G-Nαg Closed Sets and G-Ngα Closed Sets in Grill Nano Topological Spaces

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Abstract: *The purpose of this paper is to define and study certain new class of sets namely G-Nαg closed sets and G-Ngα closed sets. Also their respective closures and interiors are derived and some characterizations are given.*

Keywords: G-Nαg closed, G-Ngα closed, G-Nαg closure, G-Nαg interior.

I. INTRODUCTION

The idea of grill on a topological space was first introduced by Choquet [4] in the year 1974. The grill concept depends upon two operators namely Φ and Ψ. We observe that the grill concept is the most powerful tool for supporting many things like nets and filters. A number of theories and feature helped to expand the topological structure. This expansion of topological space is used to measure the technique of describing other than quantity, such as love, beauty and intelligence,etc... Grill concept is also used to expand the topological structure. This would change in lower approximation, upper approximation and boundary region which give new goal in Nano topological spaces. The nano topology concept was given by Lellis Thivagar and Carmel Richard [11]. Many authors [2,10] introduced the concept of Nano generalized alpha closed sets, Nano alpha generlized closed sets, Nano semi generalized closed sets, Nano generalized alpha interior and Nano generalized alpha closure in Nano topological spaces. Azzam A.A [1] introduced a new concept Grill nano generalized closed sets in grill nano topological spaces. Recently in [3] Chitra.V and Jayalakshmi.S introduced G -NSg closed sets and G- NgS closed sets and studied some properties in Grill nano topological spaces. Now our aim in this paper is to introduce the concept grill in nano generalized alpha closed sets and nano alpha generalized closed sets and their respective interiors and closures are also introduced and discuss some of their basic properties in Grill nano topological spaces.

II. PRELIMINARIES

Definition 2.1 [4] A non empty sub collection G of a space X which carries topology τ is named Grill on this space if the following conditions are true:

(1) $\phi \notin G$.

(2) $A \in G$ and $A \subseteq B \subseteq X$ which implies $B \in G$.

(3)If $A \cup B \in G$ for A, $B \subseteq X$ then $A \in G$ or $B \in G$. Since the grill depends on the two mappings Ψ and Φ which generates a unique grill topological space finer than τ on the space X denoted by τ_G on X is discussed in [5,8] .

Definition: 2.2 [11] Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U, R) is said to be the approximation space.

Let $X \subseteq U$. Then

i) The lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by $L_R(X)$. That is, $L_R(X) = \cup_{x \in U}\{R(x): R(x) \subseteq X\}$, where R(x) denotes the equivalence class determined by x.

ii) The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by $U_R(X)$. That is, $U_R(X) = \cup_{x \in U}\{R(x): R(x) \cap X \neq \phi\}$.

iii) The boundary region of X with respect to R is the set of all objects, which can be classified neither as X nor as not X with respect to R and it is denoted by $B_R(X)$. That is, $B_R(X) = U_R(X)-L_R(X)$.

Proposition 2.1[11] If (U, R) is an approximation space and X, Y$\subseteq$ U then

(1)$L_R(X) \subseteq X \subseteq U_R(X)$.

(2)$L_R(\phi) = U_R(\phi) = \phi$.

(3)$L_R(U) = U_R(U) = U$.

(4)$U_R(X \cup Y) = U_R(X) \cup U_R(Y)$.

(5)$U_R(X \cap Y) \subseteq U_R(X) \cap U_R(Y)$.

(6)$L_R(X \cup Y) \subseteq L_R(X) \cup L_R(Y)$.

(7)$L_R(X \cap Y) = L_R(X) \cap L_R(Y)$.

(8)$L_R(X) \subseteq L_R(Y)$ and $U_R(X) \subseteq U_R(Y)$ whenever $X \subseteq Y$.

(9)$U_R(X^C) = [L_R(X)]^C$ and $L_R(X^C) = [U_R(X)]^C$.

(10)$U_R\, U_R(X) = L_R\, UR(X) = U_R(X)$.

(11)$L_R\, LR(X) = U_R\, L_R(X) = L_R(X)$.

