Pathway Fractional Integral Operator and Its Composition with Gimel Functions

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ABSTRACT
Fractional integral operators are extensively used in a large number of areas of mathematical analysis. This paper provides the images of the products of multivariable Gimel-function and generalized multivariable polynomial under the Pathway fractional integral operator and its compositions. The main results are quite general in nature. Further, some interesting special cases are given.

Keywords: Fractional integral operator, Multivariable Gimel-function, Srivastava polynomial, Pathway model.

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1. Introduction

Pathway fractional integral operator is based on Riemann-Liouville fractional integral operator and Pathway model. Pathway operator is introduced in the paper of Nair [8] and defined as follows :

If $f(x) \in L(a,b), \gamma \in \mathbb{C}, Re(\gamma) > 0, a > 0$ and $\beta < 1$, then

$$(P_{0_+}^{(\gamma,\beta)} f)(x) = x^{\gamma} \int_0^{\left[\frac{x}{a(1-\beta)}\right]} \left[1 - \frac{a(1-\beta)t}{x}\right]^{\frac{\gamma}{1-\beta}} f(t)\mathrm{d}t \tag{1.1}$$

where β is called pathway parameter. For Pathway model, we refer to Mathai [7], Mathai and Haubold ([5], [6]). The pathway model transforms into three different types of densities, type-1 beta, type-2 beta and gamma in statistics. Using the Pathway parameter β, this operator can reduce to various fractional integral operators, related to different probability density functions and applications in statistics.

If $\beta \rightarrow 1$, $\left[1 - \frac{a(1-\beta)t}{x}\right]^{\frac{\gamma}{1-\beta}} \rightarrow e^{-\frac{a\gamma}{x}t}$ and hence Pathway operator switches to the Laplace integral transform of function f with parameter $\frac{a\gamma}{x}$:

$$(P_{0_+}^{(\gamma,1)} f)(x) = x^{\gamma} \int_0^{\infty} e^{-\frac{a\gamma}{x}t} f(t)\mathrm{d}t = x^{\gamma} L_f\left(\frac{a\gamma}{x}\right)$$

Taking $\beta = 0$ and $a = 1$, then replacing γ by $\gamma - 1$ in pathway operator (1.1), it transforms to the Riemann-Liouville (R-L) fractional integral operator.

Throughout this paper, let $\mathbb{C}, \mathbb{R}$ and $\mathbb{N}$ be set of complex numbers, real numbers and positive integers respectively. Also $\mathbb{N}_0 = \mathbb{N} \cup \{0\}$. We define a generalized transcendental Gimel function of several complex variables.

$$\gimel(z_1, \cdots, z_r) = \gimel_{X;p_{i_r},q_{i_r},\tau_{i_r}:R_r:Y}^{U;0,n_r:V} \left(\begin{array}{c|c} z_1 & \mathbb{A};\mathbf{A}:\ A \\ \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ z_r & \mathbb{B};\mathbf{B}:\ B \end{array} \right) = \frac{1}{(2\pi\omega)^r} \int_{L_1} \cdots \int_{L_r} \psi(s_1, \cdots, s_r) \prod_{k=1}^{r} \theta_k(s_k) z_k^{s_k} \mathrm{d}s_1 \cdots \mathrm{d}s_r \tag{1.2}$$

with $\omega = \sqrt{-1}$

About the following quantities $X, Y, U, V, \mathbb{A}, \mathbf{A}, A, \mathbb{B}, \mathbf{B}, B, \psi(s_1, \cdots, s_r)$ and $\theta_k(s_k)(k = 1, \cdots, r)$, see Ayant [2] for more informations.
The generalized polynomials defined by Srivastava ([12],p. 251, Eq. (C.1)), is given in the following manner :

$$S_{N_1,\cdots,N_s}^{M_1,\cdots,M_s}[y_1,\cdots,y_s] = \sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_s=0}^{[N_s/M_s]}\frac{(-N_1)_{M_1K_1}}{K_1!}\cdots\frac{(-N_s)_{M_sK_s}}{K_s!}A[N_1,K_1;\cdots;N_s,K_s]y_1^{K_1}\cdots y_s^{K_s} \tag{1.3}$$

we shall note $a_1 = \frac{(-N_1)_{M_1K_1}}{K_1!}\cdots\frac{(-N_s)_{M_sK_s}}{K_s!}A[N_1,K_1;\cdots;N_s,K_s]$

where $M_1,\cdots,M_s$ are arbitrary positive integers and the coefficients $A[N_1,K_1;\cdots;N_s,K_s]$ are arbitrary constants, real or complex. I we take $s=1$ in the (1.3) and denote $A[N,K]$ thus obtained by $A_{N,K}$, we arrive at general class of polynomials $S_N^M(x)$ study by Srivastava ([11],p. 1, Eq. 1).

