Skolem Difference Lucas Mean Labeling for Star Related Graphs

A.Ponmoni[1] S. Navaneetha Krishnan[2] and A. Nagarajan[3]
[1]Department of Mathematics, C.S.I.College of Engineering, Ketti – 643215
Tamilnadu, INDIA.
[2,3]Department of Mathematics, V.O.C. College, Tuticorin-628008.
Tamilnadu, INDIA.

Abstract

A graph G with p vertices and q edges is said to have Skolem difference Lucas mean labeling if it is possible to label the vertices $x \in v$ with distinct elements $f(x)$ from the set $\{1,2,\ldots,L_{p+q}\}$ in such a way that the edge $e = uv$is labelled with $\left|\frac{f(u)-f(v)}{2}\right|$ if $|f(u)-f(v)|$ is even and $\frac{|f(u)-f(v|+1}{2}$ if $|f(u)-f(v)|$ is odd, then the resulting edge labels are distinct and are from $\{L_1, L_2, \ldots, L_q\}$.A graph that admits Skolem difference Lucas mean labelling is called a Skolem difference Lucas mean graph. In this paper, we proved for some star related graphs such as $K_{1,n}^{+}$, $K_{1,m} \oslash K_{1,n}$, $K_{1,m} \odot 2P_n$, $B_{m,n}$, $< K_{1,n}^{(1)}, K_{1,n}^{(2)}, K_{1,n}^{(3)}, \ldots, K_{1,n}^{(m)} >$, are Skolem difference Lucas mean graph.

AMS subject classification: 05C78.

Key words: *Skolem Mean Labeling, Skolem difference Mean Labeling , Skolem difference Lucas Mean Labeling.*

I. INTRODUCTION

By a graph, we mean a finite, undirected graph without loops and multiples edges, for terms not defined here we refer to Harary [4].

In a labeling of a particular type, the vertices are assigned values from a given set, the edges have a prescribed induced labeling must satisfy certain properties. An excellent reference on this subject is the survey by Gallian [3].

According to Beineke and Hegde[2] labeling of discrete structure is a frontier between graph theory and numbers.

The concept of Skolem mean labeling was introduced by V.Balaji, T.Ramesh, A.Subramanian,and [1] and also Skolem difference mean labeling introduced by K.Murugan,A.Subramanian[5] and these definitions as follows

Definition 1.1

A graph $G = (V, E)$ with p vertices and q edges is said to be a Skolem mean graph if there exists a function f from the vertex set of $G\ to\ \{1,2,3,\ldots,p\}$ such that the induced map f^* from the edge set of $G\ to\ \{2,3,4,\ldots,p\}$ defined by $f^*(e = uv) =$

$\left|\frac{f(u)-f(v)}{2}\right|$ if $|f(u)-f(v)|$ is even and $\frac{|f(u)-f(v|+1}{2}$ if $|f(u)-f(v)|$ is odd then

The resulting edge labels are distinct and are from {2,3,4,…,p}. A graph that admits Skolem mean labelling is called a Skolem mean graph.

Definition 1.2

A graph $G = (V, E)$ with p vertices and q edges is said to be a Skolem difference mean graph if there exists a function f from the vertex set of $G\ to\ \{1,2,3,\ldots,p+q\}$ such that the induced map f^* from the edge set of $G\ to\ \{1,2,3,\ldots,q\}$ defined by

$f^*(e = uv) = \left|\frac{f(u)-f(v)}{2}\right|$ if $|f(u)-f(v)|$ is even and $\frac{|f(u)-f(v|+1}{2}$

if $|f(u)-f(v)|$ is odd then the resulting edge labels are distinct and are from {1,2,3,…,q}. A graph that admits Skolem difference mean labelling is called a Skolem difference mean graph.

These definition motivate to define the Skolem difference Lucas mean labeling

Definition 1.3

A graph G with p vertices and q edges is said to have Skolem difference Lucas mean labelling if it is possible to label the vertices $x \in v$ with distinct elements $f(x)$ from the set $\{1,2,\ldots,L_{p+q}\}$ in such a way that the edge $e = uv$is labelled with $\left|\frac{f(u)-f(v)}{2}\right|$ if $|f(u)-f(v)|$ is even and $\frac{|f(u)-f(v|+1}{2}$ if $|f(u)-f(v)|$ is odd, then the resulting edge labels are distinct and are from $\{L_1, L_2, \dots, L_q\}$.A graph that admits Skolem difference Lucas mean labelling is called a Skolem difference Lucas mean graph.