Definition 2.3[11] Let U be the universe, R be an equivalence relation on U and $\tau_R(X) = \{U, \phi, L_R(X), U_R(X), B_R(X)\}$ where $X \subseteq U$. Then by above property, $\tau_R(A)$ satisfies the condition of topology on U which is called Nano topology on U with respect to A. (U, $\tau_R(A)$) is called the nano topological space. Elements of Nano topological spaces are called Nano open sets in U.

Definition 2.4. [1] Suppose that $(X, \tau_R(A))$ is a Nano topology on the space X. Also, let G satisfy the condition of the grill on X. A subset B of grill Nano topological space $(X, \tau_R(A), G)$ if G-Ng closed if $\phi(B) \subseteq U$ for $B \subseteq U$ and for all U is Nano open. A subset B of X is said to be G-Ng open is X/B is G-Ng closed.

Definition 2.5.[10] A subset A of (X, $\tau_R(A)$) is called a Nano generalized α-closed set if $N\alpha\ cl\ (A) \subseteq U$ whenever $A \subseteq U$ and U is nano α-open. The complement of nano generalized α- closed set is nano generalized α- open set in (X, $\tau_R(A)$).

Definition 2.6.[10] A subset A of (X, $\tau_R(A)$) is called nano α- generalized closed set if $N\alpha\ cl(A) \subseteq U$ whenever $A \subseteq U$ and U is nano open in (X, $\tau_R(A)$).

Definition 2.7.[11] Let (X, $\tau_R(A)$) be a Nano topological space and $B \subseteq X$. Then B is said to be

(1) Nano semi open if $B \subseteq Ncl(Nint(B))$.

(2) Nano pre open if $B \subseteq Nint(Ncl(B))$.

(3) Nano α-open is $B \subseteq Nint(Ncl(Nint(B)))$.

Definition 2.8[3]. Let (X, $\tau_R(A)$) is nano topology on the space X and a set G satified the conditon of the grill on X. A subset B of a Grill nano topological space is G-NgS closed set if $\Phi_S(B) \subseteq U$ for $B \subseteq U$ and for all U is nano open.

Definition 2.9.[3] Let (X, $\tau_R(A)$,G) denotes Grill nano topological space on X. A subset B of a Grill nano topological space is said to be G-NSg closed set if $\Phi_s(B) \subseteq U$ for $B \subseteq U$ and for all U is nano semi open.

Corollary 2.1 [6] Let $(X, \tau_R(A), G)$ be a grill nano topological space. Then the following are true.

i) If G_1 and G_2 are two grills on X with $G_1 \subseteq G2$ then $\Phi_{G1}(A) \subseteq \Phi_{G2}(B)$ for all $A \subseteq X$.

ii) For any grill G on X and if A is not in G then $\Phi_G(A) = \phi$.

iii) If G is any grill on X, then ΦG is an increasing function in the sense that $A \subseteq B$ gives

$\Phi_G(A) \subseteq \Phi_G(B)$.

III. GRILL NANO GENERALIZED A-CLOSED SETS AND GRILL NANO A-GENERALIZED CLOSED SETS

Definition: 3.1. Suppose that $(X, \tau_R(A))$ is nano topology on the space X. Also a set G satisfies the conditions of the grill on X. A subset B of a grill nano topological space $(X,\tau_R(A),G)$ is Grill nano generalized α-closed set if $\Phi_\alpha(B) \subseteq U$ for $B \subseteq U$ and for all U is nano α-open.

Theorem 3.1. If A and B are G-Ngα closed sets, then $A \cup B$ is G-Ng α closed.

Proof: Let A and B are G-Ng α closed sets. Then $\Phi_\alpha(A) \subseteq U$ whenever $A \subseteq U$ and U is nano α- open and $\Phi_\alpha(B) \subseteq U$ whenever $B \subseteq U$ and U is nano α- open. Since A and B are subset of U, $A \cup B \subseteq U$ and U is nano α-open. Then $\Phi_\alpha(A \cup B) = \Phi_\alpha(A) \cup \Phi_\alpha(B) \subseteq U$ which leads $A \cup B$ is G-Ng α -closed set.

Remark 3.1. The intersection of the two grill nano generalized α-closed sets in $(X, \tau_R(A),G)$ is not grill nano generalized α-closed.

