$\approx$g(1,2)* - Continuous Maps

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Abstract

This research introduced generalized closed sets in general topology as a generalization of closed sets. This concept was found to be useful and many results in general topology were improved. Many researchers introduced ĝ-closed sets in topological spaces. In this paper, we discussed a new class of sets namely ≈ g(1,2)-closed sets in bitopological spaces. This class lies between the class of $\tau_{1,2}$-closed sets and the class of (1,2)*-ĝ-closed sets. The notion of ≈ g(1,2)*-interior is defined and some of its basic properties are studied. Also we introduce the concept of ≈ g(1,2)*-closure in bitopological spaces using the notion of ≈ g(1,2)*-closed sets, and we obtain some related results. For any $A \subseteq X$, it is proved that the complement of ≈ g(1,2)*-interior of A is the ≈ g(1,2)*-closure of the complement of A.*

Keywords: *Bitopological spaces, Continuous Maps, Closed Sets.*

I. INTRODUCTION

Several authors working in the field of general topology have shown more interest in studying the concepts of generalizations of continuous maps. A weak form of continuous maps called g-continuous maps were introduced by Balachandran et al. Recently Sheik John introduced and studied another form of generalized continuous maps called -continuous maps respectively. We first introduce $\approx$g(1,2)*-continuous maps and study their relations with various generalized (1,2)*-continuous maps . We also discuss some properties of $\approx$ g(1,2)*-continuous maps. We then introduce a new class of $\approx$g(1,2)*-open sets in bitopological spaces.

II. PRELIMINARIES

2.1 Definition

(i) A map $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is called $\approx g_\alpha$(1,2)*-continuous if $f^{-1}(V)$ is an $\approx g_\alpha$(1,2)*-closed set of X for every $\sigma_{1,2}$-closed set V of Y.
(ii) (1,2)*- -continuous if $f^{-1}(V)$ is a (1,2)*- closed set of X for every $\sigma_{1,2}$-closed set V of Y.
(iii) (1,2)*-ĝ-continuous if $f^{-1}(V)$ is a (1,2)*-ĝ-closed set of X for every $\sigma_{1,2}$-closed set V of Y.
(iv) (1,2)*-g-continuous if $f^{-1}(V)$ is a (1,2)*-g-closed set of X for every $\sigma_{1,2}$-closed set V of Y.
(v) (1,2)*- *gs* -continuous if $f^{-1}(V)$ is an (1,2)*- *gs* -closed set of X for every $\sigma_{1,2}$-closed set V of Y.
(vi) (1,2)*- g-continuous if $f^{-1}(V)$ is an (1,2)*- g-closed set of X for every $\sigma_{1,2}$-closed set V of Y.
(vii) (1,2)*-gs-continuous if $f^{-1}(V)$ is a (1,2)*-gs-closed set of X for every $\sigma_{1,2}$-closed set V of Y.
(viii) (1,2)*-gsp-continuous if $f^{-1}(V)$ is a (1,2)*-gsp-closed set of X for every $\sigma_{1,2}$-closed set V of Y.
(ix) (1,2)*-sg-continuous if $f^{-1}(V)$ is a (1,2)*-sg-closed set of X for every $\sigma_{1,2}$-closed set V of Y.
(x) (1,2)*-semi-continuous [59] if $f^{-1}(V)$ is a (1,2)*-semi-open set of X for every $\sigma_{1,2}$-open set V of Y.
(xi) (1,2)*- -continuous [60] if $f^{-1}(V)$ is an (1,2)*- -closed set of X for every $\sigma_{1,2}$-closed set V of Y.
(xii) ≈ g(1,2)*-closed if the image of every $\tau_{1,2}$-closed set in X is ≈ g(1,2)*-closed in Y.
(xiii) ≈ g(1,2)*-open if the image of every $\tau_{1,2}$-open set in X is ≈g(1,2)*-open in Y.
(xiv) (1,2)*-closed [61] if the image of every $\tau_{1,2}$-closed set in X is (1,2)*-closed in Y.
(xv) (1,2)*-continuous [61] if $f^{-1}(V)$ is a (1,2)*-closed set of X for every $\sigma_{1,2}$-closed set V of Y.