II. RESULTS

Theorem 2.1

The graph $K_{1,n}^{+}$ is Skolem difference Lucas mean graph.

Proof:

Let G be the graph $K_{1,n}^{+}$

Let $V(G) = \{u_0, u_i, v_i : 1 \le i \le n\}$

$E(G) = \{u_0u_i : 1 \le i \le n\} \cup \{u_iv_i : 1 \le i \le n$

Then $|V(G)| = 2n+1$ and $|E(G)| = 2n$

Let $f: V(G) \to \{1, 2, \dots, L_{4n+1}\}$ be defined as follows

$f(u_0) = 1$

$f(u_i) = 2L_{2i-1} + f(u_0), \quad 1 \le i \le n$

$f(v_i) = 2L_{2i} + f(u_i) - 1, \quad 1 \le i \le n$

$f^{+}(E) = \{f(u_0u_i): 1 \le i \le n\} \cup \{f(u_iv_i) : 1 \le i \le n$

$= \{f(u_0u_1), f(u_0u_2), \dots, f(u_0u_n)\} \cup \{f(u_1v_1), f(u_2v_2), \dots, f(u_nv_n)\}$

$= \left\{\left|\frac{f(u_0)-f(u_1)}{2}\right|, \left|\frac{f(u_0)-f(u_1)}{2}\right|, \dots, \left|\frac{f(u_0)-f(u_n)}{2}\right|\right\} \cup \left\{\left|\frac{f(u_1)-f(v_1)}{2}\right|, \left|\frac{f(u_2)-f(v_2)}{2}\right|, \dots, \left|\frac{f(u_n)-f(v_n)}{2}\right|\right\}$

$= \left\{\left|\frac{f(u_0)-2L_1-f(u_0)}{2}\right|, \left|\frac{f(u_0)-2L_3-f(u_0)}{2}\right|, \dots, \left|\frac{f(u_0)-2L_{2n-1}-f(u_0)}{2}\right|\right\}$

$\cup \left\{\frac{|2L_1+f(u_0)-2L_2-f(u_0)+1|+1}{2}, \frac{|2L_3+f(u_0)-2L_4-f(u_0)+1|+1}{2}, \right.$

$\left. \dots, \frac{|2L_{2n-1}+f(u_0)-2L_{2n}-f(u_0)+1|+1}{2}\right\}$

$= \{L_1, L_3, \dots, L_{2n-1}\} \cup \{L_2, L_4, \dots, L_{2n}\}$

$= \{L_1, L_2, L_3, L_4, \dots, L_{2n-1}, L_{2n}\}$

$f^{+}(E) = \{L_1, L_2, \dots, L_{2n}\}$

Thus, the induced edge labels are distinct and are $L_1, L_2, \dots, L_{2n}$.

Hence, the graph $K_{1,n}^{+}$ is Skolem difference Lucas mean graph.

Theorem 2.2

The graph $K_{1,m} \oslash K_{1,n}$ is Skolem difference Lucas mean graph for all $m, n \ge 1$.

Proof:

Let G be the graph $K_{1,m} \oslash K_{1,n}$

Let $V(G) = \{u, u_i, u_{ij} : 1 \le i \le m \text{ and } 1 \le j \le n\}$

$E(G) = \{uu_i : 1 \le i \le m\} \cup \{u_iu_{ij} : 1 \le i \le m \text{ and } 1 \le j \le n$

Then $|V(G)| = mn+m+1$ and $|E(G)| = mn+m$

Let $f: V(G) \to \{1, 2, \dots, L_{2mn+m+1}\}$ be defined as follows

$f(u) = 1$

$f(u_1) = 3$

$f(u_i) = 2L_{(n+1)(i-1)+1} + f(u), \quad 2 \le i \le m$

$f(u_{1j}) = 2L_{j+1} + f(u_1), \quad 1 \le j \le n$

$f(u_{ij}) = 2L_{(n+1)(i-1)+1+j} + f(u_i), \quad 2 \le i \le m \text{ and } 1 \le j \le n$