Example 3.1. Let X={a,b,c,d} with X/R={{a},{c},{b,d}} and A={a,b} with G={{a},{a,d},{a,c,d},X} $\tau_R(A)$={ϕ,X,{a,c}}. Then the sets {a,c},{a,b} are G-Ng α-closed but their intersection {a,c}∩ {a,b} ={a} is not G-Ng α -closed.

Theorem 3.2. Let B be a G-Ng α closed set and $B \subseteq C \subseteq \Phi_\alpha(B)$ then C is G-Ng α closed set.

Proof: Let $C \subseteq U$, whenever U is nano α -open in $\tau_R(A)$ with grill G. Then $B \subseteq C$ implies $B \subseteq U$. Since B is G-Ng α closed, $\Phi_\alpha(B) \subseteq U$. Also $C \subseteq \Phi_\alpha(B)$ which implies $\Phi_\alpha(C) \subseteq \Phi_\alpha(B)$. Thus $\Phi_\alpha(C) \subseteq U$ and C is G-Ng α closed.

Theorem 3.3. Every Nano generalized α -closed set is grill nano generalized α -closed.

Proof: Let $(X, \tau_R(A),G)$ be a grill nano topological space,and let B be a nano generalized α-closed set such that $B \subseteq U$ and U is nano α-open in $\tau_R(A)$.Since B is nano generalized α -closed set we have N α cl(B) ⊆ B and $\Phi_\alpha(B) \subseteq B$. Thus $\Phi_\alpha(B) \subseteq B \subseteq U$. Hence B is G-Ng α -closed.

Example 3.2. Let X={a,b,c,d}, A={a,b}, X/R={{b},{d},{a,c}} and $\tau_R(A)$={X, ϕ ,{b},{a,b,c},{a,c}},

G={{a},{b},{a,b},{a,b,d},X}. Then the set {a,c} is G-Ng α -closed but not Ng α -closed.

Theorem 3.4. Every grill nano generalized closed set in $(X, \tau_R(A),G)$ is grill nano generalized α -closed.

Proof: Let $(X, \tau_R(A),G)$ be a grill nano topological space. Let A be a grill nano generalized closed set. Then $\Phi(A) \subseteq V$ whenever $A \subseteq V$, V be a nano open in $\tau_R(A)$. Also since every nano open set is nano α-open and $\Phi_\alpha(A) \subseteq \Phi(A)$, then $\Phi_\alpha(A) \subseteq \Phi(A) \subseteq V$. Hence $\Phi_\alpha(A) \subseteq V$. Therefore A is grill nano generalized α-closed set.

Example 3.3. Let X={a,b,c,d}, A={a,b}, X/R={{b},{d},{a,c}} and $\tau_R(A)$={X,ϕ,{b},{a,b,c},{a,c}}, G={{a},{b},{a,b},{a,b,d},X}.Then the set {c,d} is G-Ngα-closed but not G-Ng closed.

Result 3.1. Let B be a G-Ngα closed set and suppose that F is nano α closed set. Then B∪F is a G-Ngα-closed set which is given in following example

Example 3.4. Let X={a,b,c,d}, A={a,c}, X/R={{a},{b},{c,d}} and $\tau_R(A)$={X,ϕ,{a,c,d},{c,d},{a}}, G={{a,b},{a},{b},{a,c},{c},{a,d},{d},X}.Then B={a,c,d} is a G-Ng α closed set and F={b,c,d} is nano α -closed set. Then B∪ F={a,b,c,d} is G-Ng α -closed.

Remark 3.2. Let B be a G-Ng α closed set and suppose that H is nano- α -closed set. Then B∩ H is a G-Ng α -closed set which is given in following example.

Example 3.5. Let X={a,b,c,d}, A={a,b},X/A={{b},{d},{a,c}} and $\tau_R(A)$={X, ϕ,{b}{a,b,c},{a,c}}, G={{a},{a,b},{a,b,c},X}.Let B={c,d} is a G-Ng α -closed and H={d} is a nano α - closed. Then B∩ H={d} is G-Ng α -closed.

Definition 3.2. Let (X, τ_R(A)) is a topology on the space X. Also a set G satisfies the conditions of the grill on X. A subset B of grill nano topological space. (X, τ_R(A),G) is said to be G-N α closed set if Φ_α (B) ⊆ U whenever B ⊆ U and for all U is nano open.