2.2 Definition

A map $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is called (1,2)*-sg-irresolute map if the inverse image of every (1,2)*-sg-closed set in Y is sg-closed in X.

2.3 Definition

A map $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is called pre-(1,2)*-sg-open if f(U) is (1,2)*-sg-open in Y, for each (1,2)*-sg-open set in X. pre-(1,2)*-sg-closed if f(U) is (1,2)*-sg-closed in Y, for each (1,2)*-sg-closed set U in X.

III. ≈ g(1,2)*-OPEN SETS

3.1 Definition

A subset A of X is called ≈ g(1,2)*-open in X if A^c is ≈ g(1,2)*-closed in X.

The collection of all ≈ g(1,2)*-open sets of X is denoted by ≈ (1,2)*-GO(X).

3.2 Proposition

For any bitopological space X, we have the following.
Every $\tau_{1,2}$-open set is ≈ g(1,2)*-open but not conversely.

(i) Every ≈ g(1,2)*-open set is ≈ g_α(1,2)*-open but not conversely.
(ii) Every ≈ g(1,2)*-open set is (1,2)*- -open but not conversely.
(iii) Every ≈ g(1,2)*-open set is (1,2)*-ĝ-open but not conversely.
(iv) Every ≈g(1,2)*-open set is (1,2)*-g-open but not conversely.
(v) Every ≈ g(1,2)*-open set is (1,2)*- *gs* -open but not conversely.
(vi) Every ≈ g(1,2)*-open set is (1,2)*- g-open but not conversely.
(vii) Every ≈g(1,2)*-open set is (1,2)*-gs-open but not conversely.
(viii) Every ≈g(1,2)*-open set is (1,2)*-gsp-open but not conversely.
(ix) Every ≈ g(1,2)*-open set is (1,2)*-sg-open but not conversely.

3.3 Theorem

A set A of X is ≈ g(1,2)*-open if and only if F ⊆ $\tau_{1,2}$-int(A) whenever F is (1,2)*-sg-closed and F ⊆ A.

Proof : Suppose that F ⊆ $\tau_{1,2}$-int(A), where F is (1,2)*-sg-closed and F ⊆ A. Let

A^c ⊆ U where U is (1,2)*-sg-open. Then U^c ⊆ A and U^c is (1,2)*-sg-closed. Therefore U^c ⊆ $\tau_{1,2}$-int(A) by hypothesis. Since U^c ⊆ $\tau_{1,2}$-int(A), we have $(\tau_{1,2}\text{-int}(A))^c$ ⊆ U.

That is $\tau_{1,2}$-cl(A^c) ⊆ U, since $\tau_{1,2}$-cl(A^c) = $(\tau_{1,2}\text{-int}(A))^c$. Thus A^c is ≈g(1,2)*-closed.

That is A is ≈ g(1,2)*-open.

Conversely, suppose that A is ≈g(1,2)*-open, F ⊆ A and F is (1,2)*-sg- closed. Then F^c is (1,2)*-sg-open and A^c ⊆ F^c . Therefore, $\tau_{1,2}$-cl(A^c) ⊆ F^c by definition of ≈ g(1,2)*-closedness and so F ⊆ $\tau_{1,2}$-int(A), since $\tau_{1,2}$-cl(A^c) = $(\tau_{1,2}\text{-int}(A))^c$.

IV. ≈ g(1,2)*-CONTINUOUS MAPS

We introduce the following definitions:

4.1 Definition

A map f : (X, τ_1, τ_2) → (Y, σ_1, σ_2) is called
(i) ≈ g(1,2)*-continuous if the inverse image of every $\sigma_{1,2}$-closed set in Y is g(1,2)*-closed set in X.