$f^{+}(E) = \{f(uu_i) : 1 \le i \le m\} \cup \{f(u_iu_{ij}) : 1 \le i \le m \text{ and } 1 \le j \le n$

$= \{f(uu_1), f(uu_2), \dots, f(uu_m)\} \cup \{f(u_1u_{11}), f(u_1u_{12}), \dots, f(u_1u_{1n}), f(u_2u_{21}), f(u_2u_{22}), \dots, f(u_2u_{2n}), \dots,$
$f(u_mu_{m1}), f(u_mu_{m2}), \dots, f(u_mu_{mn})\}$

$= \left\{\left|\frac{f(u)-f(u_1)}{2}\right|, \left|\frac{f(u)-f(u_2)}{2}\right|, \dots, \left|\frac{f(u)-f(u_m)}{2}\right|\right\} \cup \left\{\left|\frac{f(u_1)-f(u_{11})}{2}\right|, \left|\frac{f(u_1)-f(u_{12})}{2}\right|, \dots, \left|\frac{f(u_1)-f(u_{1n})}{2}\right|, \right.$

$\left|\frac{f(u_2)-f(u_{21})}{2}\right| \left|\frac{f(u_2)-f(u_{22})}{2}\right|, \dots, \left|\frac{f(u_2)-f(u_{2n})}{2}\right|, \dots, \left|\frac{f(u_m)-f(u_{m1})}{2}\right|,$

$\left. \left|\frac{f(u_m)-f(u_{m2})}{2}\right|, \dots, \left|\frac{f(u_m)-f(u_{mn})}{2}\right|\right\}$

$= \left\{\left|\frac{1-3}{2}\right|, \left|\frac{f(u)-2L_{n+2}-f(u)}{2}\right|, \dots, \left|\frac{f(u)-2L_{(n+1)(m-1)+1}-f(u)}{2}\right|\right\}$

$\cup \left\{ \left|\frac{f(u_1) - 2L_2 - f(u_1)}{2}\right|, \left|\frac{f(u_1) - 2L_3 - f(u_1)}{2}\right|, \dots, \left|\frac{f(u_1) - 2L_{n+1} - f(u_1)}{2}\right|,\right.$

$\left|\frac{f(u_2) - 2L_{n+3} - f(u_2)}{2}\right|, \left|\frac{f(u_2) - 2L_{n+3} - f(u_2)}{2}\right|, \dots, \left|\frac{f(u_2) - 2L_{n+4} - f(u_2)}{2}\right|,$

$\dots, \left|\frac{f(u_2) - 2L_{n+4} - f(u_2)}{2}\right|,$

$\dots, \left|\frac{f(u_m) - 2L_{(n+1)(m-1)+2} - f(u_m)}{2}\right|, \left|\frac{f(u_m) - 2L_{(n+1)(m-1)+3} - f(u_m)}{2}\right|,$

$\left.\dots, \left|\frac{f(u_m) - 2L_{(n+1)(m-1)+1+n} - f(u_m)}{2}\right|\right\}$

$= \{L_1, L_{n+2}, \dots, L_{mn-n+m}\} \cup \{L_2, L_3, \dots, L_{n+1}, L_{n+3}, L_{n+4}, \dots, L_{2n+2}, \dots, L_{mn-n+m+1}, L_{mn-n+m+2}, \dots, L_{mn+m}\}$

$= \left\{\begin{matrix} L_1, L_2, L_3, \dots, L_{n+1}, L_{n+2}, L_{n+3}, L_{n+4}, \dots, L_{2n+2}, \dots, L_{mn-n+m}, L_{mn-n+m+1}, \\ L_{mn-n+m+2}, \dots, L_{mn+m} \end{matrix}\right\}$

$f^+(E) = \{L_1, L_2, \dots, L_{mn+m}\}$

Thus, the induced edge labels are distinct and are $L_1, L_2, \dots, L_{mn+m}$.

Hence, the graph $K_{1,m} \oslash K_{1,n}$ is Skolem difference Lucas mean graph for all $m, n \geq 1$.

Theorem 2.3

The graph $K_{1,m} \odot 2P_n$ is Skolem difference Lucas mean graph for all positive integers m and n.