2. Required result.

We have the following result.

Lemma

$$\int_0^{\frac{x}{a(1-\beta)}}\left[1-\frac{a(1-\beta)t}{x}\right]^{\frac{\gamma}{1-\beta}}t^{\sigma-1}\mathrm{d}t = \frac{x^\sigma}{[a(1-\beta)]^\sigma}\frac{\Gamma(\sigma)\Gamma\left(1+\frac{\gamma}{1-\beta}\right)}{\Gamma\left(1+\sigma+\frac{\gamma}{1-\beta}\right)} \tag{2.1}$$

provided $\beta<1, Re(\gamma), Re(\sigma)>0$.

3. Main results.

Let

$$\mathbf{A}_1 = [(\mathrm{a}_{rj};\alpha_{rj}^{(1)},\cdots,\alpha_{rj}^{(r)},0;A_{rj})]_{1,n_r},[\tau_{i_r}(a_{rji_r};\alpha_{rji_r}^{(1)},\cdots,\alpha_{rji_r}^{(r)},0;A_{rji_r})]_{\mathfrak{n}+1,p_{i_r}} \tag{3.1}$$

$$\mathbf{B}_1 = [\tau_{i_r}(b_{rji_r};\beta_{rji_r}^{(1)},\cdots,\beta_{rji_r}^{(r)},0;B_{rji_r})]_{1,q_{i_r}} \tag{3.2}$$

We have the first formula

Theorem 1.

$$P_{0_+}^{(\gamma,\beta)}\left[x^{\delta-1}(x^h+c)^{-\eta}S_{N_1,\cdots,N_s}^{M_1,\cdots,M_s}\left[Z_1x^{r_1}(x^h+c)^{-\eta_1},\cdots,Z_sx^{r_s}(x^h+c)^{-\eta_s}\right]\right.$$

$$\left.\gimel\left[z_1x^{h_1}(x^h+c)^{-\phi_1},\cdots,z_rx^{h_r}(x^h+c)^{-\phi_r}\right]\right] = \frac{x^{\gamma+\delta}}{c^\eta[a(1-\beta)]^\delta}\Gamma\left(1+\frac{\gamma}{1-\beta}\right)$$

$$\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_s=0}^{[N_s/M_s]}a_1\prod_{j=1}^s\frac{Z_j^{K_j}x^{r_jK_j}}{c^{\eta_jK_j}[a(1-\beta)]^{r_jK_j}}\gimel_{X;p_{i_r}+2,q_{i_r}+2,\tau_{i_r}:R_r:Y;(0,1)}^{U;0,n_r+2:V;(1,0)}$$

$$\left(\begin{array}{c|c}\frac{z_1x^{h_1}}{c^{\phi_1}[a(1-\beta)]^{h_1}} & \mathbb{A};(1-\eta-\sum_{j=1}^s\eta_jK_j;\phi_1,\cdots,\phi_r,1;1),(1-\delta-\sum_{j=1}^s r_jK_j;h_1,\cdots,h_r,k;1),\mathbf{A}_1:A\\ \cdot & \cdot\\ \cdot & \cdot\\ \cdot & \cdot\\ \frac{z_rx^{h_r}}{c^{\phi_r}[a(1-\beta)]^{h_r}} & \cdot\\ \frac{x^k}{c[a(1-\beta)]^k} & \mathbb{B};\mathbf{B}_1,(1-\eta-\sum_{j=1}^s\eta_jK_j;\phi_1,\cdots,\phi_r,0;1),\left(1-\frac{\gamma}{1-\beta}-\delta-\sum_{j=1}^s r_jK_j;h_1,\cdots,h_r,k;1\right):B;(0,1;1)\end{array}\right) \tag{3.3}$$

provided

$\gamma,\delta,r_j,\eta_j,h_i,\phi_i\in\mathbb{C};(j=1,\cdots,s),(i=1,\cdots,r),Re\left(1+\frac{\gamma}{1-\beta}\right)>0,h,c\geq 0,\beta<1$

