3)

Every pair of vertices are strongly adjacent. Degree of each vertex is p. Hence each vertex have distinct color. The number of each vertex color in p. Otherwise is not adjacent. Then any two vertex have same color. The number of each vertex color is p-1

Theorem 3.2

If there exist a pair of non adjacent vertices (u, v) such that for all $x \in N(u)$ d (u) > d(x) and for all $y \in N(v)$ d(v) > d (y) has distinct colour all possible colouring of a graph G

Proof:

Let D be a colour class with vertex u of G. If there exist a pair of non adjacent vertices (u, v) such that for all $x \in N(u)$, d (u) > d(x) and for all $y \in N(v)$ d(v) > d(y) has distinct color all possible coloring of a graph then G. Since the vertex u cannot strongly dominated by any vertices G. The only one colour class D may be a strong dominating colour set of G But there vertex $v \notin D$, that also cannot be strongly dominated by any other vertices. For any fuzzy graph G the least possible membership vales are taken as χ_w dominating set all the in between vertices also added to χ_w similarly the largest passable membership values are taken as χ_s set we can discuss the cases by taking degrees of (v_1, v_2) and (v_p, v_{p-1})

Case (1) : d (v_1) $\geq$ d (v_2) and d (v_p) $\geq$ d (v_{p-1})

If d (v_1) $\geq$ d (v_2) and d (v_p) $\geq$ d (v_{p-1}) and for strong coloring set $v_1, v_2 \in E(C_n)$ then d (D) $\geq$ d (v-D) This ($v_1.v_p$) is also added to χ_s set. Hence χ_s = p and in weak coloring set d (D) $\leq$ d (v-D). Thus (v_2, v_{p-1}) is also added to χ_w set.

Hence χ_w = (v_2. V_3,.....v_{p-1}) = $\chi_w < \chi_s$

Case (2) : d(v_1) $\leq$ d(v_2) and d (v_p) $\leq$ d (v_{p-1})

In strong coloring set d(v_1) $\leq$ d (v_2) then (v_1. $v_{2,.....}v_{p-1}$) be in χ_s set. If weak coloring set d(v_1) $\leq$ d(v_2) and d (v_p) $\leq$ d(v_{p-1}) then (v1, v_2 . v3,.....v_{p-1}, v_p) be in χ_w set. Thus χ_w = P and $\chi_w > \chi_s$

Case (3) : d (v_p) $\leq$ d (v_{p-1})

In strong coloring set $u, v \in E(G)$ given d (u) $\geq$ d (v) then $v \in D$ given d (v_p)$\leq$ d(v_{p-1}) then χ_s = (v_2 . v_3,.....v_p) = p-1 and χ_w = (v_1, v_2 . v_3,.....v_{p-1}) = p-1 simalarly d(v_p) $\geq$ d (v_{p-1})

Theorem 3.3

If the graph G = (V, E) is odd vertex cubic fuzzy graph if each vertex has distinct color

Proof :

Let G: (σ , μ) be a fuzzy graph on G: (V. E) if d_G(V) = k for all $v \in V$. That is if each vertx has same degree k then G is a cubic fuzzy graph.

Let V = { V_1, V_2............ V_n } we assume that G is Odd vertices cubic fuzzy graph.

If all the vertex has distinct color. If the adjacent vertices V_1 be V_2 ,V_3 with degree d (v_1), d (v_2)....... d (v_n). Each vertex is adjacent to other vertex. By definition no vertex in adjacent to any vertex in that case the vertex has the same color. Otherwise each vertex has distinct color. If each vertex is adjacent it has distinct color. Then the graph is odd vertex cubic fuzzy graph

Example : 3:1

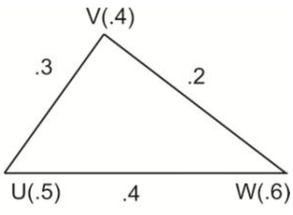

Fig 3.1

The vertex have the different colour for each because each vertex is adjacent. The odd vertex cubic fuzzy graphs always get the distinct colour in each vertex. If the adjacent vertices V_1 be V_2 , V_3 and V_2 be V_1, V_3. Also V_3 be V_1, V_2 The graph is Odd vertex cubic fuzzy graph.

THEOREM 3 :3

If the graph G = (V, E) is even vertex cubic fuzzy graph if any two vertex are same colour.

Proof:

Let G: (σ, μ) be a fuzzy graph on G (V. E) if $d_G(V) = k$ for all $v \in V$. That is if each vertx has same degree k then G is a cubic fuzzy graph. Let $V = \{V_1, V_2 \ldots\ldots V_n\}$ We assume that G is an even vertices cubic fuzzy graph if at least any two vertex has same colour. If the number of colouring is even then the graph strictly even cubic fuzzy graph. Finally if the graph is odd vertices and even vertices cubic graph then the vertex has odd number of colour is odd and even number of colour is even.

Example : 3: 2

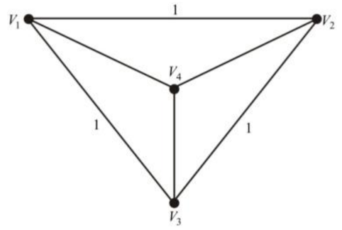

Fig 3.2

If the even vertex cubic graph alternative edges are equal. The adjacent vertex have distinct colour otherwise it has the same colour.

IV. CONCLUSION

In this paper the focus the concept of k- coloring on a cubic fuzzy graph, by taking fuzzy set of vertices and edges. Also we determine coloring of odd and even number of vertices in cubic fuzzy graph focussed on strong and weak coloring set.

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