(ii) strongly ≈g(1,2)*-continuous if the inverse image of every ≈g(1,2)*-open set in Y is $\tau_{1,2}$-open in X.

Example

Let X = Y = {a, b, c}, τ_1 = {ϕ, X, {c}} and τ_2 = {ϕ, X, {a, c}}. Then the sets in {ϕ, X, {c}, {a, c}} are called $\tau_{1,2}$-open and the sets in {ϕ, X, {b}, {a, b}} are called $\tau_{1,2}$-closed. Let σ_1 = {ϕ, Y, {b}} and σ_2 = {ϕ, Y, {a, b}}. Then the sets in {ϕ, Y, {b}, {a, b}} are called $\sigma_{1,2}$-open and the sets in {ϕ, Y, {c}, {a, c}} are called $\sigma_{1,2}$-closed. We have ≈ (1,2)*-GC(X) = {ϕ, {b}, {a, b}, X}. Let f : (X, τ_1, τ_2) → (Y, σ_1, σ_2) be the identity map. Then f is ≈g(1,2)*-continuous.

4.2 Proposition
Every (1,2)*-continuous map is ≈g(1,2)*-continuous but not conversely.

Example
The map f in Example 3.4.2 is ≈ g(1,2)*-continuous but not (1,2)*-continuous, since f^{-1}({a}) = {a} is not $\tau_{1,2}$- open in X.

4.3 Proposition
Every ≈ g(1,2)*-continuous map is ≈ g_α(1,2)*-continuous but not conversely.

Example
Let X = Y = {a, b, c}, τ_1 = {ϕ, X, {b}} and τ_2 = {ϕ, X}. Then the sets in {ϕ, X, {b}} are called $\tau_{1,2}$-open and the sets in {ϕ, X, {a, c}} are called $\tau_{1,2}$-closed.

Let σ_1 = {ϕ, Y, {b, c}} and σ_2 = {ϕ, Y}. Then the sets in {ϕ, Y, {b, c}} are called $\sigma_{1,2}$-open and the sets in {ϕ, Y, {a}} are called $\sigma_{1,2}$-closed. We have ≈(1,2)*-GC(X) = {ϕ, {a, c}, X} and ≈ (1,2)*-GαC(X) = {ϕ, {a}, {c}, {a, c}, X}. Let f : (X, τ_1, τ_2) → (Y, σ_1, σ_2) be the identity map. Then f is ≈ g_α(1,2)*-continuous but not ≈ g(1,2)*-continuous, since f^{-1}({a}) = {a} is not ≈ g(1,2)*-closed in X.

4.4 Proposition
Every ≈ g(1,2)*-continuous map is (1,2)*-continuous but not conversely.

Example
Let X = Y = {a, b, c}, τ_1 = {ϕ, X, {a}} and τ_2 = {ϕ, X}. Then the sets in {ϕ, X, {a}} are called $\tau_{1,2}$-open and the sets in {ϕ, X, {b, c}} are called $\tau_{1,2}$-closed. Let σ_1 = {ϕ, Y, {a, c}} and σ_2 = {ϕ, Y}. Then the sets in {ϕ, Y, {a, c}} are called $\sigma_{1,2}$-open and the sets in {ϕ, Y, {b}} are called $\sigma_{1,2}$-closed. We have ≈ (1,2)*-GC(X) = {ϕ, {b, c}, X} and (1,2)*- C(X) = {ϕ, {b}, {c}, {b, c}, X}. Let f : (X, τ_1, τ_2) → (Y, σ_1, σ_2) be the identity map. Then f is (1,2)*-
-continuous but not ≈g(1,2)*- continuous, since f^{-1}({b}) = {b} is not ≈ g(1,2)*-closed in X.

