Sophisticated Coalition Strategies Distinctive Hallmarks of Pathos Vertex Semi Entire Block Graph

[1]Sreejil K, [2]Rajakumari N
[1]*Research Scholar,* [2]*Assistant Professor*
Ponniyah Ramajayam Institute of Science and Technology
Tanjavur, Vallam

Abstract: *In this incredibly interesting paper studies hovers around the graphs whose vertex set is* $V(T) \cup b_t \cup_r$ *along side the two vertices are adjacent if and only if they are adjacent vertices which lies on the blocks regions and adjacent blocks; and they are categorically named as pathos vertex semi entire block graph and denoted by* $P_{v_b}(H)$. *We reveal results associated to these types of graphs and traverses deeply into absorbing wisdom to the brilliantly interpretative characterisations of graphs, whose pathos vertex semi entire block graph is planar, outer planar, Eulerian and Hamiltonian and there by embracing the emerging exegeses of the above theoretical convictions.*

Keywords: *Vertex set, block graphs, pendant pathos vertex semi entire graphs, Hamiltonian cycles.*

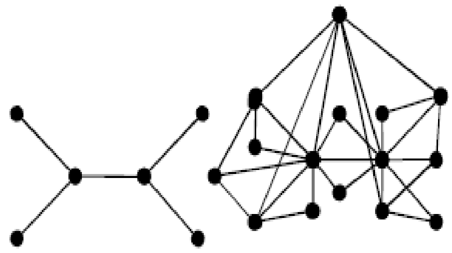
I. INTRODUCTION

The efforts of the authors served to reassert the faith in graph theory. Let H(p, q) be a connected planar graph. We show cases the terminology of [5]. The concept of pathos of a graph H was introduced by Harary [2], as a collection of minimum number of line disjoint open paths whose union is H. The path number of a graph H is the number of paths in a
pathos. A new concept of a graph valued functions called the pathos vertex semientire graph $P_{e_v}(H)$ of a plane graph H was introduced [5]. For a graph H(p, q) if $B = u_1, u_2, u_3, \cdots, u_r$; $r \geq 2$ is a block of H. Then we say that point u_1 and block B are incident with each other, as are u_2 and B and so on. If two distinct blocks B_1 and B_2 are incident with a common cut vertex then they are called adjacent blocks. All undefined terminology will confine with that in Harary[2]. All graphs considered here are finite, undirected and devoid of loops or multiple lines.

In a broader sense the edgedegree of an edge e = {a, b} is the sum of degrees of the end vertices a and b. Block degree is the number of vertices lies on a block. Blockpath is a path in which each edge in a path becomes a block. Degree of a region is the number of vertices lies on a region. A pendant pathos is a path Pi of pathos having unit length. The inner
vertex number i(H) of a planar graph H is the minimum number of vertices not belonging to the boundary of the exterior region in any embedding of H in the plane. A graph H is said to be minimally non-outer planar if i(H) = 1.

A novel concept of a graph valued functions practiced in contemporary insights called the pathos vertex semientire graph $P_{e_v}(H)$ of a plane graph H was introduced [5] and is defined as the graph whose vertex set is $V(T) \cup b_i \cup r$ and the two vertices are adjacent if and only if they are adjacent vertices, vertices lie on the path of pathos and vertices lie on the regions. Since the system of pathos for a tree is not unique, the corresponding vertex semientire block graph is also not unique. The vertex semientire block graph is introduced in [6]. The vertex semientire block graph denoted by $e_{vb}(H)$ is the graph whose vertex set is $V(T) \cup b_i \cup r$
and the two vertices are adjacent if and only if they are
adjacent vertices, vertices lie on the blocks and vertices lie on the regions.

We now incorporates the pathos vertex semientire block graph of a tree T. The pathos vertex semientire block graph of a tree T denoted by $P_{vb}(T)$ is the graph whose vertex set is the union of the vertices, regions and path of pathos of T and the two vertices are adjacent if and only if they are adjacent vertices of T, vertices lie on the blocks
of T, vertices lie on the regions of T and the adjacent blocks of T. Clearly the number of regions in a tree is one. The tree T and its pathos vertex semientire block graph Pvb(T) is depicted in the figure1.



T Figure 1. $P_{vb}(T)$

II. PRELIMINARY RESULTS

We need the following results to prove further results.

Theorem 1[4]. If H be a connected plane graph then the vertex semientire graph $e_v(H)$ is planar if and only if H is a tree.