Proof:

Let G be the graph $K_{1,m} \odot 2P_n$

Let $V(G) = \{u_0, u_i : 1 \leq i \leq m\} \cup \{v_{ij}, w_{ij} : 1 \leq i \leq m \text{ and } 1 \leq j \leq n-1\}$

$E(G) = \{u_0u_i : 1 \leq i \leq m\} \cup \{u_iv_{i1}, u_iw_{i1} : 1 \leq i \leq m\} \cup \{v_{ij}v_{i(j+1)}, w_{ij}w_{i(j+1)} : 1 \leq i \leq m \text{ and } 1 \leq j \leq n-1\}$

Then $|V(G)| = 2mn + n + 1$ and $|E(G)| = 2mn + n$

Let $f: V(G) \to \{1,2, \dots, L_{4mn+2n+1}\}$ be defined as follows

$f(u_0) = 1$

$f(u_i) = 2L_{(2n+1)(i-1)+2} + f(u_0), \quad 1 \leq i \leq m$

$f(v_{i1}) = 2L_{(2n+1)(i-1)+1} + f(u_i), \quad 1 \leq i \leq m$

$f(w_{i1}) = 2L_{(2n+1)(i-1)+3} + f(u_i), \quad 1 \leq i \leq m$

$f(v_{ij}) = 2L_{(2n+1)(i-1)+2j} + f(v_{i(j-1)}), \quad 1 \leq i \leq m \text{ and } 2 \leq j \leq n$

$f(w_{ij}) = 2L_{(2n+1)(i-1)+2j+1} + f(w_{i(j-1)}), \quad 1 \leq i \leq m \text{ and } 2 \leq j \leq n$

$f^+(E) = \{f(u_0u_i): 1 \leq i \leq m\} \cup \{f(u_iv_{i1}), f(u_iw_{i1}) : 1 \leq i \leq m\} \cup \{f(v_{ij}v_{i(j+1)}), f(w_{ij}w_{i(j+1)}) : 1 \leq i \leq m \text{ and } 1 \leq j \leq n-1\}$

$= \{f(u_0u_1), f(u_0u_2), \dots, f(u_0u_m)\}$
$\cup \{f(u_1v_{11}), f(u_2v_{21}), \dots, f(u_mv_{m1}), f(u_1w_{11}), f(u_2w_{21}), \dots, f(u_mw_{m1})\}$
$\cup \{f(v_{11}v_{12}), f(v_{12}v_{13}), \dots, f(v_{1(n-1)}v_{1n}), f(v_{21}v_{22}), f(v_{22}v_{23}), \dots, f(v_{2(n-1)}v_{2n}), \dots, f(v_{m1}v_{m2}), f(v_{m2}v_{m3}) \dots, f(v_{m(n-1)}v_{mn}), f(w_{11}w_{12}), f(w_{12}v_{13}), \dots, f(w_{1(n-1)}w_{1n}), f(w_{21}w_{22}), f(w_{22}w_{23}), \dots, f(w_{2(n-1)}w_{2n}), \dots, f(w_{m1}w_{m2}), f(w_{m2}w_{m3}) \dots, f(w_{m(n-1)}w_{mn})\}$

$= \left\{\left|\frac{f(u_0) - f(u_1)}{2}\right|, \left|\frac{f(u_0) - f(u_2)}{2}\right|, \dots, \left|\frac{f(u_0) - f(u_m)}{2}\right|\right\}$

$\cup \left\{\begin{matrix} \left|\frac{f(u_1) - f(v_{11})}{2}\right|, \left|\frac{f(u_2) - f(v_{21})}{2}\right|, \dots, \left|\frac{f(u_m) - f(v_{m1})}{2}\right|, \\ \left|\frac{f(u_1) - f(w_{11})}{2}\right|, \left|\frac{f(u_2) - f(w_{21})}{2}\right|, \dots, \left|\frac{f(u_m) - f(w_{m1})}{2}\right| \end{matrix}\right\}$

$\cup \left\{ \left|\frac{f(v_{11}) - f(v_{12})}{2}\right|, \left|\frac{f(v_{12}) - f(v_{13})}{2}\right|, \dots, \left|\frac{f(v_{1(n-1)}) - f(v_{1n})}{2}\right|,\right.$

$\left|\frac{f(v_{21}) - f(v_{22})}{2}\right|, \left|\frac{f(v_{22}) - f(v_{23})}{2}\right|, \dots, \left|\frac{f(v_{2(n-1)}) - f(v_{2n})}{2}\right|$