4.5 Proposition
Every ≈g(1,2)*-continuous map is (1,2)*-ĝ-continuous but not conversely.

Example
Let X = Y = {a, b, c, d}, τ_1 = {ϕ, X, {d}, {b, c, d}} and τ_2 = {ϕ, X, {b, c}}. Let σ_1 = {ϕ, Y, {b}} and σ_2 = {ϕ, Y}. Let f : (X, τ_1, τ_2) → (Y, σ_1, σ_2) be the identity map. Then f is (1,2)*-ĝ-continuous but not ≈g(1,2)*-continuous, since f^{-1}({a, c, d}) = {a, c, d} is not ≈ g(1,2)*-closed in X.

4.6 Proposition
Every ≈ g(1,2)*-continuous map is (1,2)*-g-continuous but not conversely.

Example
Let X = Y = {a, b, c}, τ_1 = {ϕ, X, {a}} and τ_2 = {ϕ, X, {b, c}}. Then the sets in {ϕ, X, {a}, {b, c}} are called $\tau_{1,2}$-open and the sets in {ϕ, X, {a}, {b, c}} are called $\tau_{1,2}$-closed. Let σ_1 = {ϕ, Y, {c}} and σ_2 = {ϕ, Y}. Then the sets in {ϕ, Y, {c}} are called $\sigma_{1,2}$-open and the sets in {ϕ, Y, {a, b}} are called $\sigma_{1,2}$-closed. We have (1,2)*-GC(X) = {ϕ, {a}, {b, c}, X} and (1,2)*- *G* C(X) = P(X). Let f : (X, τ_1, τ_2) → (Y, σ_1, σ_2) be the identity map. Then f is (1,2)*-g-continuous but not ≈g(1,2)*-continuous, since f^{-1}({a, b}) = {a, b} is not ≈ g(1,2)*-closed in X.

4.7 Proposition
Every ≈g(1,2)*-continuous map is (1,2)*- *gs* -continuous but not conversely.

Example
Let X = Y = {a, b, c}, τ_1 = {ϕ, X, {a}} and τ_2 = {ϕ, X, {b, c}}. Then the sets in {ϕ, X, {a}, {b, c}} are called $\tau_{1,2}$-open and the sets in {ϕ, X, {a}, {b, c}} are called $\tau_{1,2}$-closed. Let σ_1 = {ϕ, Y, {b}} and σ_2 = {ϕ, Y}. Then the sets in {ϕ, Y, {b}} are called $\sigma_{1,2}$-open and the sets in {ϕ, Y, {a, c}} are called $\sigma_{1,2}$-closed. We have (1,2)*-

GC(X) = {ϕ, {a}, {b, c}, X} and (1,2)*- *GS* C(X) = P(X). Let f : (X, τ_1, τ_2) A(Y, σ_1, σ_2) be the identity map. Then f is (1,2)*- *gs* -continuous but not ≈g(1,2)*-continuous, since f^{-1}({a, c}) = {a, c} is not ≈g(1,2)*-closed in X.

4.8 Proposition

Every ≈g(1,2)*-continuous map is (1,2)*-g-continuous but not conversely.

Example

Let X = Y = {a, b, c}, τ_1 = {ϕ, X, {a, b}} and τ_2 = {ϕ, X, {c}}. Then the sets in {ϕ, X, {a, b}, {c}} are called $\tau_{1,2}$-open and the sets in {ϕ, X, {c}, {a, b}} are called $\tau_{1,2}$-closed. Let σ_1 = {ϕ, Y, {b}} and σ_2 = {ϕ, Y}. Then the sets in {ϕ, Y, {b}} are called $\sigma_{1,2}$-open and the sets in {ϕ, Y, {a, c}} are called $\sigma_{1,2}$-closed. We have

(i) (1,2)*-GC(X) = {ϕ, {c}, {a, b}, X} and (1,2)*- *G* C(X) = P(X). Let f : (X, τ_1, τ_2)

(ii) Y, σ_1, σ_2) be the identity map. Then f is (1,2)*- g-continuous but not ≈g(1,2)*-continuous, since f^{-1}({a, c}) = {a, c} is not ≈ g(1,2)*-closed in X.