$Z_j,z_i,k\in\mathbb{R},Re(\gamma,\delta,r_j,\eta_j,p_i,\phi_i)>0$

$arg(z_ix^{h_i}(x^h+c)^{-\phi_i})<\frac{1}{2}A_i^{(k)}\pi$ where $A_i^{(k)}$ is defined by Ayant [2] where $i=1,\cdots,r$, see Braaksma [3]

Proof

To establish the theorem 1, we consider $f(t)=x^{\delta-1}(x^h+c)^{-\eta}S_{N_1,\cdots,N_s}^{M_1,\cdots,M_s}\left[Z_1x^{r_1}(x^h+c)^{-\eta_1},\cdots,Z_sx^{r_s}(x^h+c)^{-\eta_s}\right]$ $\gimel\left[z_1x^{h_1}(x^h+c)^{-\phi_1},\cdots,z_rx^{h_r}(x^h+c)^{-\phi_s}\right]$ in Pathway fractional integral operator (1.1). Express multivariable Gimel-function and generalized polynomial with the help of the equations (1.2) and (1.3) respectively. Further, express $(t^h+c)^{-\eta+\sum_{j=1}^s\eta_iK_i-\sum_{i=1}^r\phi_i\delta_i}$in the form of the Mellin-Barnes type contour integral by Srivastava et al. [13]. Interchange the order of integration under the permissible conditions and evaluate the t – integral using the lemma. Then interpreting the multiple integrals contour in term of the Gimel-function of r-variables. We obtain the theorem 1 after algebraic manipulations.

Let

$$\mathbf{A}_2=[(\mathrm{a}_{rj};\alpha_{rj}^{(1)},\cdots,\alpha_{rj}^{(r)},0,0;A_{rj})]_{1,n_r},[\tau_{i_r}(a_{rji_r};\alpha_{rji_r}^{(1)},\cdots,\alpha_{rji_r}^{(r)},0,0;A_{rji_r})]_{\mathfrak{n}+1,p_{i_r}} \tag{3.4}$$

$$\mathbf{B}_2=[\tau_{i_r}(b_{rji_r};\beta_{rji_r}^{(1)},\cdots,\beta_{rji_r}^{(r)},0,0;B_{rji_r})]_{1,q_{i_r}} \tag{3.5}$$

We have the second formula

Theorem 2.

$$P_{0+,y}^{(\gamma',\beta')}\left[P_{0+,x}^{(\gamma,\beta)}\left[x^{\delta-1}y^{\delta'-1}(x^h+c)^{-\eta}(y^{h'}+c')^{-\eta'}S_{N_1,\cdots,N_s}^{M_1,\cdots,M_s}\left[Z_1x^{r_1}y^{r'_1}(x^h+c)^{-\eta_1}(y^{h'}+c')^{-\eta'_1},\cdots,\right.\right.\right.$$

$$\left.\left.\left.Z_sx^{r_s}y^{r'_s}(x^h+c)^{-\eta_s}(x^{h'}+c')^{-\eta'_s}\right]\gimel\left[z_1x^{h_1}y^{h'_1}(x^h+c)^{-\phi_1}(y^{h'}+c')^{-\phi'_1},\cdots,z_rx^{h_r}(x^h+c)^{-\phi_r}y^{h'_r}(y^{h'}+c')^{-\phi'_r}\right]\right]\right]$$

$$=\frac{x^{\gamma+\delta}}{c^\eta[a(1-\beta)]^\delta}\Gamma\left(1+\frac{\gamma}{1-\beta}\right)\frac{y^{\gamma'+\delta'}}{c^{\eta'}[a'(1-\beta')]^{\delta'}}\Gamma\left(1+\frac{\gamma'}{1-\beta'}\right)\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_s=0}^{[N_s/M_s]}a_1$$

$$\prod_{j=1}^s\frac{Z_j^{K_j}x^{r_jK_j}y^{r'_jK_j}}{c^{\eta_jK_j}c'^{\eta'_jK_j}[a(1-\beta)]^{r_jK_j}[a'(1-\beta')]^{r'_jK_j}}\gimel_{X;p_{i_r}+4,q_{i_r}+4,\tau_{i_r}:R_r:Y;(0,1);(0,1)}^{U;0,n_r+4:V;(1,0);(1,0)}$$

