A study on Energy of Certain Graphs using MATLAB Program

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Abstract *- The Energy E(G)of a graph G is the sum of the absolute value of eigenvalues of its adjacency matrix. In this paper, we have obtained the energy of Cartesian product of paths and cycles. We have also obtained the energy of complete bipartite graph, shell graph and double star using MATLAB program.*

Keywords - *MATLAB program, Cartesian product, energy of graph, shell graph, double star graph.*

I. INTRODUCTION

In this paper simple, finite and undirected graphs only are considered. For notations and terminology, we follow Douglas B.West[2].

The energy of a graph was first defined by Ivan Gutman [5] in 1978. Gutman [3] briefly outlined the connection between the energy of a graph and the total π- electron energy of organic molecules. He also presented some fundamental results on energy, the relation between energy E(G) of the graph G and the characteristic polynomial of G, lower and upper bounds for energy E which depending on number of vertices, edges, the graph extremal with respect to energy and n- vertex graphs for which $E(G) \geq E(K_n)$. H.Liu et al [6] found the two sharp upper bounds for the energy of G in terms of number of vertices, edges, sum of degrees of the vertices adjacent to v_i and sum of the 2-degree of vertices adjacent to v_i. He also obtained the sharp bound for the energy of a forest from which some known results of trees can be improved. Gopalapillai et al [4] obtained bounds for the distance spectral radius and D-energy of graphs of diameter 2. They also constructed pairs of equiregular D-equienergetic graphs of diameter 2. C.Adiga et al [1] introduced the concept of maximum degree energy $E_M(G)$ of a graph G and obtained the bounds for $E_M(G)$. They also proved that the maximum degree energy of certain graphs are less than the maximum energy of complete graph. Mohammadreza Jooyandeh et al [7] defined the incidence energy $IE(G)$ of graph G as the sum of the singular value of its incidence matrix. They also obtained a bipartite graph Ĝ for any graph G such that $IE(G) = E(\hat{G})/2$. They proved that the incidence energy of the graph is greater than the energy of its proper subgraph in the same paper. Sangeeta Gupta et al [8] investigated the MATLAB program to find the energy of complete, pan, cycle path cycle graphs of order n, n≥10. This paper motivated us to find the energy of certain graphs using MATLAB program.

The energy of the graph can be obtained in two ways using MATLAB program.

i. By giving the entries of adjacency matrix of the graph.
ii. By giving the adjacency of the vertices.

A shell graph $C_{(n,n-3)}$ is defined as a cycle C_n with (n-3) chords sharing a common end point called the apex.

The double star graph $K_{1,n,n}$ is a tree obtained from the star $K_{1,n}$ by adding a new pendent edge of the existing n pendent vertices. It has 2n+1 vertices and 2n edges.

The cartesian graph product $G=G_1\times G_2$ of graph G_1 and G_2 with disjoint vertex sets V_1 and V_2 and edge set X_1 and X_2, is the graph with vertex set $V_1 \times V_2$ and $u=(u_1,u_2)$ adjacent with $v=(v_1,v_2)$ whenever $u_1=v_1$ and u_2 is adjacent to v_2 in G_2 or $u_2=v_2$ and u_1 adjacent to v_1 in G_1.

The energy E(G) of a graph G is the sum of absolute values of the eigenvalues of G. That is,

$$E(G) = \sum_{i=1}^{n} |\lambda_i|$$

Where $\lambda_1, \lambda_2, \ldots \lambda_n$ are the eigenvalues for the adjacency matrix of the graph.

