

Level Set of Measurable Conjugate Flexible Fuzzy Soft Bi-Ideals Over Near-Ring Structures

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Abstract - In this paper, we introduce the notion of conjugate flexible fuzzy soft bi-ideals in near-ring structures, level sets and given some characterizations of flexible fuzzy soft ideals in near-ring approximations. Also, we investigate the structure of normal flexible fuzzy soft subgroup.

Keywords - soft set, level set, fuzzy set, flexible fuzzy soft set, conjugate, epimorphism, normal, complete, soft bi-ideal.

I. INTRODUCTION

Soft set theory was introduced by Molodtsov [16] for modeling vagueness and uncertainty and it has been received much attention in the field of set theory. Maji et al [13, 14] explains the applications of soft sets in decision making problems. Ali et al [4] defined some new operations in soft set theory and Sezgin and Atagun [20] introduced and studied operations of soft sets. Soft set theory has also potential applications especially in decision making as in [20]. This theory has started to progress in the mean of algebraic structures, since Aktas and Cagman [3] defined and studied soft groups. Since then, soft substructures of rings, fields and modules, union soft substructures of near-rings and near-ring modules [21], normalistic soft groups [15] are defined and studied in detail. In 1999, Molodtsov's [16] proposed an approach for Modeling, Vagueness and uncertainty, called soft set theory, since its inception, works on soft set theory have been progressing rapidly with a wide range applications especially in the mean of algebraic structures as in [3].

Zadeh's classical paper [27] of 1965 introduced the concepts of fuzzy sets and fuzzy set operations. Foster [7] combined the structure of a fuzzy topological space. The study of the fuzzy algebraic structures started with the introduction of concepts of fuzzy subgroups and fuzzy (left, right) ideals in the pioneering paper of Rosenfeld [17]. Anthony and Sherwood [1] redefined fuzzy subgroups using the concept of triangular norm. Several mathematicians [6,8,10] followed the Rosenfeld approach in investigating fuzzy algebra where given ordinary algebraic structure on a given set X is assumed then introducing the fuzzy algebraic structure as a fuzzy subset A of X satisfying some suitable conditions. Throughout this paper, G will denote a group and " e " will denote its identity element. Let \sup , \inf , \min , \max will denote the supremum, infimum, minimum, maximum respectively. Flexible fuzzy soft subgroups in various algebraic structures defined in [23-26]. Many researchers Abd-Allah et al [2], Chengyi [5], Dib and Hassan [6], Tang and Zhang [22], Syransu and Ruy [19], Massa'deh [12] studied the properties of groups and subgroups by the definition of fuzzy subgroups. In this paper, we introduce the notion of flexible fuzzy soft ideals in near-ring structures, level sets and given some characterizations of flexible fuzzy soft ideals in near-ring approximations.

II. PRELIMINARIES

In this section, we recall some basic notions relevant to group theory.

2.1 Definition [23]: Let X be a set. Then a mapping $\mu: X \rightarrow [0, 1]$ is called a fuzzy subset of X .

2.2 Definition [16]: Let U be an initial universe. Let $P(U)$ be the power set of U , E be the set of all parameters and $A \subseteq E$. A soft set (f_A, E) on the universe U is defined by the set of order pairs $(f_A, E) = \{(e, (e)): e \in E, f_A \in P(U)\}$ where $f_A: E \rightarrow P(U)$ such that $f_A(e) = \phi$ if $e \notin A$. Here f_A is called an approximate function of the soft set.

2.3 Definition: [23] Let X be a set. Then a mapping $\mu: X \rightarrow M^*([0, 1])$ is called flexible fuzzy subset of X , where $M^*([0, 1])$ denotes the set of all non empty subset $[0, 1]$.

2.4 Definition: Let N be a near-ring. A flexible fuzzy soft set A of N is called a (t_1, t_2) -flexible fuzzy soft subnear-ring of N if for all $x, y \in N$,

- (i) $\sup\{A(x-y, t_1)\} \geq \inf\{A(x), A(y), t_2\}$
- (ii) $\sup\{A(xy, t_1)\} \leq \inf\{A(x), A(y), t_2\}$.