4.9 Proposition

Every ≈ g(1,2)*-continuous map is (1,2)*-gs-continuous but not conversely.

Example

Let X = Y = {a, b, c}, τ_1 = {ϕ, X, {a}} and τ_2 = {ϕ, X}. Then the sets in {ϕ, X, {a}} are called $\tau_{1,2}$-open and the sets in {ϕ, X, {b, c}} are called $\tau_{1,2}$-closed. Let σ_1 = {ϕ, Y, {a, b}} and σ_2 = {ϕ, Y}. Then the sets in {ϕ, Y, {a, b}} are called $\sigma_{1,2}$-open and the sets in {ϕ, Y, {c}} are called $\sigma_{1,2}$-closed. We have ≈ (1,2)*-GC(X)= {ϕ, {b, c}, X} and (1,2)*- *GS* C(X) = {ϕ, {b}, {c}, {a, b}, {a, c}, {b, c}, X}.

(i) (X, τ_1, τ_2) → (Y, σ_1, σ_2) be the identity map. Then f is (1,2)*-gs-continuous

(ii) g(1,2)*-continuous, since f^{-1}({c}) = {c} is not ≈g(1,2)*-closed in X.

4.10 Proposition

Every ≈g(1,2)*-continuous map is (1,2)*-gsp-continuous but not conversely.

Example

Let X = Y = {a, b, c}, τ_1 = {ϕ, X, {b}} and τ_2 = {ϕ, X}. Then the sets in {ϕ, X, {b}} are called $\tau_{1,2}$-open and the sets in {ϕ, X, {a, c}} are called $\tau_{1,2}$-closed. Let σ_1 = {ϕ, Y, {a, b}} and σ_2 = {ϕ, Y}. Then the sets in {ϕ, Y, {a, b}} are called $\sigma_{1,2}$-open and the sets in {ϕ, Y, {c}} are called $\sigma_{1,2}$-closed. We have ≈ (1,2)*-GC(X) = {ϕ, {a, c}, X} and (1,2)*- *GSP* C(X) = {ϕ, {a}, {c}, {a, b}, {a, c}, {b, c}, X}. Let (X, τ_1, τ_2) → (Y, σ_1, σ_2) be the identity map. Then f is (1,2)*-gsp-continuous but not g(1,2)*-continuous, since f^{-1}({c}) = {c} is not ≈ g(1,2)*-closed in X.

4.11 Proposition

Every ≈ g(1,2)*-continuous map is (1,2)*-sg-continuous but not conversely.

Example

Let X = Y = {a, b, c}, τ_1 = {ϕ, X, {a}} and τ_2 = {ϕ, X, {b, c}}. Then the sets in {ϕ, X, {a}, {b, c}} are called $\tau_{1,2}$-open and the sets in {ϕ, X, {a}, {b, c}} are called $\tau_{1,2}$-closed. Let σ_1 = {ϕ, Y, {a, b}} and σ_2 = {ϕ, Y}. Then the sets in {ϕ, Y, {a, b}} are called $\sigma_{1,2}$-open and the sets in {ϕ, Y, {c}} are called $\sigma_{1,2}$-closed. We have ≈ (1,2)*-GC(X) = {ϕ, {b, c}, X} and (1,2)*- *SG* C(X) = {ϕ, {b}, {c}, {b, c}, X}. Let (X, τ_1, τ_2) → (Y, σ_1, σ_2) be the identity map. Then f is (1,2)*-sg-continuous but not g(1,2)*-continuous, since f^{-1}({c}) = {c} is not ≈ g(1,2)*-closed in X.

Remark

The following Examples show that ≈g(1,2)*-continuity is independent of (1,2)*- -continuity and (1,2)*-semi-continuity.

