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On Different Continuities of Digital Images

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ABSTRACT. In this paper we study properties of different continuities from $Z^n \to Z^n$.

Key words: Khalimsky continuity, digital (k_0, k_1) continuity, (k_0, k_1) continuity, Khalimsky (k_0, k_1) continuity

1. INTRODUCTION

In this paper we compare several kind of continuities such as $k-(k_0, k_1)$, (k_0, k_1), KD (k_0, k_1) continuities and Khalimsky continuity and digital version of pasting theorem is also established.

2. PRELIMINARIES

Definition 2.1 (Digital (k_0, k_1) continuty)**.** Let (X, k_0) and (Y, k_1) be two discrete topological spaces in Z^{n_0} and Z^{n_1} respectively. A function $f : X \to Y$ is (k_0, k_1) continuous if and only if for every $x_0 \in X$, $\epsilon \in N$ and $N_{k_1}(f(x_1), \epsilon) \subset Y$, there is a $\delta \in N$ such that the corresponding $N_{k_0}(x_0, \delta) \subset X$ satisfies

$$f(N_{k_0}(x_0, \delta)) \subset N_{k_1}(f(x), \epsilon).$$

Definition 2.2 (Khalimsky continuty)**.** [2] For two spaces $(X, T_X^{n_0})$ and $(Y, T_Y^{n_1})$, a function $f : X \to Y$ is said to be Khalimsky continuous at a point $x \in X$, if f is continuous at the point x with the Khalimsky product topology.

Definition 2.3. [2] Let X_{n_0,k_0} and Y_{n_1,k_1} be two topological spaces with k_i adjacency $i \in \{0, 1\}$ and $x \in X$. A map $f : X \to Y$ is Khalimsky (k_0, k_1) continuous [$KD(k_0, k_1)$ continuous] at the point x if

(1) f is Khalimsky continuous at the point x and

(2) for $N_{k_1}(f(x),1) \subset Y$, there is $N_{k_0}(x,1) \subset X$ such that

$$f(N_{k_0}(x,1)) \subset N_{k_1}(f(x),1).$$

Definition 2.4 ((k_0,k_1) continuty)**.** For two spaces X_{n_0,k_0} and Y_{n_1,k_1}, a function $f : X \to Y$ is said to be (k_0,k_1) continuous at a point $x \in X$ if for any $N^*_{k_1}(f(x),\epsilon) \subset Y$, there is $N^*_{k_0}(x,\delta) \subset X$ such that

$$f(N^*_{k_0}(x,\delta)) \subset N^*_{k_1}(f(x),\epsilon)$$

for some $\epsilon \in N$.

Result 2.5 (Khalimsky (k_0,k_1) continuty)**.** *For two spaces X_{n_0,k_0} and Y_{n_1,k_1}, a function $f : X \to Y$ is said to be K-(k_0,k_1) continuous at a point $x \in X$, if*

(1) f is Khalimsky continuous at the point x and

(2) f is (k_0,k_1) continuous at the point x.

Example 2.6. Consider a map $f : X_{1,2} \to Y_{n,k}$. Then

(1) $K(2,k)$ continuity of f implies KD-$(2,k)$ continuity of f, but the converse does not hold.

(2) None of $(2,k)$ continuity of f and KD-$(2,k)$ continuity of f implies the other.

Consider the map $f : A_{2,4} \to Y_{1,2}$ given by $f(a_1) = 1$ and $f(a_2) = 2$ where $A = \{a_1 = (0,1), a_2 = (1,1)\}$ and $Y = \{1,2\} \subset Z$ The map f is (4,2) continuous but it can not be Khalimsky continuous at the point a_1 because $\{1\} \in T_Y$ and $\{a_1\} \notin T^2_A$.