2.5 Definition: Let N be a near-ring. A flexible fuzzy soft set A of N is called a (t_1, t_2) -flexible fuzzy soft bi-ideal of N if for all $x, y, z \in N$,

- (i) $\sup\{A(x-y, t_1)\} \geq \inf\{A(x), A(y), t_2\}$
- (ii) $\sup\{A(xyz, t_1)\} \geq \inf\{A(x), A(z), t_2\}$.

2.6 Definition: A family of flexible fuzzy soft set $\{A_i / i \in \Gamma\}$ is a near-ring N , the union $\bigvee A_i$ of $\{A_i / i \in \Gamma\}$ is defined by $(\bigvee A_i)(x) = \max\{A_i(x) / i \in \Gamma\}$ for each $x \in N$.

2.7 Definition: A family of flexible fuzzy soft set $\{A_i / i \in \Gamma\}$ is a near-ring N , the intersection $\bigcap A_i$ of $\{A_i / i \in \Gamma\}$ is defined by $(\bigcap A_i)(x) = \min\{A_i(x) / i \in \Gamma\}$ for each $x \in N$.

2.8 Definition: Let N and N' be two near-rings and f be a function of N into N' .

- (i) If λ is a flexible fuzzy soft set in N' , then the pre-image of λ under f is the flexible fuzzy soft set N defined by $f^{-1}(\lambda)(x) = \lambda(f(x))$ for each $x \in N$.
- (ii) If A is a flexible fuzzy soft set of N , then the image of A under f is the flexible fuzzy soft set in N' defined by

$$f(A)(y) = \begin{cases} \sup\{A(x), t_1\}, & \text{if } f^{-1}(y) \neq \Phi \text{ where } x \in f^{-1}(y) \\ \text{otherwise} \end{cases}$$

$$f^{-1}(A)(y) = \begin{cases} \sup\{A(x), t_1\}, & \text{if } f^{-1}(y) \neq \Phi \text{ where } x \in f^{-1}(y) \\ \text{otherwise} \end{cases}$$

A flexible fuzzy soft bi-ideal A of a near-ring N is said to be normal if $A(0) = 1$.

2.10 Definition: A flexible fuzzy soft bi-ideal A of a near-ring N is said to be complete if it is normal and there exists $z \in N$ such that $A(z) = 0$.

III FLEXIBLE FUZZY SOFT BI-IDEALS IN NEAR-RING

3.1 Definition: Let N be a near-ring. A flexible fuzzy soft set A of N is called a (t_1, t_2) -flexible conjugate fuzzy soft ideal of N if for all $x, y, z \in N$,

- (i) $\sup\{A(x-y, t_1)\} \leq \inf\{A(x), A(y), t_2\}$
- (ii) $\sup\{A(xyz, t_1)\} \leq \inf\{A(x), A(z), t_2\}$.

3.2 Example: Let $N = \{0, a, b, c\}$ be the Klein's 4 group. Define addition and multiplication in N as follows

+	0	A	B	c
0	0	A	B	c
a	a	0	C	b
b	b	C	0	a
c	c	B	A	0

•	0	A	B	c
0	0	0	0	0
a	0	B	0	b
b	0	0	0	0
c	0	B	0	b

Then $(N, +, \cdot)$ is a near-ring. Define a flexible fuzzy soft set $A : N \rightarrow P^*[0,1]$ by $A(0) = 0.7, A(a) = 0.3, A(b) = 0.6, A(c) = 0.8$. It is easy to verify that A is (t_1, t_2) -conjugate flexible fuzzy soft ideal of N . But A is not a flexible fuzzy soft ideal of N since $A(0) = A(b-b) \not\geq \inf\{A(b), A(b)\}$.

