Level Operators on Generalized Intuitionistic Fuzzy Sets

D.Sadhanaa[1], P.Prabakaran[2]
[1] Department of Mathematics, Sri Vasavi College, Erode, Tamilnadu, India.
[2] Full-Time Research scholar, Department of Mathematics, Sri Vasavi College, Erode, Tamilnadu, India.

Abstract

In this paper, we have introduced new level operators over generalized intuitionistic fuzzy sets.Some of the basic properties of the new level operators are discussed.

Keywords: *Fuzzy set, Intuitionistic Fuzzy set, Generalized Intuitionistic Fuzzy set, Level operators.*

I. INTRODUCTION

The concept of fuzzy setwas introduced by Lotfi.A.Zadeh[1].It is an extension of the classical sets. In [2] and [3], Atanassov introduced the concept of intuitionistic fuzzy set(IFS), using a degree of membership and a degree of non-membership, under the constraint that the sum of the two degrees does not exceed one. Modal operators, topological operators, level operators, negation operators and aggregation operators are different groups of operators over the IFS due to Atanassov[2]. Atanassov[4] defined level operators $P_{\alpha,\beta}$ and $Q_{\alpha,\beta}$ over IFS. In 2008, Atanassov[5] studied some relations between intuitionistic fuzzy negations and intuitionistic fuzzy level operators $P_{\alpha,\beta}$ and $Q_{\alpha,\beta}$ .Parvathi and Geetha[6] defined some level operators, max-min implication operators and $P_{\alpha,\beta}$ and $Q_{\alpha,\beta}$ operators on temporal intuitionistic fuzzy sets.In [7], T.K.Mondal and S.K.Samanta introduced the concept of generalized intuitionistic fuzzy set(GIFS) $M = \{\langle x, \mu_M(x), \nu_M(x)\rangle : x \in E\}$ where $\mu_M: E \to [0,1]$ and $\nu_M: E \to [0,1]$ which satisfies the condition $\mu_M(x) \wedge \nu_M(x) \leq 0.5, \forall\, x \in E$. E.BalouiJamkhaneh and Nadarajah[8] defined an extension of generalized intuitionistic fuzzy set. In 2017, BalouiJamkhaneh[9] defined level operators $P_{\alpha,\beta}$ and $Q_{\alpha,\beta}$ over GIFS.BalouiJamkhaneh and NadiGhara[10] defined four new level operators over GIFS and established some of their properties.In this paper, we introduce new level operators $P^*_{\alpha,\beta}$ and $Q^*_{\alpha,\beta}$ over GIFS. In section 2, we give some basic definitions and in section 3. We derive the properties of the operators $P^*_{\alpha,\beta}$ and $Q^*_{\alpha,\beta}$.

II. PRELIMINARIES

Definition 2.1 [1] Let E be an universal set.A Fuzzy Set M in E is defined by,

$$M = \{\langle x, \mu_M(x)\rangle : x \in E\}$$

Where the function $\mu_M: E \to [0,1]$ defines the degree of membership function of the element $x \in E$ and satisfying $0 \leq \mu_M(x) \leq 1$ for each $x \in E$.

Definition 2.2 [2] Let E be a non-empty set. An Intuitionistic Fuzzy set M in E is defined by,

$$M = \{\langle x, \mu_M(x), \nu_M(x)\rangle : x \in E\},$$

Where $\mu_M: E \to [0,1]$ and $\nu_M: E \to [0,1]$ define the degree of membership and degree of non-membership functions of the element $x \in E$ respectively andsatisfying $0 \leq \mu_M(x) + \nu_M(x) \leq 1$.

Definition 2.3 [8] Let E be a non-empty set. A Generalized Intuitionistic Fuzzy Set (GIFS) Min E, is defined by

$$M = \{\langle x, \mu_M(x), \nu_M(x)\rangle : x \in E\}$$

Where the functions $\mu_M: E \to [0,1]$ and $\nu_M: E \to [0,1]$ define the degree of membership and degree of non-membership functions of the element $x \in E$ respectively and satisfying $0 \leq \mu_M(x)^\delta + \nu_M(x)^\delta \leq 1$where $\delta = n \text{ or } \frac{1}{n}$, n is a natural number greater than 0.