$, \dots, \left|\frac{f(v_{m1}) - f(v_{m2})}{2}\right|, \left|\frac{f(v_{m2}) - f(v_{m3})}{2}\right|, \dots, \left|\frac{f(v_{m(n-1)}) - f(v_{mn})}{2}\right|,$

$$\left|\frac{f(w_{11})-f(w_{12})}{2}\right|,\left|\frac{f(w_{12})-f(w_{13})}{2}\right|,\dots,\left|\frac{f(w_{1(n-1)})-f(w_{1n})}{2}\right|,$$
$$\left|\frac{f(w_{21})-f(w_{22})}{2}\right|,\left|\frac{f(w_{22})-f(w_{23})}{2}\right|,\dots,\left|\frac{f(w_{2(n-1)})-f(w_{2n})}{2}\right|$$
$$,\dots,\left|\frac{f(w_{m1})-f(w_{m2})}{2}\right|,\left|\frac{f(w_{m2})-f(w_{m3})}{2}\right|,\dots,\left|\frac{f(w_{m(n-1)})-f(w_{mn})}{2}\right|\}$$
$$=\left\{\left|\frac{f(u_0)-2L_2-f(u_0)}{2}\right|,\left|\frac{f(u_0)-2L_{2n+3}-f(u_0)}{2}\right|,\dots,\left|\frac{f(u_0)-2L_{2nm-2n+m+1}-f(u_0)}{2}\right|\right\}$$
$$\cup\left\{\begin{array}{c}\left|\frac{f(u_1)-2L_1-f(u_1)}{2}\right|,\left|\frac{f(u_2)-2L_{2n+2}-f(u_2)}{2}\right|,\dots,\\ \left|\frac{f(u_m)-2L_{2nm-2n+m}-f(u_m)}{2}\right|,\\ \left|\frac{f(u_1)-2L_3-f(u_1)}{2}\right|,\left|\frac{f(u_2)-2L_{2n+4}-f(u_2)}{2}\right|,\dots,\\ \left|\frac{f(u_m)-2L_{2nm-2n+m+2}-f(u_m)}{2}\right|\end{array}\right\}$$
$$\cup\{\left|\frac{f(v_{11})-2L_4-f(v_{11})}{2}\right|,\left|\frac{f(v_{12})-2L_6-f(v_{12})}{2}\right|,\dots,$$
$$\left|\frac{f(v_{1(n-1)})-2L_{2n}-f(v_{1(n-1)})}{2}\right|,$$
$$\left|\frac{f(v_{21})-2L_{2n+5}-f(v_{21})}{2}\right|,\left|\frac{f(v_{22})-2L_{2n+7}-f(v_{22})}{2}\right|,\dots,$$
$$\left|\frac{f(v_{2(n-1)})-2L_{2n+2m+1}-f(v_{2(n-1)})}{2}\right|,\dots,\left|\frac{f(v_{m1})-2L_{2nm-2n+m+3}-f(v_{m1})}{2}\right|$$
$$\left|\frac{f(v_{m2})-2L_{2nm-2n+m+5}-f(v_{m2})}{2}\right|,\dots,\left|\frac{f(v_{m(n-1)})-2L_{2nm+m-1}-f(v_{m(n-1)})}{2}\right|,$$
$$\left|\frac{f(w_{11})-2L_5-f(w_{11})}{2}\right|,\left|\frac{f(w_{12})-2L_7-f(w_{12})}{2}\right|,\dots,$$
$$\left|\frac{f(w_{1(n-1)})-2L_{2n+1}-f(w_{1(n-1)})}{2}\right|,\left|\frac{f(w_{21})-2L_{2n+6}-f(w_{21})}{2}\right|$$
$$,\left|\frac{f(w_{22})-2L_{2n+8}-f(w_{22})}{2}\right|,\dots,\left|\frac{f(w_{2(n-1)})-2L_{2n+2m+2}-f(w_{2(n-1)})}{2}\right|$$
$$,\dots,\left|\frac{f(w_{m1})-2L_{2nm-2n+m+4}-f(w_{m1})}{2}\right|,\left|\frac{f(w_{m2})-2L_{2nm-2n+m+6}-f(w_{m2})}{2}\right|,$$
$$\dots,\left|\frac{f(w_{m(n-1)})-2L_{2nm+m}-f(w_{m(n-1)})}{2}\right|\}$$
$$=\{L_2,L_{2n+3},\dots,L_{2nm-2n+m+1}\}\cup\{L_1,L_{2n+2},\dots,L_{2nm-2n+m}\}\cup\{L_4,L_6,\dots,L_{2n},L_{2n+5},L_{2n+7},\dots,L_{2n+2m+1},\dots,L_{2nm-2n+m+3},$$
$$L_{2nm-2n+m+5},\dots,L_{2nm+m-1},L_5,L_7,\dots,L_{2n+1},L_{2n+6},L_{2n+8},\dots,L_{2n+2m+2},\dots,$$
$$L_{2nm-2n+m+4},L_{2nm-2n+m+6},\dots,L_{2nm+m}\}$$
$$=\left\{\begin{array}{c}L_1,L_2,L_3,L_4,L_5,L_6,L_7,\dots,L_{2n+1},L_{2n+2},\\ L_{2n+3},L_{2n+4},L_{2n+5},L_{2n+6},L_{2n+7},L_{2n+8},\dots,L_{2n+2m+1}L_{2nm-2n+m},L_{2nm-2n+m+1}\\ ,L_{2n+2m+2},\dots,L_{2nm-2n+m+2},L_{2nm-2n+m+3},L_{2nm-2n+m+4},L_{2nm-2n+m+5},\\ L_{2nm-2n+m+6},\dots L_{2nm+m-1},L_{2nm+m}\end{array}\right\}$$
$$f^+(E)=\{L_1,L_2,\dots,L_{2mn+m}\}$$