3.3 Theorem: Let $\Phi : N \rightarrow N'$ be an onto homomorphism of near-rings.

- (i) If λ is a (t_1, t_2) -flexible fuzzy soft bi-ideal in N' , then $\Phi^{-1}(\lambda)$ is a (t_1, t_2) -flexible fuzzy soft bi-ideal in N .
- (ii) If δ is a (t_1, t_2) -flexible fuzzy soft bi-ideal in N , then $\Phi(\delta)$ is a (t_1, t_2) -flexible fuzzy soft bi-ideal in N' .

Proof: (i) Let λ be a (t_1, t_2) -flexible fuzzy soft ideal of N' .

For any $x, y \in N$,

$$\begin{aligned} \sup\{f^{-1}(\lambda)(x-y), t_1\} &= \sup\{\lambda(f(x-y)), t_1\} \\ &= \sup\{\lambda(f(x)-f(y)), t_1\} \\ &\geq \inf\{\lambda(f(x)), \lambda(f(y)), t_2\} \end{aligned}$$

$= \inf\{f^1(\lambda)(x), f^1(\lambda)(y), t_2\}$
 Therefore, $\sup\{f^1(\lambda)(x-y), t_1\} \geq \inf\{f^1(\lambda)(x), f^1(\lambda)(y), t_2\}$ and
 $\sup\{f^1(\lambda)(xyz), t_1\} = \sup\{\lambda(f(xyz)), t_1\}$
 $= \sup\{\lambda(f(x)f(y)f(z)), t_1\}$
 $\geq \inf\{\lambda(f(x)), \lambda(f(z)), t_2\}$
 $= \inf\{f^1(\lambda)(x), f^1(\lambda)(z), t_2\}$
 Thus, $\sup\{f^1(\lambda)(xyz), t_1\} \geq \inf\{f^1(\lambda)(x), f^1(\lambda)(z), t_2\}$. Hence $f^1(\lambda)$ is a flexible fuzzy soft bi-ideal in N.

(ii) Let δ be (t_1, t_2) -flexible fuzzy soft ideal in N.

Let $y_1, y_2, y_3 \in N'$. Then we have $\{x/x \in f^1(y_1-y_2)\}$, $\{x_1-x_2/x_1 \in f^1(y_1) \text{ and } x_2 \in f^1(y_2)\}$ and hence
 $\sup\{f(\delta)(y_1-y_2), t_1\} = \sup\{\delta(x)/x \in f^1(y_1-y_2), t_1\}$
 $\geq \sup\{\inf\{\delta(x_1-x_2)/x_1 \in f^1(y_1) \text{ and } x_2 \in f^1(y_2), t_2\}\}$
 $\geq \sup\{\inf\{\delta(x_1), \delta(x_2), t_2\}/x_1 \in f^1(y_1) \text{ and } x_2 \in f^1(y_2)\}$
 $= \inf\{\sup\{\delta(x_1)/x_1 \in f^1(y_1), t_2\} \text{ and } \sup\{\delta(x_2)/x_2 \in f^1(y_2), t_2\}\}$
 $= \inf\{f(\delta)(y_1), f(\delta)(y_2), t_2\}$
 Thus, $\sup\{f(\delta)(y_1-y_2), t_1\} \geq \inf\{f(\delta)(y_1), f(\delta)(y_2), t_2\}$.

Let $y_1, y_2, y_3 \in N'$. Then we have,
 $\sup\{f(\delta)(y_1y_2y_3), t_1\} = \sup\{\delta(x)/x \in f^1(y_1y_2y_3), t_1\}$
 $\geq \sup\{\inf\{\delta(x_1x_2x_3), t_2\}/x_1 \in f^1(y_1) \text{ and } x_2 \in f^1(y_2), x_3 \in f^1(y_3)\}$
 $\geq \sup\{\inf\{\delta(x_1), \delta(x_3), t_2\}/x_1 \in f^1(y_1) \text{ and } x_3 \in f^1(y_3)\}$
 $= \inf\{\sup\{\delta(x_1)/x_1 \in f^1(y_1), t_2\} \text{ and } \sup\{\delta(x_3)/x_3 \in f^1(y_3), t_2\}\}$
 $= \inf\{f(\delta)(y_1), f(\delta)(y_3), t_2\}$

Thus, $\sup\{f(\delta)(y_1y_2y_3), t_1\} \geq \inf\{f(\delta)(y_1), f(\delta)(y_3), t_2\}$. Hence $f(\delta)$ is a (t_1, t_2) -flexible fuzzy soft bi-ideal of N' .