Definition 2.4 [8] Let M and N be two GIFS such that $M = \{\langle x, \mu_M(x), \nu_M(x)\rangle : x \in E\}$ and $N = \{\langle x, \mu_N(x), \nu_N(x)\rangle : x \in E\}$,define the following relations and operations on M and N

(i) $M \subset N$ *if and only if* $\mu_M(x) \leq \mu_N(x)$ *and* $\nu_M(x) \geq \nu_N(x), \forall x \in E,$
(ii) $M = N$ *if and only if* $\mu_M(x) = \mu_N(x)$ *and* $\nu_M(x) = \nu_N(x), \forall x \in E,$
(iii) $M \cup N = \{\langle x, max(\mu_M(x), \mu_N(x)), min(\nu_M(x), \nu_N(x))\rangle : x \in E\},$
(iv) $M \cap N = \{\langle x, min(\mu_M(x), \mu_N(x)), max(\nu_M(x), \nu_N(x))\rangle : x \in E\},$

(v) $M+N=\left\{\langle x,\mu_M(x)^\delta+\mu_N(x)^\delta-\mu_M(x)^\delta.\mu_N(x)^\delta,\nu_M(x)^\delta\nu_N(x)^\delta\rangle : x\in E\right\}$,
(vi) $M.N=\left\{\langle x,\mu_M(x)^\delta.\mu_N(x)^\delta,\nu_M(x)^\delta+\nu_N(x)^\delta-\nu_M(x)^\delta.\nu_N(x)^\delta\rangle : x\in E\right\}$,
(vii) $\bar{M}=\{\langle x,\nu_M(x),\mu_M(x)\rangle : x\in E\}$,
(viii) $M@N=\left\{\langle x,\frac{\mu_M(x)+\mu_N(x)}{2},\frac{\nu_M(x)+\nu_N(x)}{2}\rangle : x\in E\right\}$,
(ix) $M\$N=\left\{\langle x,\sqrt{\mu_M(x).\mu_N(x)},\sqrt{\nu_M(x).\nu_N(x)}\rangle : x\in E\right\}$.

Definition 2.5 Let E be a non-empty set and $M\in GIFS$,as$M=\{\langle x,\mu_M(x),\nu_M(x)\rangle : x\in E\}$.BalouiJamkhaneh and Nadarajah[8] introduced the following operators over $GIFS$:

(i) $\square M=\left\{\langle x,\mu_M(x),\left(1-\mu_M(x)^\delta\right)^{\frac{1}{\delta}}\rangle : x\in E\right\}$*(modal logic: the necessity operator),*

(ii) $\lozenge M=\left\{\langle x,\left(1-\nu_M(x)^\delta\right)^{\frac{1}{\delta}},\nu_M(x)\rangle : x\in E\right\}$*(modal logic: the possibility operator),*

(iii) $C(M)=\{\langle x,K,L\rangle : x\in E\}$, where $K=max_{y\in E}\,\mu_M(y)$, $L=min_{y\in E}\,\nu_M(y)$,

(iv) $I(M)=\{\langle x,k,l\rangle : x\in E\}$, where$k=min_{y\in E}\,\mu_M(y)$,$l=max_{y\in E}\,\nu_M(y)$.

Definition 2.6 [10] For any $GIFS, M$ defined as $M=\{\langle x,\mu_M(x),\nu_M(x)\rangle : x\in E\}$ and $\alpha,\beta\in[0,1]$. Baloui Jamkhaneh introduced the following level operators

(i) $P_{\alpha,\beta}(M)=\left\{\langle x,max\left(\alpha^{\frac{1}{\delta}},\mu_M(x)\right),min\left(\beta^{\frac{1}{\delta}},\nu_M(x)\right)\rangle : x\in E\right\}$,

(ii) $Q_{\alpha,\beta}(M)=\left\{\langle x,min\left(\alpha^{\frac{1}{\delta}},\mu_M(x)\right),max\left(\beta^{\frac{1}{\delta}},\nu_M(x)\right)\rangle : x\in E\right\}$.

Here$\alpha+\beta\leq 1$ and $\delta=n$ or $\frac{1}{n}$, where n is a natural number greater than 0.

III. MAIN RESULTS

In this section, we will introduce two new level operators over $GIFS$ and discuss its properties.