Thus, the induced edge labels are distinct and are $L_1,L_2,\dots,L_{2mn+m}$.
Hence, the graph $K_{1,m}\odot 2P_n$ is Skolem difference Lucas mean graph for all positive integers m *and* n.

Theorem 2.4

The bistar $B_{m,n}$ is Skolem difference Lucas mean graph for all $m,n\geq 1$.

Proof:

Let G be the graph $B_{m,n}$

Let $V(G) = \{u, v, u_i, v_j : 1 \leq i \leq m \ and \ 1 \leq j \leq n\}$
$E(G) = \{uv, uu_i, vv_j : 1 \leq i \leq m \ and \ 1 \leq j \leq n\}$
Then $|V(G)| = m + n + 2$ and $|E(G)| = m + n + 1$
Let $f: V(G) \to \{1,2, \dots, L_{2m+2n+3}\}$ be defined as follows
$f(u) = 1$
$f(u_i) = 2L_i + f(u), \quad 1 \leq i \leq m$
$f(v) = 2L_{m+1} + f(u),$
$f(v_j) = 2L_{m+1+j} + f(v), \quad 1 \leq j \leq n$
$f^+(E) = \{f(uv), f(uu_i), f(vv_j) : 1 \leq i \leq m \ and \ 1 \leq j \leq n\}$
$= \{ f(uv), f(uu_1), f(uu_2), \dots, f(uu_m), f(vv_1), f(vv_2), \dots, f(vv_n)\}$

$$= \left\{ \begin{array}{c} \left|\frac{f(u) - f(v)}{2}\right|, \left|\frac{f(u) - f(u_1)}{2}\right|, \left|\frac{f(u) - f(u_2)}{2}\right|, \dots, \left|\frac{f(u) - f(u_m)}{2}\right|, \\ \left|\frac{f(v) - f(v_1)}{2}\right|, \left|\frac{f(v) - f(v_2)}{2}\right|, \dots, \left|\frac{f(v) - f(v_n)}{2}\right| \end{array} \right\}$$

$$= \left\{ \begin{array}{c} \left|\frac{f(u) - 2L_{\mathrm{m+1}} - f(u)}{2}\right|, \left|\frac{f(u) - 2L_1 - f(u)}{2}\right|, \left|\frac{f(u) - 2L_2 - f(u)}{2}\right|, \dots, \\ \left|\frac{f(u) - 2L_{\mathrm{m}} - f(u)}{2}\right|, \left|\frac{f(v) - 2L_{\mathrm{m+2}} - f(v)}{2}\right|, \left|\frac{f(v) - 2L_{\mathrm{m+3}} - f(v)}{2}\right|, \dots, \\ \left|\frac{f(v) - 2L_{\mathrm{m+n+1}} - f(v)}{2}\right| \end{array} \right\}$$

$= \{L_{m+1}, L_1, L_2, \dots, L_m, L_{m+2}, L_{m+3}, \dots, L_{m+n+1}\}$
$f^+(E) = \{L_1, L_2, \dots, L_{m+n+1}\}$
Thus, the induced edge labels are distinct and are $L_1, L_2, \dots, L_{m+n+1}$.
Hence, the bistar $B_{m,n}$ is Skolem difference Lucas mean graph for all $m, n \geq 1$.