3.4 Theorem: Let N be a near-ring and A be a flexible fuzzy soft set in N. Then A is (t_1, t_2) -conjugate flexible fuzzy soft bi-ideal in N if and only if A^C is a (t_1, t_2) -flexible fuzzy soft ideal in N.

Proof: Let N be near-ring and A be a (t_1, t_2) -conjugate flexible fuzzy soft bi-ideal in N.

For $x, y \in N$,
 $\sup\{A^C(x-y), t_1\} = \sup\{1 - A(x-y), t_1\}$
 $\geq \inf\{1 - \sup\{A(x), A(y)\}, t_2\}$
 $= \inf\{1 - A(x), 1 - A(y), t_2\}$
 $= \inf\{A^C(x), A^C(y), t_2\}$
 Therefore, $\sup\{A^C(x-y), t_1\} \geq \inf\{A^C(x), A^C(y), t_2\}$.

For any $x, y, z \in N$,
 $\sup\{A^C(xyz), t_1\} = \sup\{1 - A(xyz), t_2\}$
 $\geq \inf\{1 - \sup\{A(x), A(z)\}, t_2\}$
 $= \inf\{1 - A(x), 1 - A(z), t_2\} = \inf\{A^C(x), A^C(z), t_2\}$
 Therefore, $\sup\{A^C(xyz), t_1\} \geq \inf\{A^C(x), A^C(z), t_2\}$. Hence A^C is (t_1, t_2) -bi-ideal in N.

Conversely, Suppose that A^C is a (t_1, t_2) -bi-ideal in N.

For any $x, y \in N$,
 $\sup\{A(x-y), t_1\} = \sup\{1 - A^C(x-y), t_1\}$
 $\leq 1 - \sup\{1 - A^C(x), 1 - A^C(y), t_2\}$
 $= \inf\{A(x), A(y), t_2\}$
 Therefore, $\sup\{A(x-y), t_1\} \leq \inf\{A(x), A(y), t_2\}$.

For any $x, y, z \in N$,
 $\sup\{A(xyz), t_1\} = \sup\{1 - A^C(xyz), t_1\}$
 $\leq 1 - \sup\{1 - A^C(x), 1 - A^C(z), t_2\}$
 $= \inf\{A(x), A(z), t_2\}$
 Therefore, $\sup\{A(xyz), t_1\} \leq \inf\{A(x), A(z), t_2\}$. Hence A is (t_1, t_2) -conjugate flexible fuzzy soft bi-ideal in N.

3.5 Proposition: Let A be a flexible fuzzy soft set in a near-ring N. Then A is an (t_1, t_2) -conjugate flexible fuzzy soft bi-ideal of N if and only if the lower level cut $L(A; \alpha)$ of N is a bi-ideal of N for each $\alpha \in [A(0), 1]$.

Proof: Let A be (t_1, t_2) -conjugate flexible fuzzy soft bi-ideal of N. Let $x, y \in L(A; \alpha)$. Then $\sup\{A(x), t_1\} \leq \alpha$ and $\sup\{A(y), t_1\} \leq \alpha$.