Definition 3.1 Letting $\alpha,\beta\in[0,1]$ and$\alpha+\beta\leq 1$. For every $GIFS$ as $M=\{\langle x,\mu_M(x),\nu_M(x)\rangle : x\in E\}$,

We define the level operators as follows

(i) $P^*_{\alpha,\beta}(M)=\left\{\langle x,max\left((1-\alpha)^{\frac{1}{\delta}},\mu_M(x)\right),min\left((1-\beta)^{\frac{1}{\delta}},\nu_M(x)\right)\rangle : x\in E\right\}$,

(ii) $Q^*_{\alpha,\beta}(M)=\left\{\langle x,min\left((1-\alpha)^{\frac{1}{\delta}},\mu_M(x)\right),max\left((1-\beta)^{\frac{1}{\delta}},\nu_M(x)\right)\rangle : x\in E\right\}$.

Definition 3.2 Let M and N be two $GIFS$ in a universal set E such that$N=\{\langle x,\mu_M(x),\nu_M(x)\rangle : x\in E\}andN=\{\langle x,\mu_N(x),\nu_N(x)\rangle : x\in E\}$, we define the following relations and operations on M and N

$(i). P^*_{\alpha,\beta}(M)\subseteq P^*_{\alpha,\beta}(N) iff max\left((1-\alpha)^{\frac{1}{\delta}},\mu_M(x)\right)\leq max\left((1-\alpha)^{\frac{1}{\delta}},\mu_N(x)\right)$and

$$min\left((1-\beta)^{\frac{1}{\delta}},\nu_M(x)\right)\geq min\left((1-\beta)^{\frac{1}{\delta}},\nu_N(x)\right),\qquad \forall\ x\in E,$$

$(ii). P^*_{\alpha,\beta}(M)=P^*_{\alpha,\beta}(N) iff max\left((1-\alpha)^{\frac{1}{\delta}},\mu_M(x)\right)=max\left((1-\alpha)^{\frac{1}{\delta}},\mu_N(x)\right)$and

$$min\left((1-\beta)^{\frac{1}{\delta}},\nu_M(x)\right)=min\left((1-\beta)^{\frac{1}{\delta}},\nu_N(x)\right),\qquad \forall\ x\in E,$$

$(iii). P^*_{\alpha,\beta}(M\cap N)=\{\langle x,max\left((1-\alpha)^{\frac{1}{\delta}},min(\mu_M(x),\mu_N(x))\right),$

$$min\left((1-\beta)^{\frac{1}{\delta}},max(\nu_M(x),\nu_N(x))\right)\rangle : x\in E\},$$

$(iv). P^*_{\alpha,\beta}(M\cap N)=\{\langle x,min\left((1-\alpha)^{\frac{1}{\delta}},min(\mu_M(x),\mu_N(x))\right),$

$$max\left((1-\beta)^{\frac{1}{\delta}},max(\nu_M(x),\nu_N(x))\right)\rangle : x\in E\},$$

$(v). P^*_{\alpha,\beta}(C(M)) = \left\{\langle x, max\left((1-\alpha)^{\frac{1}{\delta}}, \max_{y\in E} \mu_M(y)\right), min\left((1-\beta)^{\frac{1}{\delta}}, \min_{y\in E} \nu_M(y)\right)\rangle : x \in E\right\},$

$(vi). P^*_{\alpha,\beta}(I(M)) = \left\{\langle x, max\left((1-\alpha)^{\frac{1}{\delta}}, \min_{y\in E} \mu_M(y)\right), min\left((1-\beta)^{\frac{1}{\delta}}, \max_{y\in E} \nu_M(y)\right)\rangle : x \in E\right\},$

$(vii). P^*_{\alpha,\beta}(\overline{M}) = \left\{\langle x, max\left((1-\alpha)^{\frac{1}{\delta}}, \nu_M(x)\right), min\left((1-\beta)^{\frac{1}{\delta}}, \mu_M(x)\right)\rangle : x \in E\right\}.$

Theorem 3.1 For every $M, N \in GIFS$ and $\alpha, \beta \in [0,1]$, where $\alpha + \beta \leq 1$, we have