Theorem 2.5

The graph $< K_{1,n}^{(1)}, K_{1,n}^{(2)}, K_{1,n}^{(3)}, \dots, K_{1,n}^{(m)} >$ is Skolem difference Lucas mean graph.

Proof:

Let G be the graph $< K_{1,n}^{(1)}, K_{1,n}^{(2)}, K_{1,n}^{(3)}, \dots, K_{1,n}^{(m)} >$
Let $V(G) = \{v_i : 1 \leq i \leq m\} \cup \{u_i : 1 \leq i \leq m - 1\} \cup \{v_{ij} : 1 \leq i \leq m \ and \ 1 \leq j \leq n\}$
$E(G) = \{v_i v_{ij} : 1 \leq i \leq m \ and \ 1 \leq j \leq n\} \cup \{v_i u_i : 1 \leq i \leq m - 1\} \cup \{u_{i-1} v_i : 2 \leq i \leq m\}$
Then $|V(G)| = mn + 2m - 1$ and $|E(G)| = mn + 2m - 2$
Let $f: V(G) \to \{1,2, \dots, L_{2mn+4m-3}\}$ be defined as follows
$f(v_1) = 1$
$f(v_i) = 2L_{(n+2)(i-1)} + f(u_{i-1}), \quad 2 \leq i \leq m$
$f(u_i) = 2L_{(n+2)i-1} + f(v_i), \quad 1 \leq i \leq m - 1$
$f(v_{ij}) = 2L_{n(i-1)+2i-2+j} + f(v_i), \quad 1 \leq i \leq m \ and \ 1 \leq j \leq n$
$f^+(E) = \{f(v_i v_{ij}) : 1 \leq i \leq m \ and \ 1 \leq j \leq n\} \cup \{f(v_i u_i) : 1 \leq i \leq m - 1\} \cup \{f(u_{i-1} v_i) : 2 \leq i \leq m\}$
$= \{f(v_1 v_{11}), f(v_1 v_{12}), \dots, f(v_1 v_{1n}), f(v_2 v_{21}), f(v_2 v_{22}), \dots, f(v_2 v_{2n}), \dots, f(v_m v_{m1}), f(v_m v_{m2}), \dots, f(v_m v_{mn})\} \cup \{f(v_1 u_1), f(v_2 u_2), \dots, f(v_{m-1} u_{m-1})\} \cup \{f(u_1 v_2), f(u_2 v_3), \dots, f(u_{m-1} v_m)\}$

$$= \{\left|\frac{f(v_1) - f(v_{11})}{2}\right|, \left|\frac{f(v_1) - f(v_{12})}{2}\right|, \dots, \left|\frac{f(v_1) - f(v_{1n})}{2}\right|, \left|\frac{f(v_2) - f(v_{21})}{2}\right|, \left|\frac{f(v_2) - f(v_{22})}{2}\right|, \dots, \left|\frac{f(v_2) - f(v_{2n})}{2}\right|, \dots, \left|\frac{f(v_m) - f(v_{m1})}{2}\right|, \left|\frac{f(v_m) - f(v_{m2})}{2}\right|, \dots, \left|\frac{f(v_m) - f(v_{mn})}{2}\right|\} \cup \{\left|\frac{f(v_1) - f(u_1)}{2}\right|, \left|\frac{f(v_2) - f(u_2)}{2}\right|, \dots, \left|\frac{f(v_{m-1}) - f(u_{m-1})}{2}\right|\} \cup \{\left|\frac{f(u_1) - f(v_2)}{2}\right|, \left|\frac{f(u_2) - f(v_3)}{2}\right|, \dots, \left|\frac{f(u_{m-1}) - f(v_m)}{2}\right|\}$$

$$= \{\left|\frac{f(v_1) - 2L_1 - f(v_1)}{2}\right|, \left|\frac{f(v_1) - 2L_2 - f(v_1}{2}\right|, \dots, \left|\frac{f(v_1) - 2L_n - f(v_1}{2}\right|,$$