Theorem 2[3]. Every maximal outerplanar graph H with p vertices has 2p – 3 edges.

Theorem 3 [4]. For any (p, q) graph Hwith b blocks and r

regions vertex semientire block graph $e_{vb}(H)$ has (p + b +r) vertices and $q+\sum_{i=1}^{k} d(b_i)+\sum_{j}^{l} d(r_j)$ edges, where

$d(b_i)$ is

the block degree of a block b_i and $d(r_j)$ is the degree of a region r_j.

III. PRIME RESULTS

We start with a preliminary result.

Remark 1. For any graph H, H$\subseteq$ e_{vb} (H) $\subseteq$ P_{vb} (T) .

In the following theorem we obtain the number of vertices and edges in pathos vertex semientire block graph.

Theorem 4. For any (p, q) graph T with b blocks and r regions pathos vertex semientire block graph $P_{vb}(T)$ has (2p +k) vertices and 4p – 3 + $q+\sum_{i=1}^{k} v(p_i)$ edges, where $v(p_i)$ be the number of vertices lies on the path pi.

Proof. By the Theorem 3, the number of vertices in $e_{vb}(T)$ is (p + b +r). By the definition of pathos vertex semientire block graph $P_{ev}(T)$ it follows that the number of vertices is the union of the vertices, blocks, the regions and the path of pathos of T. Since in a tree T, each edge is a block and it contains only one region. Hence p+b+r+k=p+q+1+k

= p+p-1+1+k

=2p+k vertices. Hence the number of vertices in pathos vertex semientire block graph $P_{vb}(T)$ is (2p + k).

Further, by Theorem 3, the number of edges vertex semientire block graph $e_{vb}(H)$ is $q+\sum_{i=1}^{k} d(b_i)+\sum_{j}^{l} d(r_j)$. By the Remark 1 it follows that $e_{vb}(H)$ is subgraph of $P_{vb}(T)$. Also the number of edges in $P_{vb}(T)$ is the sum of the edges in $e_{vb}(H)$ and the edges formed by the pathos vertices , which is Σ $v(p_i)$Hence the number of edge in

$$P_{vb}(H)=q+\sum_{i=1}^{k} d(b_i)+\sum_{j}^{l} d(r_j)+\sum_{i=1}^{k} v(p_i)$$

$$=q+2q+p+\sum_{i=1}^{k} v(p_i)=3p-3+p+\sum_{i=1}^{k} v(p_i)$$

$$=4p-3+\sum_{i=1}^{k} v(p_i).$$

Theorem 5. For any tree T, pathos vertex semientire block graph $P_{vb}(T)$ is always nonseparable.

Proof. We have the following two cases.

Case 1. Assume T be a path. All internal vertices of T are the cut vertices C_i. These cut vertices lies on the region as well as on two blocks. Clearly C_i is not a cut vertex in $P_{vb}(T)$. Hence $P_{vb}(T)$is nonseparable.

Case 2. Assume T be any tree. Since cut-vertex C_i lies on at least two blocks and one region. Hence in $P_{vb}(T)$, C_i becomes non-cut vertex. Also the pathos vertex is adjacent to all vertices v_i of T. Hence $P_{vb}(T)$ is always nonseparable.

Theorem 6. For any tree T, pathos vertex semientire block graph $P_{vb}(T)$ is planar.

Proof. Assume a graph T be a tree. By definition of vertex semientire block graph, for each edge of a tree H, there is a $K_4 - e$ in $e_{vb}(H)$. Hence in $P_{vb}(T)$, the pathos vertices are adjacent to the vertices those are lies on the path. Clearly $P_{vb}(T)$ is a graph which is homeomorphic to K_4. Hence $P_{vb}(T)$ is planar.

Theorem 7. For any tree T the pathos vertex semientire block graph $P_{vb}(H)$ always non-outer planar.

Proof. Consider a tree T be a path P_n. Suppose n=2. Since each edge is a block and both end vertices lies on a block. These end vertices and a block vertex form a graph K_3 in $e_{vb}(H)$. Further the region vertex vertex is adjacent to all vertices of H to form K_4-x. Also the pathos vertex is adjacent to all vertices of $e_{vb}(T)$ to form a graph with one inner vertex, which is non- outerplanar. Hence $P_{vb}(T)$ is always non-outerplanar.

Theorem 8. For any tree T, pathos vertex semientire block graph $P_{vb}(T)$ is minimally non- outerplanar if and only if T is a path P_2.

Proof. Proof follows from the Theorem 7.