Example

Let X = Y = {a, b, c}, τ_1 = {ϕ, X, {a, b}} and τ_2 = {ϕ, X}. Then the sets in {ϕ, X, {a, b}} are called $\tau_{1,2}$-open and the sets in {ϕ, X, {c}} are called $\tau_{1,2}$-closed. Let σ_1 = {ϕ, Y, {a}} and σ_2 = {ϕ, Y}. Then the sets in {ϕ, Y, {a}} are called $\sigma_{1,2}$-open and the sets in {ϕ, Y, {b, c}} are called $\sigma_{1,2}$-closed. We have ≈ (1,2)*-GC(X) = {ϕ, {c}, {a, c}, {b, c}, X} and (1,2)*- C(X) = *S* C(X) = {ϕ, {c}, X}. Let (X, τ_1, τ_2) → (Y, σ_1, σ_2) be the identity map. Then f is ≈ g(1,2)*-continuous but it is neither (1,2)*- -continuous nor (1,2)*-semi-continuous, since f^{-1}({b, c}) = {b, c} is neither (1,2)*- -closed nor (1,2)*-semi-closed in X.

Example

Let X = Y = {a, b, c}, τ_1 = {ϕ, X, {a}} and τ_2 = {ϕ, X}. Then the sets in {ϕ, X, {a}} are called $\tau_{1,2}$-open and the sets in {ϕ, X, {b, c}} are called $\tau_{1,2}$-closed. Let σ_1 = {ϕ, Y, {a, b}} and σ_2 = {ϕ, Y}. Then the sets in {ϕ, Y, {a, b}} are called $\sigma_{1,2}$-open and the sets in {ϕ, Y, {c}} are called $\sigma_{1,2}$-closed. We have ≈ (1,2)*-GC(X)= {ϕ, {b, c}, X}, SC(X) = (1,2)*- C(X) = {ϕ, {b}, {c}, {b, c}, X}. Let f : (X, τ_1, τ_2) → (Y, σ_1, σ_2) be the identity map. Then f is both (1,2)*- -continuous and (1,2)*-semi-continuous but it is not ≈g(1,2)*-continuous, since f^{-1}({c}) = {c} is not ≈ g(1,2)*-closed in X.

4.12 Proposition

A map f : (X, τ_1, τ_2) → (Y, σ_1, σ_2) is ≈ g(1,2)*-continuous if and only if and (1,2)*-semi-continuous but it is not ≈g(1,2)*-continuous, since f^{-1}({c}) = {c} is not ≈ g(1,2)*-closed in X.

4.13 Proposition

A map f : (X, τ_1, τ_2) → (Y, σ_1, σ_2) is ≈g(1,2)*- continuous if and only if f^{-1}(U) is ≈g(1,2)*-open in X for every $\sigma_{1,2}$-open set U in Y.

Proof

Let f : (X, τ_1, τ_2) → (Y, σ_1, σ_2) be ≈ g(1,2)*-continuous and U be an $\sigma_{1,2}$-open set in Y. Then U^c is $\sigma_{1,2}$-closed in Y and since f is ≈g(1,2)*-continuous, $f^{-1}(U^c)$ is g(1,2)*-closed in X. But $f^{-1}(U^c) = (f^{-1}(U))^c$ and so f^{-1}(U) is ≈ g(1,2)*-open in X.

Conversely, assume that f^{-1}(U) is ≈ g(1,2)*-open in X for each $\sigma_{1,2}$-open set U in Y. Let F be a $\sigma_{1,2}$-closed set in Y. Then F^c is $\sigma_{1,2}$-open in Y and by assumption, $f^{-1}(F^c)$ is ≈ g(1,2)*-open in X. Since $f^{-1}(F^c) = (f^{-1}(F))^c$, we have f^{-1}(F) is ≈ g(1,2)*-closed in X and so f is ≈ g(1,2)*-continuous.