II. ENERGY OF $P_2 \times P_N$

This section deals with the energy of $P_2\times P_n$ for any values of n using MATLAB program

A. MATLAB program to generate the energy of general $P_2 \times P_n$

```
% 'A' is the adjacency matrix of a graph
% 'K' is the eigenvalue of the matrix
% 'E' is the energy of the graph
m= input (' Enter the number of vertices:');
A= Zeros(m);
for i=1:m/2
A(i,i+(m/2))=1;
A(i+(m/2),i)=1;
end
for i=1:(m/2)-1
A(i,i+1)=1;
A(i+1,i)=1;
end
for i=(m/2)+1:m-1
A(i,i+1)=1;
A(i+1,i)=1;
end
A
K=eig(A);
E=sum(abs(K))
```

Using the above MATLAB program ,the energy of $P_2 \times P_3$ is obtained in the following example

B. Example

```
% 'A' is the adjacency matrix of a graph
% 'K' is the eigenvalue of the matrix
% 'E' is the energy of the graph
m= input(' Enter the number of the vertices:');
Enter the number of the vertices:
6
A=zeros(6);
for i=1:6/2
A(i,i+(6/2))=1;
A(i+(6/2),i)=1;
end
for i=1:(6/2)-1
A(i,i+1)=1;
A(i+1,i)=1;
end
for i=(6/2)+1:6-1
A(i,i+1)=1
A(i+1,i)=1
end
A
A= 0 1 0 1 0 0
   1 0 1 0 1 0
   0 1 0 0 0 1
   1 0 0 0 1 0
   0 1 0 1 0 1
   0 0 1 0 1 0
K= eig(A);
E=sum(abs(K))
E=7.6569
```

III.ENERGY OF $P_2 \times C_N$

The energy of $P_2 \times C_n$ for $n \geq 3$ using MATLAB program is discussed in this section.

A. MATLAB program to generate the energy of general $P_2 \times P_n$

```
% 'A' is the adjacency matrix of a graph
```

```
% 'K' is the eigenvalue of the matrix
% 'E' is the energy of the graph
m= input(' Enter the number of the vertices');
A=zeros(m);
 for i=1:m/2;
 A(i,i+(m/2))=1;
 A(i+(m/2),i)=1;
end
for i=1:(m/2)-1
A(i,i+1)=1;
A(i+1,i)=1;
end
for i=(m/2)+1:m-1
A(i,i+1)=1;
A(i+1,i)=1;
end
A(1,m/2)=1;
A(m/2,1)=1;
A((m/2)+1,m)=1;
A(m,(m/2)+1)=1;
A
K=eig(A);
E=sum(abs(K))
```

By using the above MATLAB program, the energy of $P_2 \times C_3$ is obtained in the following example.

B. *Example*

```
% 'A' is the adjacency matrix of a graph
% 'K' is the eigenvalue of the matrix
% 'E' is the energy of the graph
m= input(' Enter the number of the vertices');
Enter the number of the vertices:
6
A=zeros(6);
for i=1:6/2
A(i,i+(6/2))=1;
A(i+(6/2),i)=1;
end
for i=1:(6/2)-1
A(i,i+1)=1;
A(i+1,i)=1;
end
for i=(6/2)+1:6-1
A(i,i+1)=1;
A(i+1,i)=1;
end
A(1,6/2)=1;
A(6/2,1)=1;
A((6/2)+1,6)=1;
A(6,(6/2)+1)=1;
A
A= 0 1 1 1 0 0
   1 0 1 0 1 0
   1 1 0 0 0 1
   1 0 0 0 1 1
   0 1 0 1 0 1
   0 0 1 1 1 0
K=eig(A);
E=sum(abs(K))
E=8
```

IV. ENERGY OF COMPLETE BIPARTITE GRAPH $K_{M,N}$

This section deals with the energy of complete bipartite graph using MATLAB program.

A. *MATLAB program to generate the energy of general complete bipartite graph $K_{m,n}$*

```
% ‘A’ is the adjacency matrix of a graph
% ‘K’ is the eigenvalue of the matrix
% ‘E’ is the energy of the graph
p= input (‘ Enter the number of vertices:’);
A=zeros(p);
for j=1:m
for i=1:n
A(j,i+m)=1;
A(i+m,j)=1;
end
end
A
K=eig(A);
E=sum(abs(K))
```

Using the above MATLAB program, the energy of $K_{3,4}$ is obtained in the following example.