Theorem 9. For any tree T, pathos vertex semientire block graph $P_{vb}(H)$ is Eulerian if and only if T is a path P_n for n is even.

Proof. Assume $P_{vb}(H)$ is Eulerian. Assume that a tree T be a path P_n for n is odd. By the definition of $P_{vb}(T)$, the region vertex is adjacent to all vertices of T, the pathos vertex is adjacent to all vertices of T. Also each edge is a block and the block vertex is adjacent of exactly two vertices . Lastly the pathos vertex is adjacent to all vertices of T.

Clearly each vertex v_i is adjacent to the corresponding block vertex b_i, region vertex r_1 and the pathos vertex p_1. It follows that degree of each v_k is even. Since the region vertex is adjacent to all vertices v_i for i= 1,2,…n which is odd. Hence degree of region vertex becomes odd. Similarly the degree of pathos vertex becomes odd. Hence $P_{vb}(T)$ is non- Eulerian, a contradiction. Conversely suppose T be a path P_n for n is even. By the definition of $P_{vb}(T)$. The degree of all v_i in $P_{vb}(T)$ becomes even. Since the region vertex is adjacent to all even number of vi such that degree of Region vertex becomes even. Lastly the pathos vertex is adjacent to all even numbers of vertices such that degree of region vertex becomes even. Clearly all vertices is of even degree. Hence $P_{vb}(H)$ is Eulerian.

Theorem 10. For any tree T, the pathos vertex semientire block graph $P_{vb}(T)$ is non-Hamiltonian.

Proof. Assume T be a tree. Without loss of generality consider a path P_n for n=2. Let v_1 and v_2 be the vertices of T. By the definition of vertex edge semientire graph $e_{vb}(T)$, the vertices v_1 and v_2 are adjacent to the block vertex b_1 and the region vertex r_1. Clearly $e_{vb}(P_2) = K_4$-x, for any edge x. In pathos vertex edge semientire graph $P_{vb}(T)$, the pathos vertex P_1 is adjacent to v_1 and v_2 . Clearly v_1 , b_1, v_2, r_1, v_1 form a Hamiltonian cycle in $e_{vb}(T)$. But in $P_{vb}(T)$, the vertex P_1 is not lies on the Hamiltonian cycle. Hence very edge forms this type of graph and it is non- Hamiltonian.

IV. CONCLUSION

The narrative of this paper is to introduce and in fuse its relationship between the line graph and the pathos vertex semientire block graph. Further it embraces an eclectic range of condition for establishing planarity, Hamiltonian and Eulerian depicted through the exemplarary proofs and infused theorems in the above paper, hierarchically.

ACKNOWLEDGEMENT

This journal paper could not have been written without the help of numerous people who have been kind enough to comment on various transformation aspects of the paper.My former guide Mrs Radha rugmini and aide has guided me with affection in the pursuit by which she asked me always to point out the areas that she agrees with my depiction of concepts , but not with my rejection of mathematical faiths. Dr Ramdass, the dean and HOD of our department undoubtedly India's most popular interpreter of mathematical diaspora came up with a number of pertinent suggestions, on an earlier draft which have helped, influenced ,inspired and motivated me immensely in my final research version paper. Mr Paneerselvam sir , the everbrightening star have been a superb trend setter for me and it is his intelligence, vision and humanity that have accompanied me through many of my papers in the process of upbringing my mathematical skills. Mrs Rajakumari.N, the professor an intellectual moonlighting as guide raised a number of questions I have summarised in the note and offered other insights into the faith.While many minds have therefore contributed to the contents of the volume, the final responsibility for the arguments and interpretations in this rests with me .If after reading this paper , mathematicians and non mathematicians come away with a new appreciation of the faith I cherish and the

challenges it currently dealing with in the contemporary planet, why I am a researcher would have served its purpose.

REFERENCES

[1] Haray F., Annals of New York, Academy of Sciences, (1977)175, 198.
[2] Harrary, F.,Graph Theory, Addison- Wesley Reading Mass, (1969)., p.72, 107.
[3] Kulli. V R., On minimally nonouterplanar graphs, proceedings of the Indian National Science Academy, (1975)., 41A.
[4] Kulli. V. R. and Akka. D. G., Journal mathematical Science, Vol. 14, No. 6, (1980)., P. 585- 588.
[5] Sedlacek J., Some properties of interchange graphs. The Graphs and the applications. Academic press, New York (1962).
[6] Venkanagouda. M. Goudar, "On Pathos vertex semientire graph of a tree", international journal of applied mathematical