Remark

The composition of two ≈ g(1,2)*-continuous maps need not be ≈ g(1,2)*-continuous and this is shown by the following example.

Example

Let X = Y = Z = {a, b, c}, τ_1 = {ϕ, X, {a}, {a, b}} and τ_2 = {ϕ, X, {a, c}}. Then the sets in {ϕ, X, {a}, {a, b}, {a, c}} are called $\tau_{1,2}$-open and the sets in {ϕ, X,{b}, {c}, {b, c}} are called $\tau_{1,2}$-closed. Let σ_1 = {ϕ, Y, {a, b}} and σ_2 = {ϕ, Y}. Then the sets in {ϕ, Y, {a, b}} are called $\sigma_{1,2}$-open and the sets in {ϕ, Y, {c}} are called $\sigma_{1,2}$-closed. Let η_1 = {ϕ, Z, {b}} and η_2 = {ϕ, Z}. Then the sets in {ϕ, Z, {b}} are called $\eta_{1,2}$-open and the sets in {ϕ, Z, {a, c}} are called $\eta_{1,2}$-closed. Let f : (X, τ_1, τ_2) → (Y, σ_1, σ_2) and g : (Y, σ_1, σ_2) → (Z, η_1, η_2) be the identity maps. Then f and g are ≈g(1,2)*-continuous but their g o f : (X, τ_1, τ_2) → (Z, η_1, η_2) is not ≈g(1,2)*-continuous, because V = {a, c} is $\eta_{1,2}$-closed in Z but $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V)) = f^{-1}(g^{-1}(\{a, c\}) = f^{-1}(\{a, c\}) = \{a, c\}$, which is not ≈g(1,2)*-closed in X.

4.14 Proposition

Let (X, τ_1, τ_2) and (Z, η_1, η_2) be bitopological spaces and (Y, σ_1, σ_2) be a T $\approx g_{(1,2)*}$-space. Then the composition g o f : (X, τ_1, τ_2) → (Z, η_1, η_2) of the g(1,2)*-continuous maps f : (X, τ_1, τ_2) → (Y, σ_1, σ_2) and g : (Y, σ_1, σ_2) → (Z, η_1, η_2) is ≈g(1,2)*-continuous.

Proof

Let F be any $\eta_{1,2}$-closed set of (Z, η_1, η_2). Then g^{-1}(F) is ≈g(1,2)*-closed in (Y, σ_1, σ_2), since g is ≈ g(1,2)*-continuous. Since Y is a T ≈ $g_{(1,2)*}$-space, g^{-1}(F) is $\sigma_{1,2}$-closed in Y. Since f is ≈ g(1,2)*-continuous, $f^{-1}(g^{-1}(F))$ is ≈ g(1,2)*-closed in X. But $f^{-1}(g^{-1}(F)) = (g \circ f)^{-1}(F)$ and so g o f is ≈ g(1,2)*-continuous.

4.15 Proposition

Let F be any $\eta_{1,2}$-closed set of (Z, η_1, η_2). Then g^{-1}(F) is ≈g(1,2)*-closed in (Y, σ_1, σ_2), since g is ≈ g(1,2)*-continuous. Since Y is a T ≈ $g_{(1,2)*}$-space, g^{-1}(F) is $\sigma_{1,2}$- closed in Y. Since f is ≈ g(1,2)*-continuous, $f^{-1}(g^{-1}$(F)) is ≈ g(1,2)*-closed in X. But $f^{-1}(g^{-1}$(F)) = $(g \circ f)^{-1}$(F) and so g o f is ≈ g(1,2)*-continuous.