$\left|\frac{f(v_2)-2L_{n+3}-f(v_2)}{2}\right|, \left|\frac{f(v_2)-2L_{n+4}-f(v_2)}{2}\right|, \dots, \left|\frac{f(v_2)-2L_{2n+2}-f(v_2)}{2}\right|, \dots,$
$\left|\frac{f(v_m)-2L_{mn+2m-n-1}-f(v_m)}{2}\right|, \left|\frac{f(v_m)-2L_{mn+2m-n}-f(v_m)}{2}\right|, \dots,$
$\left|\frac{f(v_m)-2L_{mn+2m-2}-f(v_m)}{2}\right| \cup \{\left|\frac{f(v_1)-2L_{n+1}-f(v_1)}{2}\right|, \left|\frac{f(v_2)-2L_{2n+3}-f(v_2)}{2}\right|$
$\left|\frac{f(v_{m-1})-2L_{mn+2m-n-3}-f(v_{m-1})}{2}\right|\} \cup \{\left|\frac{f(u_1)-2L_{n+2}-f(u_1)}{2}\right|$
$\left|\frac{f(u_2)-2L_{2n+4}-f(u_2)}{2}\right|, \dots, \left|\frac{f(u_{m-1})-2L_{mn+2m-n-2}-f(u_{m-1})}{2}\right|\}$
$=$
$\{L_1, L_2, \dots, L_n, L_{n+3}, L_{n+4}, \dots, L_{2n+2}, \dots, L_{mn+2m-n-1}, L_{mn+2m-n}, \dots, L_{mn+2m-2}\} \cup$
$\{L_{n+1}, L_{2n+3}, \dots, L_{mn+2m-n-3}\} \cup \{L_{n+2}, L_{2n+4}, \dots, L_{mn+2m-n-2}\}$
$= \begin{Bmatrix} L_1, L_2, L_3, \dots, L_n, L_{n+1}, L_{n+2}, L_{n+3}, L_{n+4}, \dots, L_{2n+2}, L_{2n+3}, \dots, L_{mn+2m-n-3}, \\ L_{mn+2m-n-2}, L_{mn+2m-n-1}, L_{mn+2m-n}, \dots, L_{mn+2m-2} \end{Bmatrix}$
$f^+(E) = \{L_1, L_2, \dots, L_{mn+2m-2}\}$

Thus, the induced edge labels are distinct and are $L_1, L_2, \dots, L_{mn+2m-2}$.

Hence, the graph $< K_{1,n}^{(1)}, K_{1,n}^{(2)}, K_{1,n}^{(3)}, \dots, K_{1,n}^{(m)} >$ is Skolem difference Lucas mean graph.

III. CONCLUSION

It is very interesting and challenging as well to investigate skolem difference Lucas mean labeling for the graph or graph families which admit skolem difference Lucas mean labeling.Here we have proved for some star related graphs such as $K_{1,n}^{+}$, $K_{1,m} \oslash K_{1,n}$, $K_{1,m} \odot 2P_n$, $B_{m,n}$, $< K_{1,n}^{(1)}, K_{1,n}^{(2)}, K_{1,n}^{(3)}, \dots, K_{1,n}^{(m)} >$, are Skolem difference Lucas mean graph. In the subsequent paper we will prove skolem difference Lucas mean labeling for path related graphs.

REFERENCES

[1] V.Balaji , D.S.T.Ramesh , A, Subramanian , Skolem Mean Labeling ; Bulletin of Pure and Applied Sciences , Vol.26E(2)(2007),245-248.

[2] L.W. Beineke and S.M. Hedge, Strongly Multiplicative graphs, Discuss. Math. Graph Theory, vol. 21, 63-75, and 2001.

[3] J.A. Gallian, A Dynamic survey of graph labeling, The electronic Journal of combinatorics, 16(2009), #DS6, pp 219.

[4] Frank Harary, Graph theory, Narosa publishing house pvt.Ltd. 10th reprint, 2001.

[5] K. Murugan, A. Subramanian, Skolem difference mean Labeling of H-graphs , International Journal of Mathematics and Soft Computing, Vol, No.1(2011),115-129.