Theorem 3.5. If A is Grill nano α -generalized closed in (X, τ_R(A),G) then it is grill nano generalized α -closed set.

Proof: Let A be a Grill nano α generalized closed set then Φ_α (A) ⊆ U whenever A ⊆ U and U be a nano open in τR(X). Since every nano open set is nano α - open, U is nano α -open in (τ_R(X)). Therefore A is grill nano generalized α -closed.

Example 3.6. Let X={1,2,3,4}, A={1,2} and X/R={{2},{4},{1,3}} with G={{1,3},{1,2,3},{1,3,4},X}. Then the set {1,3,4} is grill nano generalized α -closed set, but not grill nano α -generalized closed

Theorem 3.6. If A is Grill nano generalized α -closed in (X, τ_R(X), G) then it is grill nano generalized semi closed.

Proof: Let A be a Grill nano generalized α -closed set and A ⊆ C, C is nano open in U. We have Φ_α (A) ⊆ C, C is nano open. Since every nano open set is nano semi open. Hence A is grill nano generalized semi closed.

Example 3.7. Let X={1,2,3,4}, A={1,2} and X/R={{2},{4},{1,3}} with G={{1,3},{1,2,3},{1,3,4},X}. Then the set {1, 2,3} is grill nano semi generalized closed set, but not grill nano generalized α -closed.

Theorem 3.7. The union of two grill nano α - generalized closed sets in (X, τ_R(A),G) is a grill nano α -generalized closed set in (X, τ_R(A),G).

Proof: Assume that C and D are grill nano α -generalized closed set in (X, τ_R(A),G). Let V be a nano open in (X, τ_R(A),G) Such that (C∪ D) ⊆ V then C⊆ V and D⊆ V. Since C and D are grill nano α -generalized closed in (X, τ_R(A),G) Φ_α (C) ⊆V and Φ_α (D) ⊆ V . Hence Φ_α (C∪ D)= Φ_α (C) ∪ Φ_α (D) ⊆ V. Then Φ_α (C∪ D) ⊆ V. Hence C ∪ D is a grill nano generalized closed set.

Remark 3.3. The intersection of two grill nano α -generalized closed set in (X, τ_R(A),G) is not grill nano α -generalized closed set in (X, τ_R(A),G).

Example 3.8. Let X={a,b,c,d} with X/R={{b},{d},{a,c}} and A={a,b}. Then the nano topology τ_R(A)={X, ϕ,{b},{a,b,c},{a,c}}. Then the grill nano α - generalized closed sets are ϕ,{d},{a,d},{b,d},{c,d},{b,c},{a,b,d},{a,c,d},{b,c,d},X. Hence here the intersection of {b,c} and {b,d} is {b} which is not grill nano α -generalized closed set.

Theorem 3.8. Let C be a grill nano α - generalized closed subset of X. If C⊆ D⊆ Φ_α (C). Then D is also a grill nano α -generalized closed subset of τ_R(A).

Proof: Let V be nano open set of τ_R(A) such that D ⊆ V, since C is a grill nano α - generalized closed Φ_α (C) ⊆ V whenever C⊆V. Since C⊆ D Φ_α (C), Φ_α (C) ⊆ Φ_α (D) ⊆ V. Hence D is also a grill nano α - generalized closed subset of τ_R(A).

Theorem 3.9. Every grill nano α - generalized closed set is grill nano generalized semi closed set.

Proof: The proof is obvious from the definition.

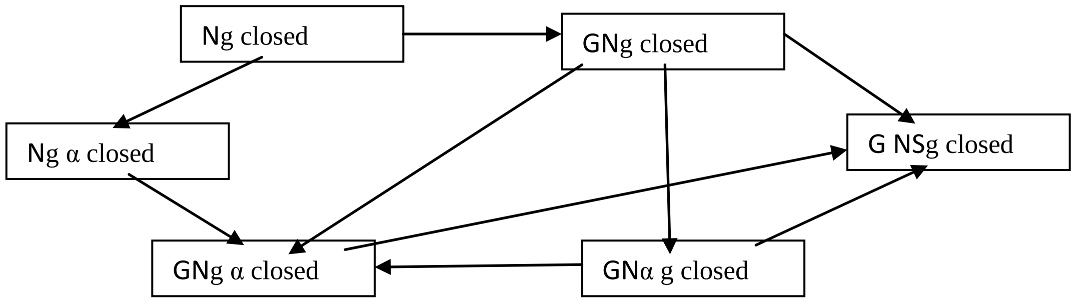
Converse need not be true is shown in the following example.