Now $\sup\{A(x-y), t_1\} \leq \inf\{A(x), A(y), t_2\} = \alpha$ which implies that $A(x-y) \leq \alpha$ and so $x-y \in L(A; \alpha)$. Hence $L(A; \alpha)$ is a subgroup of N. Let $x, z \in L(A; \alpha)$ and $y \in N$. Then $A(x) \leq \alpha$ and $A(z) \leq \alpha$. Now $\sup\{A(xyz), t_1\} \leq \inf\{A(x), A(z), t_2\} \leq \alpha$ which implies that $A(xyz) \leq \alpha$ and hence $xyz \in L(A; \alpha)$. Hence, $L(A; \alpha)$ is a bi-ideal of N.

Conversly, suppose that $L(A ; \alpha)$ is a bi-ideal of N . Suppose that $x,y \in N$ and $\sup\{A(x-y), t_1\} \geq \inf\{A(x), A(y), t_2\}$. Choose α such that $\sup\{A(x-y), t_1\} \geq \alpha \geq \inf\{A(x), A(y), t_2\}$. Then, we get $x,y \in L(A ; \alpha)$, a contradiction. Hence $\sup\{A(x-y), t_1\} \leq \inf\{A(x), A(y), t_2\}$. Similarly, we can prove that $\sup\{A(xyz), t_1\} \leq \inf\{A(x), A(z), t_2\}$. Hence, A is a (t_1, t_2) -conjugate flexible fuzzy soft bi-ideal of N .

3.6Theorem: Let $\Phi : N \rightarrow N'$ be an onto homomorphism of near-rings. Then we have that

- (i) If λ is a (t_1, t_2) - conjugate flexible fuzzy soft bi-ideal in N' , then $\Phi^{-1}(\lambda)$ is a (t_1, t_2) - conjugate flexible fuzzy soft bi-ideal in N .
- (ii) If δ is a (t_1, t_2) - conjugate flexible fuzzy soft bi-ideal in N , then $\Phi(\delta)$ is a (t_1, t_2) - conjugate flexible fuzzy soft bi-ideal in N' .

Proof: (i) Let λ be a (t_1, t_2) -conjugate flexible fuzzy soft ideal of N' .

For any $x,y \in N$,

$$\begin{aligned} \sup\{f^{-1}(\lambda)(x-y), t_1\} &= \sup\{\lambda(f(x-y), t_1)\} \\ &= \sup\{\lambda(f(x) - f(y), t_1)\} \\ &\leq \inf\{\lambda(f(x)), \lambda(f(y)), t_2\} \\ &= \inf\{f^{-1}(\lambda)(x), f^{-1}(\lambda)(y), t_2\} \\ \text{Therefore, } \sup\{f^{-1}(\lambda)(x-y), t_1\} &\leq \inf\{f^{-1}(\lambda)(x), f^{-1}(\lambda)(y), t_2\} \text{ and} \\ \sup\{f^{-1}(\lambda)(xyz), t_1\} &= \sup\{\lambda(f(xyz), t_1)\} \\ &= \sup\{\lambda(f(x) f(y) f(z), t_2)\} \\ &\leq \inf\{\lambda f(x), \lambda f(z), t_2\} \\ &= \inf\{f^{-1}(\lambda)(x), f^{-1}(\lambda)(z), t_2\} \end{aligned}$$

Thus, $\sup\{f^{-1}(\lambda)(xyz), t_1\} \leq \inf\{f^{-1}(\lambda)(x), f^{-1}(\lambda)(z), t_2\}$. Hence $f^{-1}(\lambda)$ is a (t_1, t_2) -conjugate flexible fuzzy soft bi-ideal in N .

(ii) Let δ be a (t_1, t_2) -conjugate flexible fuzzy soft bi-ideal in N .