(i) $\overline{P^*_{\alpha,\beta}(\overline{M})} = Q^*_{\alpha,\beta}(M),$
(ii) $P^*_{\alpha,\beta}(M \cap N) = P^*_{\alpha,\beta}(M) \cap P^*_{\alpha,\beta}(N),$
(iii) $P^*_{\alpha,\beta}(M \cup N) = P^*_{\alpha,\beta}(M) \cup P^*_{\alpha,\beta}(N),$
(iv) $Q^*_{\alpha,\beta}(M \cap N) = Q^*_{\alpha,\beta}(M) \cap Q^*_{\alpha,\beta}(N),$
(v) $Q^*_{\alpha,\beta}(M \cup N) = Q^*_{\alpha,\beta}(M) \cup Q^*_{\alpha,\beta}(N).$

Proof:

(i) Consider, $P^*_{\alpha,\beta}(\overline{M}) = \left\{\langle x, max\left((1-\alpha)^{\frac{1}{\delta}}, \nu_M(x)\right), min\left((1-\beta)^{\frac{1}{\delta}}, \mu_M(x)\right)\rangle : x \in E\right\}$

$\overline{P^*_{\alpha,\beta}(\overline{M})} = \left\{\langle x, min\left((1-\beta)^{\frac{1}{\delta}}, \mu_M(x)\right), max\left((1-\alpha)^{\frac{1}{\delta}}, \nu_M(x)\right)\rangle : x \in E\right\}$ ……….(I)

$Q^*_{\beta,\alpha}(M) = \left\{\langle x, min\left((1-\beta)^{\frac{1}{\delta}}, \mu_M(x)\right), max\left((1-\alpha)^{\frac{1}{\delta}}, \nu_M(x)\right)\rangle : x \in E\right\}$……(II)

From (I) and (II), we have $\overline{P^*_{\alpha,\beta}(\overline{M})} = Q^*_{\beta,\alpha}(M)$.

(ii) Consider,

$$P^*_{\alpha,\beta}(M \cap N) = \{\langle x, max\left((1-\alpha)^{\frac{1}{\delta}}, min(\mu_M(x), \mu_N(x))\right), min\left((1-\beta)^{\frac{1}{\delta}}, max(\nu_M(x), \nu_N(x))\right)\rangle: x \in E\}$$

$$= \{\langle x, min\left(max\left((1-\alpha)^{\frac{1}{\delta}}, \mu_M(x)\right), max\left((1-\alpha)^{\frac{1}{\delta}}, \mu_N(x)\right)\right),$$

$$max\left(min\left((1-\beta)^{\frac{1}{\delta}}, \nu_M(x)\right), min\left((1-\beta)^{\frac{1}{\delta}}, \nu_N(x)\right)\right)\rangle: x \in E\}$$

$$= \{\langle x, max\left((1-\alpha)^{\frac{1}{\delta}}, \mu_M(x)\right), min\left((1-\beta)^{\frac{1}{\delta}}, \nu_M(x)\right)\rangle : x \in E\} \cap$$

$$\{\langle x, max\left((1-\alpha)^{\frac{1}{\delta}}, \mu_N(x)\right), min\left((1-\beta)^{\frac{1}{\delta}}, \nu_N(x)\right)\rangle : x \in E\}$$

$$= P^*_{\alpha,\beta}(M) \cap P^*_{\alpha,\beta}(N)$$

Hence $P^*_{\alpha,\beta}(M \cap N) = P^*_{\alpha,\beta}(M) \cap P^*_{\alpha,\beta}(N)$.

(iii) Consider,

$$P^*_{\alpha,\beta}(M \cup N) = \{\langle x, max\left((1-\alpha)^{\frac{1}{\delta}}, max(\mu_M(x), \mu_N(x))\right), min\left((1-\beta)^{\frac{1}{\delta}}, min(\nu_M(x), \nu_N(x))\right)\rangle: x \in E\}$$

$$= \{\langle x, max\left(max\left((1-\alpha)^{\frac{1}{\delta}}, \mu_M(x)\right), max\left((1-\alpha)^{\frac{1}{\delta}}, \mu_N(x)\right)\right),$$

$$min\left(min\left((1-\beta)^{\frac{1}{\delta}}, \nu_M(x)\right), min\left((1-\beta)^{\frac{1}{\delta}}, \nu_N(x)\right)\right)\rangle: x \in E\}$$