Example 3.9. Let X={a,b,c,d}, A={a,d} and X/R= {{a},{d},{b,c}} with G={{a},{a,c},X}. Then the set {a,c} is grill nano generalized semi closed set but not grill nano α generalized closed.

Remark 3.4. In a Grill nano topological space, Ng α -closed set and G-Ng closed set are independent.

Example 3.10. Let X={a,b,c,d}, X/R={{b},{a,c}} and A={a,b} with G={{a},{a,b,c},U}. Then {b} is G-Ng closed but not Ng α -closed. And the set {d} is Ng closed but not G-Ng closed.

Remark 3.5. Summing up the above implications and results in [1, 10] we have the following diagram. However the converse implications are not true.



IV. G- Ngα CLOSURE AND G-Ngα INTERIOR:

Definition 4.1. Let $(X,\tau_R(A),G)$ be a grill nano topological space. Let C be a subset of X. Then C is called G-Ngα-nbd of y of X, if there exist a G-Ngα-open set D that contains yε D⊆ C.

Definition 4.2. Let $(X, \tau_R(A),G)$ be a grill nano topological space. A subset C of X is called G-Ngα-interior of C if the union of all G-Ngα open sets contained in C. i.e., G-Ngα-int(C)=∪{D:D⊆ C, D is a G-Ngα-open}.

Definition 4.3. Let B be a subset of X in a grill nano topological space $(X, \tau_R(A),G)$. Then the G-Ngα- closure of C is defined as the intersection of all G-Ngα-closed sets containing C, and is denoted by G-Ngα-cl(C).

Theorem 4.1. Let $(X, \tau_R(A), G)$ be a grill nano topological space. Let C and D are subsets of a grill nano topological space. Then the following properties hold:

i)G-Ngα-int(X)=X and G-Ngα-int(ϕ)=ϕ.

ii) G-N-gα-int(C)⊆ C.

iii) If D is G-Ngα-open set contained in C, then D⊆ G-Ngα-int(C).

iv)If C ⊆ D then G-Ngα-int(C)⊆ G-Ngα-int(D).

Proof: i) Since ϕ and X are G-Ngα-open set and by the definition we have G-Ngα- int(X)=∪{D:D⊆ X, D is G-Ngα- open}=union of all G-Ngα-open sets =X. Also it is clear that G-Ngα-int(ϕ)=ϕ.

ii) Suppose y be a point in G-Ngα-int(C). Then y is a G-Ngα- interior point of C. Since y is G-Ngα-interior point, we have C is a G-Ngα- nbd of y. Thus y∊ C.

iii) Suppose D is G-Ngα-open contained in C. Now we have to prove that D⊆ G-Ngα-int(C). So let y∊ D. By our assumption D is G-Ngα-open that contained in C. Thus y is a G-Ngα- interior point of C. Hence y ∊ G-Ngα-int(C), and hence D⊆ G-Ngα-int(C).

iv) Let C and D be the subsets of a grill nano topological space $(X, \tau_R(A),G)$. Suppose C⊆ D. Let y∊ G-Ngα-int(C). Then y is the G-Ngα-nbd of C. Given C⊆ D. Therefore y is also a G-Ngα-nbd of D. Hence y∊ G-Ngα-int(D).

Theorem 4.2. If a subset C of a grill nano topological space $(X, \tau_R(A),G)$ is G-Ngα- open then G-Ngα-int(C)=C.

Proof: Suppose C is a subset of a grill nano topological space $(X, \tau_R(A), G)$. We know that G-Ngα-int(C) ⊆ C. Also C is G-Ngα-open which is contained in C itself. Therefore by above theorem (iii) we have C ⊆ G-Ngα-int(C). Hence G-Ngα-int(C)=C.