$$\left(\begin{array}{c|c}\frac{Z_1x^{h_1}y^{h'_1}}{c^{\phi_1}c'^{\phi'_1}[a(1-\beta)]^{h_1}[a'(1-\beta')]^{h'_1}} & \mathbb{A};(1\text{-}\eta-\sum_{j=1}^s\eta_jK_j;\phi_1,\cdots,\phi_r,1,0;1),\\ \cdot & \cdot\\ \cdot & \cdot\\ \frac{Z_rx^{h_r}y^{h'_r}}{c^{\phi_r}c'^{\phi'_r}[a(1-\beta)]^{h_r}[a'(1-\beta')]^{h'_r}} & \cdot\\ \frac{x^k}{c[a(1-\beta)]^k} & \cdot\\ \frac{y^{k'}}{c'[a'(1-\beta')]^{k'}} & \mathbb{B};\mathbf{B}_2,(1-\eta-\sum_{j=1}^s\eta_jK_j;\phi_1,\cdots,\phi_r,0,0;1),\end{array}\right.$$

$$(1\text{-}\delta-\textstyle\sum_{j=1}^sr_jK_j;h_1,\cdots,h_r,k,0;1),(1-\delta'-\sum_{j=1}^sr'_jK_j;h'_1,\cdots,h'_r,0,k';1),$$

$$\cdot$$
$$\cdot$$
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$$\left(-\delta-\tfrac{\gamma}{1-\beta}-\textstyle\sum_{j=1}^sr_jK_j;h_1,\cdots,h_r,k,0;1\right),\left(-\delta'-\tfrac{\gamma'}{1-\beta'}-\sum_{j=1}^sr'_jK_j;h'_1,\cdots,h'_r,0,k';1\right),$$

$$\left.\begin{matrix} (1\text{-}\eta' - \sum_{j=1}^{s}\eta'_j K_j;\phi'_1,\cdots,\phi'_r,0,1;1),\mathbf{A}_2 : A \\ \vdots \\ (1\text{-}\eta' - \sum_{j=1}^{s}\eta'_j K_j;\phi'_1,\cdots,\phi'_r,0,0;1) : B;(0,1;1);(0,1;1) \end{matrix}\right) \tag{3.6}$$

provided

$\gamma,\gamma',\delta,\delta',r_j,\eta_j,h_i,\phi_i,\eta'_j,r'_j,h'_i,\phi'_i \in \mathbb{C};(j=1,\cdots,s),(i=1,\cdots,r), Re\left(1+\frac{\gamma}{1-\beta}\right)>0, Re\left(1+\frac{\gamma'}{1-\beta'}\right)>0$

$Z_j,z_i,k \in \mathbb{R}, Re(\gamma,\delta,r_j,\eta_j,p_i,\phi_i,\gamma',\delta',r'_j,\eta'_j,p'_i,\phi'_i)>0, h,c,h',c' \geq 0, \beta<1,\beta'<1, Z_j,k,k' \in \mathbb{R}$

$arg\left(z_i x^{h_i}(x^h+c)^{-\phi_i} x'^{h'_i}(x'^{h'}+c')^{-\phi'_i}\right) < \frac{1}{2}A_i^{(k)}\pi$ where $i=1,\cdots,r$.see Braaksma [3]

Proof

The theorem 2 can be proved if we apply the theorem 1 twice, first with respect to the variable y, and then with respect to the variable x ; x and y are independent variables.

4.Particular cases.

In this section, we shall see two particular cases concerning the theorem 2.

Let $S_{N_1,\cdots,N_s}^{M_1,\cdots,M_s}[y_1,\cdots,y_s] \rightarrow S_N^M(x)$, see [11] ,we obtain the following formula.