$$= \{\langle x, max\left((1-\alpha)^{\frac{1}{\delta}}, \mu_M(x)\right), min\left((1-\alpha)^{\frac{1}{\delta}}, \nu_M(x)\right)\rangle : x \in E\} \cup$$

$$\{\langle x, max\left((1-\alpha)^{\frac{1}{\delta}}, \mu_N(x)\right), min\left((1-\alpha)^{\frac{1}{\delta}}, \nu_N(x)\right)\rangle : x \in E\}$$

$$= P^*_{\alpha,\beta}(M) \cup P^*_{\alpha,\beta}(N)$$

Hence $P^*_{\alpha,\beta}(M \cup N) = P^*_{\alpha,\beta}(M) \cup P^*_{\alpha,\beta}(N)$.

(iv) Consider,

$$Q^*_{\alpha,\beta}(M \cap N) = \{\langle x, min\left((1-\alpha)^{\frac{1}{\delta}}, min(\mu_M(x), \mu_N(x))\right), max\left((1-\beta)^{\frac{1}{\delta}}, max(\nu_M(x), \nu_N(x))\right)\rangle : x \in E\}$$

$$= \{\langle x, min\left(min\left((1-\alpha)^{\frac{1}{\delta}}, \mu_M(x)\right), min\left((1-\alpha)^{\frac{1}{\delta}}, \mu_N(x)\right)\right),$$

$$max\left(max\left((1-\beta)^{\frac{1}{\delta}}, \nu_M(x)\right), max\left((1-\beta)^{\frac{1}{\delta}}, \nu_N(x)\right)\right)\rangle : x \in E\}$$

$$= \{\langle x, min\left((1-\alpha)^{\frac{1}{\delta}}, \mu_M(x)\right), max\left((1-\beta)^{\frac{1}{\delta}}, \nu_N(x)\right)\rangle : x \in E\} \cap$$

$$\{\langle x, min\left((1-\alpha)^{\frac{1}{\delta}}, \mu_T(x)\right), max\left((1-\beta)^{\frac{1}{\delta}}, \nu_T(x)\right)\rangle : x \in E\}$$

$$= Q^*_{\alpha,\beta}(M) \cap Q^*_{\alpha,\beta}(N)$$

Hence $Q^*_{\alpha,\beta}(M \cap N) = Q^*_{\alpha,\beta}(M) \cap Q^*_{\alpha,\beta}(N)$.

(v) Consider,

$$Q^*_{\alpha,\beta}(M \cup N) = \{\langle x, min\left((1-\alpha)^{\frac{1}{\delta}}, max(\mu_M(x), \mu_N(x))\right), max\left((1-\beta)^{\frac{1}{\delta}}, min(\nu_M(x), \nu_N(x))\right)\rangle : x \in E\}$$

$$= \{\langle x, max\left(\min\left((1-\alpha)^{\frac{1}{\delta}}, \mu_M(x)\right), \min\left((1-\alpha)^{\frac{1}{\delta}}, \mu_N(x)\right)\right),$$

$$min\left(max\left((1-\beta)^{\frac{1}{\delta}}, \nu_M(x)\right), max\left((1-\beta)^{\frac{1}{\delta}}, \nu_N(x)\right)\right)\rangle : x \in E\}$$

$$= \{\langle x, min\left((1-\alpha)^{\frac{1}{\delta}}, \mu_M(x)\right), max\left((1-\beta)^{\frac{1}{\delta}}, \nu_M(x)\right)\rangle : x \in E\} \cup$$

$$\{\langle x, min\left((1-\alpha)^{\frac{1}{\delta}}, \mu_N(x)\right), max\left((1-\beta)^{\frac{1}{\delta}}, \nu_N(x)\right)\rangle : x \in E\}$$

$$= Q^*_{\alpha,\beta}(M) \cup Q^*_{\alpha,\beta}(N)$$

Hence $Q^*_{\alpha,\beta}(M \cup N) = Q^*_{\alpha,\beta}(M) \cup Q^*_{\alpha,\beta}(N)$.