Theorem 4.3. If C and D are subsets of a grill nano topological space $(X, \tau_R(A), G)$, then G-Ngα- int(C) ∪ G-Ngα- int(D)⊆eq G-Ngα-int(C ∪ D).

Proof: We know that C ⊆ C ∪ D and D⊆ C ∪ D. Then G-Ngα-int(C) ⊆ G-Ngα- int(C∪ D) and G-Ngα- int (D) ⊆ G-Ngα-int (C∪ D). Thus G-Ngα-int (C) ∪ G-Ngα- int (D) ⊆eq G-Ngα- int (C∪ D).

Theorem 4.4. If A and C are subsets of $(X, \tau_R(A), G)$, then G-Ngα-int(A ∩ C)= G-Ngα-int(A) ∩ G-Ngα-int(C).

Proof: We know that A ∩ C ⊆A and A ∩ C ⊆ C. Then we have that G-Ngα-int(A∩ C)⊆ G-Ngα- int(A) and G-Ngα- int(A ∩ C)⊆ G-Ngα- int(C). Thus G-Ngα- int (A ∩ C)⊆ G-Ngα- int(A) ∩ G-Ngα-int (C). Suppose let y∊ G-Ngα-int (A) ∩ G-Ngα-int (C).Then y ∊ G-Ngα-int (A) and y ∊ G-Ngα-int (C). Thus y is a G-Ngα-interior point of each of set A and C. This implies A and C are G-Ngα-neighborhoods of y. Then their intersection A ∩ C is also a G-Ngα neighbourhood of y. Hence y ∊ G-Ngα-int (A∩ C). Thus G-Ngα- int (A) ∩ G-Ngα- int (C) ⊆ G-Ngα (A ∩ C). Thus G-Ngα- int (A∩ C) = G-Ngα-int(A) ∩ G-Ngα-int(C).

Theorem 4.5. Let A and B are subsets of grill nano topological space (X, τ_R(A), G). Then the properties hold.

i) G-Ngα-cl(X)=X and G-Ngα-cl(ϕ)=ϕ.

ii) B ⊆ G-Ngα-cl(B).

iii) If B is the GNgα- closed set containing A, then GNgα-cl(A)⊆ B.

Proof: i) From the definition of G-Ngα-closure, we say X is the only G-Ngα-closed set that contains X. Also ϕ is the only G-Ngα-closed set that contained in ϕ. Thus G-Ngα-cl(X)=X and G-Ngα-cl(ϕ)=ϕ.

ii) Let y ∊ B and B is the G-Ngα- nbd of y. Then the by definition B is a G-Ngα- interior point of y. Thus y ∊ Ngα-int(B) implies B⊆ Ngα-int(B).

iii) Let B be the G-Ngα-closed set that contains A. Then by definition of G-Ngα-cl(A) which is contained in every G-Ngα-closed set that contain A. Hence G-Ngα-cl(A)⊆ B.

Theorem 4.6. Let A and C be subsets of X in a grill nano topological space, (X, τ_R(A), G), then G-Ngα-cl(A∪ C)=G-Ngα- cl(A)∪ G-Ngα-cl(C).

Proof: Let A and C be subset of X. Then A ⊆ A∪ C and C ⊆ A ∪ C. We have G-Ngα- cl(A) ∪ G-Ngα- cl(C)⊆ G-Ngα-cl(A ∪ C). To prove: G-Ngα-cl(A∪ C)⊆ G-Ngα-cl(A)∪ G-Ngα-cl(C). Let y ∊ G-Ngα- cl (A ∪ C). Suppose y ∉ G-Ngα-cl(A)∪ G-Ngα-cl(C). Then there exist G-Ngα closed sets A_1 and C_1 with A A_1,C ⊆ C_1 and y ∉ A_1 ∪ C_1. Then we have A ∪ C ⊆ A_1 ∪ C_1 and A_1 ∪ C_1 is G-Ngα-closed such that y ∉ A_1 ∪ C_1. Thus y∉ G-Ngα-cl(A ∪ C), a contradiction. Hence y ∊ G-Ngα-cl(A ∪ C).Thus G-Ngα-cl(A) ∪ C ⊆ G-Ngα-cl(A) ∪ G-Ngα-cl(C). Therefore we have G-Ngα- cl(A ∪ C)=G-Ngα- cl(A)∪ G-Ngα- cl(C).