Corollary 1.

$$P_{0+,y}^{(\gamma',\beta')}\left[P_{0+,x}^{(\gamma,\beta)}\left[x^{\delta-1}y^{\delta'-1}(x^h+c)^{-\eta}(y^{h'}+c')^{-\eta'}S_N^M\left[Zx^r y^{r'}(x^h+c)^{-\eta}(y^{h'}+c')^{-\eta'}\right]\right.\right.$$

$$\left.\left.\daleth\left[z_1x^{h_1}y^{h'_1}(x^h+c)^{-\phi_1}(y^{h'}+c')^{-\phi'_1},\cdots,z_rx^{h_r}(x^h+c)^{-\phi_r}y^{h'_r}(y^{h'}+c')^{-\phi'_r}\right]\right]\right]$$

$$=\frac{x^{\gamma+\delta}}{c^{\eta}[a(1-\beta)]^{\delta}}\Gamma\left(1+\frac{\gamma}{1-\beta}\right)\frac{y^{\gamma'+\delta'}}{c^{\eta'}[a'(1-\beta')]^{\delta'}}\Gamma\left(1+\frac{\gamma'}{1-\beta'}\right)\sum_{K=0}^{[N/M]}A_{N,K}$$

$$\frac{Z^K x^{rK}y^{r'K}}{c^{\eta K}c'^{\eta' K}[a(1-\beta)]^{rK}[a'(1-\beta')]^{r'K}}\daleth_{X;p_{i_r}+4,q_{i_r}+4,\tau_{i_r}:R_r:Y;(0,1);(0,1)}^{U;0,n_r+4:V;(1,0);(1,0)}$$

$$\left(\begin{matrix} \frac{Z_1x^{h_1}y^{h'_1}}{c^{\phi_1}c'^{\phi'_1}[a(1-\beta)]^{h_1}[a'(1-\beta')]^{h'_1}} \\ \vdots \\ \frac{Z_rx^{h_r}y^{h'_r}}{c^{\phi_r}c'^{\phi'_r}[a(1-\beta)]^{h_r}[a'(1-\beta')]^{h'_r}} \\ \frac{x^k}{c[a(1-\beta)]^k} \\ \frac{y^{k'}}{c'[a'(1-\beta')]^{k'}} \end{matrix}\ \middle|\ \begin{matrix} \mathbb{A};(1\text{-}\eta-\eta K;\phi_1,\cdots,\phi_r,1,0;1), \\ \vdots \\ \mathbb{B};\mathbf{B}_2,(1-\eta-\eta K;\phi_1,\cdots,\phi_r,0,0;1), \end{matrix}\right.$$

$$\begin{matrix} (1\text{-}\delta-rK;h_1,\cdots,h_r,k,0;1),(1-\delta'-r'K;h'_1,\cdots,h'_r,0,k';1), \\ \vdots \\ \left(-\delta-\frac{\gamma}{1-\beta}-rK;h_1,\cdots,h_r,k,0;1\right),\left(-\delta'-\frac{\gamma'}{1-\beta'}-r'K;h'_1,\cdots,h'_r,0,k';1\right), \end{matrix}$$

$$\left.\begin{matrix} (1\text{-}\eta'-\eta'K;\phi'_1,\cdots,\phi'_r,0,1;1),\mathbf{A}_2:A \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ (1\text{-}\eta'-\eta'K;\phi'_1,\cdots,\phi'_r,0,0;1):B;(0,1;1);(0,1;1) \end{matrix}\right) \tag{4.1}$$

under the same existence conditions that (3.2)

On suitably specializing the coefficients $A_{N,K}$, $S_N^M(x)$ yields a number of known polynomials, these include the Jacobi polynomials, Laguerre polynomials and others polynomials ([16],p. 158-161).

Let $\beta,\beta'\rightarrow 1$, then $\frac{\gamma}{1-\beta},\frac{\gamma'}{1-\beta'}\rightarrow\infty$, we obtain the following formula

Corollary 2

$$\lim_{(\beta,\beta')\rightarrow(1,1)} P_{0+,y}^{(\gamma',\beta')}\left[P_{0+,x}^{(\gamma,\beta)}\left[x^{\delta-1}y^{\delta'-1}(x^h+c)^{-\eta}(y^{h'}+c')^{-\eta'}S_{N_1,\cdots,N_s}^{M_1,\cdots,M_s}\left[Z_1x^{r_1}y^{r'_1}(x^h+c)^{-\eta_1}(y^{h'}+c')^{-\eta'_1},\cdots,\right.\right.\right.$$