Theorem 3.2 For every $M \in GIFS$ and $\alpha, \beta \in [0,1]$, where $\alpha + \beta \leq 1$ we have

(i) $P^*_{\alpha,\beta}(C(M)) = C\left(P^*_{\alpha,\beta}(M)\right)$,

(ii) $P^*_{\alpha,\beta}(I(M)) = I\left(P^*_{\alpha,\beta}(M)\right)$,

(iii) $Q^*_{\alpha,\beta}(C(M)) = C\left(Q^*_{\alpha,\beta}(M)\right)$,

(iv) $Q^*_{\alpha,\beta}(I(M)) = I\left(Q^*_{\alpha,\beta}(M)\right)$.

Proof:

(i) Consider,

$$P^*_{\alpha,\beta}(C(M)) = \left\{\langle x, max\left((1-\alpha)^{\frac{1}{\delta}}, \max_{y\in E} \mu_M(y)\right), min\left((1-\beta)^{\frac{1}{\delta}}, \min_{y\in E} \nu_M(y)\right)\rangle : x \in E\right\}$$

$$= \left\{\langle x, \max_{y\in E}\left(\max((1-\alpha)^{\frac{1}{\delta}}, \mu_M(y))\right), \min_{y\in E}\left(\min((1-\beta)^{\frac{1}{\delta}}, \nu_M(y))\right)\rangle : x \in E\right\}$$

$$= C\left(P^*_{\alpha,\beta}(M)\right)$$

Hence $P^*_{\alpha,\beta}\left(C(M)\right) = C\left(P^*_{\alpha,\beta}(M)\right)$.

(ii) Consider,

$$P^*_{\alpha,\beta}(I(M)) = \left\{\langle x, max\left((1-\alpha)^{\frac{1}{\delta}}, \min_{y\in E}\mu_M(y)\right), min\left((1-\beta)^{\frac{1}{\delta}}, \max_{y\in E}\nu_M(y)\right)\rangle : x \in E\right\}$$

$$= \left\{\langle x, \min_{y\in E}\left(max\left((1-\alpha)^{\frac{1}{\delta}}, \mu_M(y)\right)\right), \max_{y\in E}\left(min\left((1-\beta)^{\frac{1}{\delta}}, \nu_M(y)\right)\right)\rangle : x \in E\right\}$$

$$= I\left(P^*_{\alpha,\beta}(M)\right)$$

Hence $P^*_{\alpha,\beta}\left(I(M)\right) = I\left(P^*_{\alpha,\beta}(M)\right)$.

(iii) Consider,

$$Q^*_{\alpha,\beta}(C(M)) = \left\{\langle x, min\left((1-\alpha)^{\frac{1}{\delta}}, \max_{y\in E}\mu_M(y)\right), max\left((1-\beta)^{\frac{1}{\delta}}, \min_{y\in E}\nu_M(y)\right)\rangle : x \in E\right\}$$

$$= \left\{\langle x, \max_{y\in E}\left(\min((1-\alpha)^{\frac{1}{\delta}}, \mu_M(y))\right), \min_{y\in E}\left(\max((1-\beta)^{\frac{1}{\delta}}, \nu_M(y)\right)\rangle : x \in E\right\}$$

$$= C\left(Q^*_{\alpha,\beta}(M)\right)$$

Hence $Q^*_{\alpha,\beta}\left(C(M)\right) = C\left(Q^*_{\alpha,\beta}(M)\right)$.

(iv) Consider,

$$Q^*_{\alpha,\beta}\left(I(M)\right) = \left\{\langle x, min\left((1-\alpha)^{\frac{1}{\delta}}, \min_{y\in E}\mu_M(y)\right), max\left((1-\beta)^{\frac{1}{\delta}}, \max_{y\in E}\nu_M(y)\right)\rangle : x \in E\right\}$$

$$= \left\{\langle x, \min_{y\in E}\left(\min((1-\alpha)^{\frac{1}{\delta}}, \mu_M(y))\right), \max_{y\in E}\left(\max((1-\beta)^{\frac{1}{\delta}}, \nu_M(y)\right)\rangle : x \in E\right\}$$

$$= I\left(Q^*_{\alpha,\beta}(M)\right)$$

Hence $Q^*_{\alpha,\beta}\left(I(M)\right) = I\left(Q^*_{\alpha,\beta}(M)\right)$.

Theorem 3.3 For every $M \in GIFS$ and $\alpha, \beta \in [0,1]$, where $\alpha + \beta \leq 1$ we have

(i) $P^*_{1-\alpha,1-\beta}(M) = P_{\alpha,\beta}(M)$,
(ii) $Q^*_{1-\alpha,1-\beta}(M) = Q_{\alpha,\beta}(M)$.