Let $y_1, y_2, y_3 \in N'$. Then we have $\{x/x \in f^{-1}(y_1-y_2)\}, \{x_1-x_2/x_1 \in f^{-1}(y_1) \text{ and } x_2 \in f^{-1}(y_2)\}$ and hence

$$\begin{aligned} \sup\{f(\delta)(y_1-y_2), t_1\} &= \inf\{\delta(x)/x \in f^{-1}(y_1-y_2), t_1\} \\ &\leq \inf\{\sup\{\delta(x_1-x_2)/x_1 \in f^{-1}(y_1) \text{ and } x_2 \in f^{-1}(y_2), t_2\} \\ &\leq \inf\{\sup\{\delta(x_1), \delta(x_2)\}/x_1 \in f^{-1}(y_1) \text{ and } x_2 \in f^{-1}(y_2), t_2\} \\ &= \sup\{\inf\{\delta(x_1)/x_1 \in f^{-1}(y_1), t_2\} \text{ and } \inf\{\delta(x_2)/x_2 \in f^{-1}(y_2), t_2\}\} \\ &= \inf\{f(\delta)(y_1), f(\delta)(y_2), t_2\} \\ \text{Thus, } \sup\{f(\delta)(y_1-y_2), t_1\} &\leq \inf\{f(\delta)(y_1), f(\delta)(y_2), t_2\}. \end{aligned}$$

Let $y_1, y_2, y_3 \in N'$. Then we have,

$$\begin{aligned} \sup\{f(\delta)(y_1 y_2 y_3), t_1\} &= \inf\{\delta(x)/x \in f^{-1}(y_1 y_2 y_3), t_1\} \\ &\leq \inf\{\sup\{\delta(x_1 x_2 x_3)/x_1 \in f^{-1}(y_1) \text{ and } x_2 \in f^{-1}(y_2), x_3 \in f^{-1}(y_3), t_2\}\} \\ &\leq \inf\{\sup\{\delta(x_1), \delta(x_3)\}/x_1 \in f^{-1}(y_1) \text{ and } x_3 \in f^{-1}(y_3), t_2\} \\ &= \sup\{\inf\{\delta(x_1)/x_1 \in f^{-1}(y_1), t_2\} \text{ and } \inf\{\delta(x_3)/x_3 \in f^{-1}(y_3), t_2\}\} \\ &= \inf\{f(\delta)(y_1), f(\delta)(y_3), t_2\} \end{aligned}$$

Thus, $\sup\{f(\delta)(y_1 y_2 y_3), t_1\} \leq \inf\{f(\delta)(y_1), f(\delta)(y_3), t_2\}$. Hence $f(\delta)$ is (t_1, t_2) -conjugate fuzzy soft bi-ideal of N' .

3.7Proposition: If $\{A_i / i \in \Lambda\}$ is a family of (t_1, t_2) -conjugate flexible fuzzy soft bi-ideals of a near-ring N , then $\cup A_i$ is (t_1, t_2) -conjugate flexible fuzzy soft bi-ideal.

3.8Proposition: If $\{A_i / i \in \Lambda\}$ is a family of (t_1, t_2) -conjugate flexible fuzzy soft bi-ideals of a near-ring N , then $\cap A_i$ is (t_1, t_2) -conjugate flexible fuzzy soft bi-ideal.

3.9Theorem: Let A be (t_1, t_2) -conjugate flexible fuzzy soft bi-ideal of a near-ring N and N^* be a flexible fuzzy soft set in N defined by $A^*(x) = A(x) + 1 - A(0)$ for all $x \in N$. Then A^* is a normal (t_1, t_2) -conjugate flexible fuzzy soft bi-ideal of N containing A .

Proof: Let A be (t_1, t_2) -conjugate flexible fuzzy soft bi-ideal of a near-ring N .

For any $x,y \in N$,

$$\begin{aligned} \sup\{A^*(x-y), t_1\} &= \sup\{A(x-y) + 1 - A(0), t_1\} \\ &\leq \inf\{\{A(x), A(y)\} + 1 - A(0), t_2\} \\ &= \inf\{A(x) + 1 - A(0), A(y) + 1 - A(0), t_2\} \\ &= \inf\{A^*(x), A^*(y), t_2\} \end{aligned}$$

Therefore $\sup\{A^*(x-y), t_1\} \leq \inf\{A^*(x), A^*(y), t_2\}$.

