Blended Dominance Property Embedded in Graphs

[1]Sreejil K, [2]Rajakumari N
[1]Research Scholar, [2]Assistant Professor
Ponniyah Ramajayam Institute of Science and Technology
Tanjavur, Vallam

Abstract**:** *Domination in graphs has been the core competency study topic at the outbreak of this era. Its profound and exclusive study of domination is graphs kick started around1850, coupled with the challenges of placing the least number of queens on an n x n chess board. So as to cover, there by dominating every square. Neverthless the challenges till date remains unanswered and clueless, the domination of graphs and its rise, dwells at length on graph theory introduced by Ore and Berge, they delves deep into researches most significant schools of thought and innovations. Berge presents the challenges of five queens namely place five queens on the chess board, so that unsparingly every square is covered by at least one queen. The solutions to these problems are dominating sets in graph, whose vertices are the queens of the chess board and vertices u, v are adjacent if a queen move from u to v is single move. This paves the way to domination in graphs. Starting with a close examination of one's own concept of domination in graphs ranges far and wide to give specific answers to all the challenges poping up.*

Keywords: *Graph theory, domination in graph.*

I. INTRODUCTION

The domination theory of graphs has been the scintillating essence of booming research activities, in the operation of graph theory in modern era. It dwelves deeply into the myriad manifestations and wide range of innovative parameters that can be emerged from basic definitions [11]. The adherence of study of dominating sets is graph theory commenced around 1960's irrespective of the theory lacking its significant roots is 1862. When De Jacnish, studied its problem of determining the least number of queens necessary to cover or dominate n x n chess board. In 1958 Berge defined the concept of domination number of a graph. Coining them as the "Coefficient of External stability". In 1962 Ore used the terminology "dominating set" and precisely domination number for the same concept. In 1977 Cockayne and Hedetniemi made a profound and exemplarory survey of the results at that time about dominating sets is graphs [10, 9]. In the segmal the following definitions are needed.

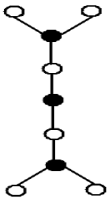
Definition 1: Let H be a finite connected graph with vertex set V = V (H) and edge set E = E(H)[8,7]. The neighborhood of a vertex v ∈ V (H), denoted by N(v), is the set of vertices adjacent to v. The open neighbourhood of a vertex v in a graph G is defined as the set $N_G(v)$ = { u ∈ V (H) : uv ∈ E(H)}.The closed neighborhood of v denoted by N[v] = N(v) ∪ {v} and the degree of v in Hby d(v). If S is a subset of V , we set $N(S) = U_{v \in S} N(v)$ and $N[S] = U_{v \in S} N[v]$ = S ∪ N(S). We use |X| for the cardinality of a set X. Let Δ(H) and δ(H) denote the maximum degree and the minimum degree of H[6].
We now introduce the concept of dominating sets in graphs[5].

Definition 2: A set D ⊆ V of vertices in a graph H = (V,E) is a dominating set if every vertex v ∈ V is an element of D or adjacent to an element of D[4]. Alternatively, we can say that D ⊆ V is a dominating set of H if N[D] = V (H).

Definition 3**:** A dominating set D is a minimal dominating set if no proper subset D′ ⊂ D is a dominating set[3]. The domination number γ(H) of a graph G is the minimum cardinality of a dominating set of G. We call such a set a γ-set of H[2].

Definition 4: For a graph H = (V,E) and S ⊆ V a vertex v ∈ D is an enclave of D if N[v] ⊆ D. For D ⊆ V a vertex v ∈ D is an isolate of D if N(v) ⊆ V − D. We say that a set is enclaveless if it does not contain any enclaves. Note that S is a dominating set of a graph H = (V,E) if and only if V − D is enclaveless[1].

Example
D = Shaded vertices



Theorem 1 . A dominating set T of a graph H is a minimal dominating set if and only if for any u ∈ T,
1. u is an isolate of T, or 2. There is v ∈ V − T for which N[v] ∩ T = {u}.

Proof . Let T be a γ-set of H. Then for every vertex u ∈ T, T −{u} is not a dominating set of H. Thus, there is a vertex v ∈ (V − T) ∪ {u} that is not dominated by any vertex T −{u}. Now, either v = u, which implies u is an isolate of T; or v ∈ V − T, in which case v is not dominated by T − {u}, and is dominated by T. This shows that N[v] ∩ T = {u}. In order to prove the converse, we assume T is a dominating set and for all u ∈ T, either u is an isolate of T or there is v ∈ V − T for which N[v] ∩ T = {u}. We assume to the contrary that T is not a γ- set of H. Thus, there is a vertex u ∈ T such that T − {u} is a dominating set of H. Hence, u is adjacent to at least one vertex in T − {u}, so condition (1) does not hold. Also, if T − {u} is a dominating set, then every vertex in V − T is adjacent to at least one vertex in T −{u}, so condition (2) does not hold for u. Therefore, neither (1) nor (2) holds, contradicting our assumption.