Proof:

(i) Consider,

$$P^*_{1-\alpha,1-\beta}(M) = \left\{\langle x, max\left((1-(1-\alpha))^{\frac{1}{\delta}}, \mu_M(x)\right), min\left((1-(1-\beta))^{\frac{1}{\delta}}, \nu_M(x)\right)\rangle : x \in E\right\}$$

$$= \left\{\langle x, max\left(\alpha^{\frac{1}{\delta}}, \mu_M(x)\right), min\left(\beta^{\frac{1}{\delta}}, \nu_M(x)\right)\rangle : x \in E\right\}$$

$$= P_{\alpha,\beta}(M)$$

Hence $P^*_{1-\alpha,1-\beta}(M) = P_{\alpha,\beta}(M)$.

(ii) Consider,

$$Q^*_{1-\alpha,1-\beta}(M) = \left\{\langle x, min\left((1-(1-\alpha))^{\frac{1}{\delta}}, \mu_M(x)\right), max\left((1-(1-\beta))^{\frac{1}{\delta}}, \nu_M(x)\right)\rangle : x \in E\right\}$$

$$= \left\{\langle x, min\left(\alpha^{\frac{1}{\delta}}, \mu_M(x)\right), max\left(\beta^{\frac{1}{\delta}}, \nu_M(x)\right)\rangle : x \in E\right\}$$

$$= Q_{\alpha,\beta}(M)$$

Hence $Q^*_{1-\alpha,1-\beta}(M) = Q_{\alpha,\beta}(M)$.

Theorem 3.4 For every $M, N \in GIFS$ and $\alpha, \beta \in [0,1]$, where $M \subseteq N$ we have

(i) $P^*_{\alpha,\beta}(M) \subseteq P^*_{\alpha,\beta}(N)$,
(ii) $Q^*_{\alpha,\beta}(M) \subseteq Q^*_{\alpha,\beta}(N)$.

Proof:

(i) Since$P^*_{\alpha,\beta}(M) = \left\{\langle x, max\left((1-\alpha)^{\frac{1}{\delta}}, \mu_M(x)\right), min\left((1-\beta)^{\frac{1}{\delta}}, \nu_M(x)\right)\rangle : x \in E\right\}$
Since$M \subseteq N$ iff$\mu_M(x) \le \mu_N(x)$and$\nu_M(x) \ge \nu_N(x)$.

Therefore $max\left((1-\alpha)^{\frac{1}{\delta}}, \mu_M(x)\right) \le max\left((1-\alpha)^{\frac{1}{\delta}}, \mu_N(x)\right)$ and

$$min\left((1-\beta)^{\frac{1}{\delta}}, \nu_M(x)\right) \ge min\left((1-\beta)^{\frac{1}{\delta}}, \nu_N(x)\right)$$

We have $P^*_{\alpha,\beta}(M) \subseteq P^*_{\alpha,\beta}(N)$.

(ii) Since$Q^*_{\alpha,\beta}(M) = \left\{\langle x, min\left((1-\alpha)^{\frac{1}{\delta}}, \mu_M(x)\right), max\left((1-\beta)^{\frac{1}{\delta}}, \nu_M(x)\right)\rangle : x \in E\right\}$
Since$M \subseteq N$iff$\mu_M(x) \le \mu_N(x)$and$\nu_M(x) \ge \nu_N(x)$.

Therefore $min\left((1-\alpha)^{\frac{1}{\delta}}, \mu_M(x)\right) \le min\left((1-\alpha)^{\frac{1}{\delta}}, \mu_N(x)\right)$and

$$max\left((1-\beta)^{\frac{1}{\delta}}, \nu_M(x)\right) \ge max\left((1-\beta)^{\frac{1}{\delta}}, \nu_N(x)\right)$$

We have $Q^*_{\alpha,\beta}(M) \subseteq Q^*_{\alpha,\beta}(N)$.

IV. CONCLUSION

In this paper, we have introduced two new level operators over GIFS and established some of their properties. My future study will be on level operators on GIFS and their applications(such asmedical diagnosis, pattern recognition, image processing, decision making problems and so on) in various fields. This paves way for further research.

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