For any $x,y,z \in N$,

$$\begin{aligned} \sup\{A^*(xyz), t_1\} &= \sup\{A(xyz) + 1 - A(0), t_1\} \\ &\leq \inf\{\{A(x), A(z)\} + 1 - A(0), t_2\} \\ &= \inf\{A(x) + 1 - A(0), A(z) + 1 - A(0), t_2\} \\ &= \inf\{A^*(x), A^*(z), t_2\} \end{aligned}$$

Therefore $\sup\{A^*(xyz), t_1\} \leq \inf\{A^*(x), A^*(z), t_2\}$. Clearly $A^*(0) = A(0) + 1 - A(0) = 1$ and A^* is normal. Hence A^* is a normal (t_1, t_2) -conjugate flexible fuzzy soft bi-ideal of N and obviously A is contained in A^* .

3.10 Theorem: If A is (t_1, t_2) -conjugate flexible fuzzy soft bi-ideal of a near-ring N , then $(A^*)^* = A^*$.

Proof: For any $x \in N$, we have

$$\begin{aligned} \sup\{(A^*)^*, t_1\} &= \sup\{A^*(x) + 1 - A^*(0), t_2\} \\ &= \sup\{[A(x) + 1 - A(0)] + [A(0) + 1 - A(0)], t_2\} \\ &= A(x) - A(0) + 1 - A(0) + A(0) \\ &= A(x) + 1 - A(0) = A^*(x). \end{aligned}$$

Therefore $(A^*)^* = A^*$.

3.11 Theorem: If A is normal (t_1, t_2) -conjugate flexible fuzzy soft ideal of a near-ring N if and only if $A^* = A$.

Proof: The sufficient part is obvious. To prove the necessary part, let us suppose that A is normal (t_1, t_2) -conjugate flexible fuzzy soft bi-ideal of a near-ring N . Let $x \in N$. Since A is normal, $A^*(x) = A(x) + 1 - A(0) = A^*(x) + 1 - A(0) + A(0)$. Hence $A^* = A$.

3.12 Theorem: Let A be (t_1, t_2) -conjugate flexible fuzzy soft bi-ideal of a near-ring N , and α be a fixed element of N such that $A(0) \neq A(\alpha)$. Define a flexible fuzzy soft set A^* in N by

$$A^*(x) = \begin{cases} A(x) - A(\alpha) & \text{if } x \in N. \end{cases}$$

Then A^* is a normal (t_1, t_2) -conjugate flexible fuzzy soft bi-ideal of near-ring N .

For any $x, y \in N$,

$$\begin{aligned} \sup\{A^*(x-y), t_1\} &= \frac{A(x) - A(\alpha) - (A(y) - A(\alpha))}{A(0) - A(\alpha)} \\ &\leq \frac{A(x) - A(\alpha)}{A(0) - A(\alpha)} \\ &= \inf\{A^*(x), t_2\} \\ \text{Therefore } \sup\{A^*(x-y), t_1\} &\leq \inf\{A^*(x), A^*(y), t_2\}. \end{aligned}$$

For any $x, y, z \in N$,

$$\begin{aligned} \sup\{A^*(xyz), t_1\} &= \frac{A(x) - A(\alpha) - (A(y) - A(\alpha)) - (A(z) - A(\alpha))}{A(0) - A(\alpha)} \\ &\leq \frac{A(x) - A(\alpha)}{A(0) - A(\alpha)} \\ &= \inf\{A^*(x), t_2\} \end{aligned}$$

Therefore $\sup\{A^*(xyz), t_1\} \leq \inf\{A^*(x), A^*(z), t_2\}$.

Also $A^*(0) = A(0) - A(\alpha) + A(\alpha) = 1$, A^* is normal.

IV. CONCLUSION

We introduce the notion of conjugate flexible fuzzy soft bi-ideals in near-ring structures, level sets and given some characterizations of flexible fuzzy soft ideals in near-ring approximations. Also, we investigate very interesting structures of normal flexible fuzzy soft subgroup.

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