B. *Example*

```
% ‘A’ is the adjacency matrix of a graph
% ‘K’ is the eigenvalue of the matrix
% ‘E’ is the energy of the graph
P= input (‘ Enter the number of vertices:’);
Enter the number of vertices:
7
A=zeros(7);
for j=1:3
for i=1:4
A(j,i+3)=1;
A(i+3,j)=1;
end
end
A
A=  0 0 0 1 1 1 1
    0 0 0 1 1 1 1
    0 0 0 1 1 1 1
    1 1 1 0 0 0 0
    1 1 1 0 0 0 0
    1 1 1 0 0 0 0
    1 1 1 0 0 0 0
K=eig(A);
E=sum(abs(K))
E=6.9282
```

V. ENERGY OF SHELL GRAPH $C_{(N,N-3)}$

This section deals with the energy of shell graph using MATLAB program.

A. *MATLAB program to generate the energy of general shell graph$C_{(n,n-3)}$*

```
% ‘v’ be the number of vertices
% ‘e’ be the number of edges
% ‘A’ be the adjacency matrix of the graph
% ‘K’ be the eigenvalue of the matrix
% ‘E’ be the energy of the graph
v=[1:n];
e=[2:n,1];
G=graph(v,e);
for i=1:n-3
```

```
G=addedge(G,1,i+2);
end
A=adjacency(G);
K=eig(A);
E=sum(abs(K))
```

Using the above MATLAB program , the energy of shell graph $C_{(6,6-3)}$ is obtained in the following example.

B. Example

```
% ‘v’ be the number of vertices
% ‘e’ be the number of edges
% ‘A’ be the adjacency matrix of the graph
% ‘K’ be the eigenvalue of the matrix
% ‘E’ be the energy of the graph
v=[1:6];
e=[2:6,1];
G=graph(v,e);
for i=1:6-3
G=addedge(G,1,i+2);
end
A=adjacency(G);
K=eig(A);
E=sum(abs(K))
E=8.6703
```

VI. ENERGY OF DOUBLE STAR GRAPH $K_{1,N,N}$

This section deals with the energy of double star graph using MATLAB program.

A. MATLAB program to generate the energy of double star graph $K_{1,n,n}$

```
% ‘A’ is the adjacency matrix of the graph
% ‘K’ is the eigenvalue of the matrix
% ‘E’ is the energy of the graph
m=input(‘Enter the number of vertices:’);
A=zeros(m);
for i=1:n
A(1,i+1)=1;
A(i+1,1)=1;
A(i+1,i+(n+1))=1;
A(i+(n+1),i+1)=1;
end
A
K=eig(A);
E=sum(abs(K))
```

Using the above MATLAB program, the energy of double star $K_{1,4,4}$ is obtained in the following example.

B. Example

```
% ‘A’ is the adjacency matrix of the graph
% ‘K’ is the eigenvalue of the matrix
% ‘E’ is the energy of the graph
m=input(‘Enter the number of vertices:’);
Enter the number of vertices:
9
A=zeros(9);
For i=1:4
A(1,i+1)=1;
A(i+1,1)=1;
A(i+1,i+(4+1))=1;
A(i+(4+1),i+1)=1;
End
```

```
A
A= 0  1  1  1  1  0  0  0  0
   1  0  0  0  0  1  0  0  0
   1  0  0  0  0  0  1  0  0
   1  0  0  0  0  0  0  1  0
   1  0  0  0  0  0  0  0  1
   0  1  0  0  0  0  0  0  0
   0  0  1  0  0  0  0  0  0
   0  0  0  1  0  0  0  0  0
   0  0  0  0  1  0  0  0  0
K=eig(A);
E=sum(abs(K))
E=10.4721
```

VII. CONCLUSION

In this paper, the energy of Cartesian product of paths, path and cycle, complete bipartite graph, shell graph, double star graph are obtained for any values of n using MATLAB program.

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