4.16 Proposition

Let (X, τ_1, τ_2) and (Z, η_1, η_2) be bitopological spaces and (Y, σ_1, σ_2) be a (1,2)*-$T_{1/2}$-space (resp. (1,2)*-T_b-space, (1,2)*- T_b-space). Then the composition g o(X, τ_1, τ_2) → (Z, η_1, η_2) of the ≈ g(1,2)*-continuous map f : (X, τ_1, τ_2) → (Y, σ_1, σ_2) and the (1,2)*-g-continuous (resp. (1,2)*-gs-continuous, (1,2)*-αg-continuous) map (Y, σ_1, σ_2) → (Z, η_1, η_2) is ≈g(1,2)*-continuous.

4.17 Theorem

If f : (X, τ_1, τ_2) → (Y, σ_1, σ_2) is ≈ g(1,2)*-continuous and pre-(1,2)*-sg-closed and if A is an ≈ g(1,2)*-open (or ≈ g(1,2)*-closed) subset of Y, then f^{-1}(A) is g(1,2)*-open (or ≈ g(1,2)*-closed) in X.

Proof

Let A be an ≈g(1,2)*-open set in Y and F be any (1,2)*-sg-closed set in X such that F ⊆ f^{-1}(A). Then f(F) ⊆ A. By hypothesis, f(F) is (1,2)*-sg-closed and A is ≈ g(1,2)*-open in Y. Therefore, f(F) ⊆ $\sigma_{1,2}$-int(A) by the theorem and so ⊆ $f^{-1}(\sigma_{1,2}$-int(A)). Since f is ≈g(1,2)*-continuous and $\sigma_{1,2}$-int(A) is $\sigma_{1,2}$-open in Y, $f^{-1}(\sigma_{1,2}$-int(A)) is ≈ g(1,2)*-open in X. Thus F ⊆ $\tau_{1,2}$-int($f^{-1}(\sigma_{1,2}$-int(A))) ⊆ $\tau_{1,2}$-int(f^{-1}(A)). That is F ⊆ $\tau_{1,2}$-int(f^{-1}(A)) and by the theorem, f^{-1}(A) is ≈g(1,2)*-open in X. By taking complements, we can show that if A is ≈g(1,2)*-closed in Y, f^{-1}(A) is ≈g(1,2)*-closed in X.

4.18 Theorem

Suppose the collection of all ≈g(1,2)*-open sets of X is closed under arbitrary unions. Let f : (X, τ_1, τ_2) → (Y, σ_1, σ_2) be a map from a bitopological space (X, τ_1, τ_2) into a bitopological space (Y, σ_1, σ_2). Then the following statements are equivalent.

(i) The function f is ≈g(1,2)*-continuous.
(ii) The inverse of each $\sigma_{1,2}$-open set is ≈g(1,2)*-open.
(iii) For each point x in X and each $\sigma_{1,2}$-open set V in Y with f(x) ∈ V, there is an g(1,2)*-open set U in X such that x ∈ U, f(U) ⊆ V.
(iv) The inverse of each $\sigma_{1,2}$-closed set is ≈ g(1,2)*-closed.
(v) For each x in X, the inverse of every neighborhood of f(x) is an ≈ g(1,2)*- nbhd of x.
(vi) For each x in X and each neighborhood N of f(x), there is an ≈ g(1,2)*-nbhd G of x such that f(G) ⊆ N.
(vii) For each subset A of X, f(≈g(1,2)*-cl(A)) ⊆ $\sigma_{1,2}$-cl(f(A)).
(viii) For each subset B of Y, ≈g(1,2)*-cl(f^{-1}(B)) ⊆ $f^{-1}(\sigma_{1,2}$-cl(B)).

V. CONCLUSION

General topology plays vital role in many fields of applied sciences as well as in all branches of mathematics. In reality it is used in data mining, computational topology for geometric design and molecular design, computer-aided design, computer-aided geometric design, digital topology, information systems, particle physics and quantum physics etc.

The notions of sets and functions in bitopological spaces and fuzzy topological spaces are extensively developed and used in many engineering problems, information systems, particle physics, computational topology and mathematical sciences.

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