Theorem 4.7. Let A and B be subsets of X in a grill nano topological space (X, τ_R(A), G).Then G- Ngα-cl(A∩ B)=G-Ngα- cl(A)∩ G-Ngα-cl(B).

Proof: Let A and B be subsets of X. Then A ⊆ A ∩ B and B ⊆ A ∩ B. We have G-Ngα-cl(A) ∩ G-Ngα-cl(B) ⊆ GNgα-cl(A ∩ B). To prove :G-Ngα-cl (A∩ B)=G-Ngα-cl(A)∩ G-Ngα-cl(B). Let y ∊ G-Ngα-cl(A ∩ B). Suppose y ∉ G-Ngα-cl(A)∩ G-Ngα-cl(B). Then there exist G-Ngα- closed sets A_1 and B_1 with A ⊆A_1, B⊆ B_1, and y∉ A_1∩ B_1. Then we have A ∩ B ⊆ A_1 ∩ B_1 and A_1 ∩ B_1 is G-Ngα-closed such that y∉ A_1∩ B_1. Thus y∉ G-Ngα cl(A ∩ B), which is a contradiction. Hence y∊ G-Ngα- cl(A ∩ B). Thus G-Ngα-cl(A ∩ B)⊆ G-Ngα-cl(A)∩ G-Ngα-cl(B). Therefore GNgα-cl(A ∩ B)=GNgα-cl(A)∩ GNgα-cl(B).

Theorem 4.8. In a grill nano topological space (X, τ_R(A), G) , for any y∊ X, y ∊ G-Ngα-cl(C) if and only if W∩ C≠ϕ for each G-Ngα-closed set W contains y.

Proof : i) Suppose y ∊ X, y ∊ G-Ngα-cl(C).We have to prove W∩ C≠ϕ for every G-Ngα-closed set W containing y.We prove this by method of contradiction. Assume that there is a G-Ngα-open set, W that contains y such that W∩ C=ϕ.Now C⊆ X-W is G-Ngα-closed. Thus we have G-Ngα-cl(C)⊆ X-W implies y ∉ G-Ngα-cl(C) a contradiction. Hence W∩ C \not=ϕ for each G-Ngα- closed set W containing y.

Conversely: Suppose that W∩ C≠ϕ for each G-Ngα- closed set W containing y.We have to prove y∊ G-Ngα-cl(C). We prove this also by a contradiction method. Suppose y∊ G-Ngα-cl(C) .Then y ∊ X-D, X-D is G-Ngα-open Now (X-D) ∩ C=ϕ a contradiction.

Definition 4.4. Let C be a subset of X in a grill nano topological space (X,τ_R(A),G). Then G-Nα g-closure of C is defined as the intersection of all grill nano α- generalized closed sets containing C and it is denoted by G-Nα g-cl(C).

Definition 4.5. Let A be a subset of X in a grill nano topological space $(X,\tau_R(A),G)$ then the G-N α g- interior of A is defined as the union of all grill nano α- generalized open sets contained in A and is denoted by G-N α g-int(A).

Theorem 4.9. Suppose A is a subset of X in a grill nano topological space $(X,\tau_R(A),G)$, then the following conditions hold :

i) G-Nα g-int(X-A)=X-(G- N α g-cl(A)).

ii) G-Nα g-cl(X-A)=X-(G- Nα g-int (A)) .

Proof: i) G-Nα g-cl(A)= ∩{C:A⊆ C, C is G-Nα g-closed}

X-G- Nα gcl(A) = X-∩{C:A⊆C, C is G-Nα gclosed}

=∪ {X-C: A⊆ C, C is a G-Nα g -closed}

=∪ {U: U⊆ X-A is GNα g -open}

=G-Nα g-int(X-A)

ii) Proof similar.

V. CONCLUSION

We used the concept of grill to expand a topological space which helps to measure the things which are difficult to measure such as beauty, intelligence and goodness. In this paper the concept of G-Ngα- closed and G-Nα g closed are defined and properties are discussed.

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