$$\left.\left.\left.Z_sx^{r_s}y^{r'_s}(x^h+c)^{-\eta_s}(x^{h'}+c')^{-\eta'_s}\right]\gimel\left[z_1x^{h_1}y^{h'_1}(x^h+c)^{-\phi_1}(y^{h'}+c')^{-\phi'_1},\cdots,z_rx^{h_r}(x^h+c)^{-\phi_r}y^{h'_r}(y^{h'}+c')^{-\phi'_r}\right]\right]\right]$$

$$=\frac{x^{\gamma+\delta}}{c^{\eta}[a\gamma]^{\delta}}\frac{y^{\gamma'+\delta'}}{c^{\eta'}[a'\gamma']^{\delta'}}\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_s=0}^{[N_s/M_s]}a_1\prod_{j=1}^{s}\frac{Z_j^{K_j}x^{r_jK_j}y^{r'_jK_j}}{c^{\eta_jK_j}c'^{\eta'_jK_j}[a\gamma]^{r_jK_j}[a'\gamma']^{r'_jK_j}}\gimel^{U;0,n_r+4:V;(1,0);(1,0)}_{X;p_{i_r}+4,q_{i_r}+2,\tau_{i_r}:R_r:Y;(0,1);(0,1)}$$

$$\left(\begin{matrix}\frac{Z_1x^{h_1}y^{h'_1}}{c^{\phi_1}c'^{\phi'_1}[a\gamma)]^{h_1}[a'\gamma']^{h'_1}}\\ \cdot\\ \cdot\\ \frac{Z_rx^{h_r}y^{h'_r}}{c^{\phi_r}c'^{\phi'_r}[a\gamma]^{h_r}[a'\gamma']^{h'_r}}\\ \frac{x^k}{c[a\gamma]^k}\\ \frac{y^{k'}}{c'[a'\gamma']^{k'}}\end{matrix}\ \middle|\ \begin{matrix}\mathbb{A};(1\text{-}\eta-\sum_{j=1}^s\eta_jK_j;\phi_1,\cdots,\phi_r,1,0;1), & (1\text{-}\eta'-\sum_{j=1}^s\eta'_jK_j;\phi'_1,\cdots,\phi'_r,0,1;1),\\ \cdot & \cdot\\ \cdot & \cdot\\ \cdot & \cdot\\ \cdot & \cdot\\ \mathbb{B};\mathbf{B}_2,(1-\eta-\sum_{j=1}^s\eta_jK_j;\phi_1,\cdots,\phi_r,0,0;1), & \cdots\end{matrix}\right.$$

$$\left.\begin{matrix}(1\text{-}\delta-\sum_{j=1}^s r_jK_j;h_1,\cdots,h_r,k,0;1),(1-\delta'-\sum_{j=1}^s r'_jK_j;h'_1,\cdots,h'_r,0,k';1),\mathbf{A}_2:A\\ \cdot\\ \cdot\\ \cdot\\ \cdot\\ (1\text{-}\eta'-\sum_{j=1}^s\eta'_jK_j;\phi'_1,\cdots,\phi'_r;0,0;1):B;(0,1;1);(0,1;1)\end{matrix}\right) \tag{4.2}$$

under the same existence conditions that (3.2) with $\beta,\beta'\rightarrow 1$

Remarks

We obtain the same relations about the theorem 1.

We obtain the same formulae (3.1) and (3.2) concerning the multivariable Aleph- function defined by Ayant [1], the multivariable I-function defined by Prathima et al. [10], the multivariable I-function defined by Prasad [9] and the multivariable H-function defined by Srivastava and Panda [14,15]. The last case has been studied by Ghiya et al. [4].

By the similar procedure, the results of this document can be extented to product of any finite number of compositions concerning the Pathway operators.

5. CONCLUSION

In this article we provide the composition formulae of pathway fractional integral operator with generalized polynomial and Gimel-function of r variable which is expressed in Gimel-function of r+1 variables and r+2 variables. Pathway operator is related to pathway model and various fractional integral operators, generalized polynomial and multivariable Gimel-function are general in nature. By specialising the various parameters as well as variables in the generalized multivariable Gimel-function and class of multivariable polynomials, we get a several formulae involving remarkably wide variety of useful functions (or product of such functions) which are expressible in terms of E, F, G, H, I, Aleph-function of one and several variables and simpler special functions of one and several variables. Hence the formulae derived in this paper are most general in character and may prove to be useful in several interesting cases appearing in literature of Pure and Applied Mathematics and Mathematical Physics.

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