II. CONCLUSION

This paper is proud to offer such an awesome concept to the diaspora, only to offer itself as an example that the other dominating sets may or may not follow. It does not share with the entire domination graph theory faiths a desire to universalize itself. Yet it's tenets and values are universally applicable. But first dominating graph theory must be revived and reasserted, it its glorious liberalism, it's openness and acceptance it's eclecticism and universalism that resonate with meaning for every mathematicians on the planet.

ACKNOWLEDGEMENT

This journal paper could not have been written without the help of numerous people who have been kind enough to comment on various transformation aspects of the paper.My former guide Mrs Radha rugmini and aide has guided me with affection in the pursuit by which she asked me always to point out the areas that she agrees with my depiction of concepts , but not with my rejection of mathematical faiths. Dr Ramdass, the dean and HOD of our department undoubtedly India's most popular interpreter of mathematical diaspora came up with a number of pertinent suggestions, on an earlier draft which have helped, influenced ,inspired and motivated me immensely in my final research version paper. Mr Paneerselvam sir , the everbrightening star have been a superb trend setter for me and it is his intelligence, vision and humanity that have accompanied me through many of my papers in the process of upbringing my mathematical skills. Mrs Rajakumari.N, the professor an intellectual moonlighting as guide raised a number of questions I have summarised in the note and offered other insights into the faith.

While many minds have therefore contributed to the contents of the volume, the final responsibility for the arguments and interpretations in this rests with me. If after reading this paper , mathematicians and non mathematicians come away with a new appreciation of the faith I cherish and the challenges it currently dealing with in the contemporary planet, why I am a researcher would have served its purpose.

REFERENCES

[1] E.J. Cockayne, R.M. Dawes, and S. T .Hedetniemi, Total domination in graphs, Networks 10 (1980), 211–219.
[2] Kumar S.S., Rao M.R.K., Balasubramanian M.P., "Anticarcinogenic effects of indigofera gaspalathoides on 20-methylcholanthrene induced fibrosarcoma in rats", Research Journal of Medicinal Plant, ISSN : 5(6) (2011) PP. 747-755.
[3] M. Borowiecki and D.Michalak, Generalized Independence and domination in graphs, Discrete Mathematics 191 (1998), 51–56.
[4] Beula Devamalar P.M., Thulasi Bai V., Srivatsa S.K., "Design and architecture of real time web-centric tele health diabetes diagnosis expert system", International Journal of Medical Engineering and Informatics, ISSN : 1755-0661, 1(3) (2009) PP.307-317.
[5] T. W. Haynes, S. T. Hedetniemi, and P. J. Slater, Fundamentals of Domination in Graphs, Marcel Dekker, Inc., New York, 1998.
[6] Niranjan U., Subramanyam R.B.V., Khanaa V., "Developing a Web Recommendation System Based on Closed Sequential Patterns",fCommunications in Computer and Information Science, ISSN : 1865-0929, 101() (2010) PP.171-179.
[7] J. Harant and M. A. Henning, On double domination in graphs, Discussiones Mathematicae Graph Theory 25 (2005), 29–34.
[8] T. W. Haynes, S. T. Hedetniemi, and P. J. Slater, eds. Domination in Graphs: Advanced Topics, Marcel Dekker, Inc., New York, 1998.
[9] Jaikumar S., Ramaswamy S., Asokan B.R., Mohan T., Gnanavel M., "Anti ulcer activity of methanolic extract of Jatropha curcas (Linn.) on Aspirin-induced gastric lesions in wistar strain rats", Research Journal of Pharmaceutical, Biological and Chemical Sciences, ISSN : 0975- 8585, 1(4) (2010) PP.886-897.
[10] W Sharmila S., Jeyanthi Rebecca L., Saduzzaman M., "Biodegradation of domestic effluent using different solvent extracts of Murraya koenigii", Journal of Chemical and Pharmaceutical Research, ISSN : 0975 – 7384, 5(2) (2013) PP.279-282.
[11] Goddard and M. A. Henning, Generalised domination and independence in graphs, Congressus Numerantium 123 (1997), 161–171.