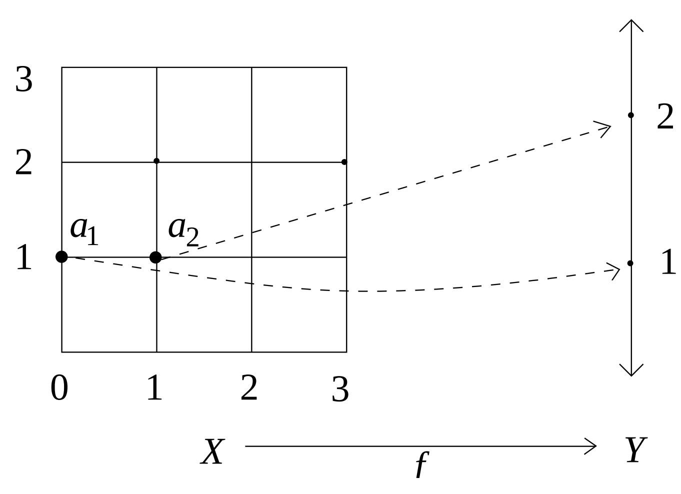

FIGURE 1

Theorem 2.7. *Let $f : X_{n_0,k_0} \to Y_{n_1,k_1}$ be a map $1 \le n_i \le 3$, i.e., $\{0,1\}$. Then K-(k_0,k_1) continuity of f implies KD-(k_0,k_1) continuity of f.*

Proof. Case I. $X_{1,2} \to Y_{1,2}$. Then $K(2,2)$ continuity of f is equivalent to $KD(2,2)$ continuity of f because for any points $x \in X$ and $y \in Y$ we have

$$N_2(x,1) = N_2^*(x,1) \quad \text{and} \quad N_2(y,1) = N_2^*(y,1).$$

Case II. If $f : X_{n,k} \to Y_{1,2}$, $2 \le n \le 3$, for any point $y \in Y$, we have $N_2(y,1) = N_2^*(y,1)$. Hence K-$(k,2)$ continuity of f implies KD-$(k,2)$ continuity of f because the existence of $N_k^*(x,r)$ implies $N_k(x,1) \subset N_k^*(x,r)$.

Case III. If $f : X_{1,2} \to Y_{n,k}$, $2 \le n \le 3$, K-(k_0,k_1) continuity of f implies KD-(k_0,k_1) continuity of f because for any point $x \in X$, $N_2^*(x,1) = N_2(x,1)$ and further for any point $y \in Y$, the existence of $N_k^*(y,s)$ implies $N_k(y,1) \subset N_k^*(y,s)$.

Case IV. $2 \le n_i \le 3$, $i \in \{0,1\}$, we can prove that K-(k_0,k_1) continuity of f implies KD-(k_0,k_1) continuity of f at each point $x_1 \in X$; if not for some point $x_1 \in X$, there exist some $K(k_0,k_1)$ continuous map $f : X_{n_0,k_0} \to Y_{n_1,k_1}$ such that $f(x_2) \notin N_{k_1}(f(x),1)$ where $x_2 \in N_{k_0}(x_1,1)$. Thus by 2.4 and 2.6 by considering a pair of distinct points $x_1, x_2 \in X$ such that x_1 and x_2 are k_0-adjacent, we investigate the following case according to the locations of $x_1 \in X$ and $f(x_1) \in Y$.

a. Assume that the two points $x_1 \in X$ and $f(x_1) \in Y$ are pure open points. Since both $\{x\}$ and $\{f(x_1)\}$ are the smallest open sets containing the points x_1 and $f(x_1)$ respectively, we have $N_{k_0}(x,1) = N_{k_0}^*(x,1)$ and

$$N_{k_1}(f(x),1) = N_{k_1}^*(f(x),1).$$

By the $K(k_0,k_1)$ continuity of f at the point x_1, we have

$$f(N_{k_0}(x_1,1)) = f(N_{k_0}^*(x_1,1)) \subset N_{k_1}^*(f(x_1),1) = N_{k_1}(f(x_1),1)$$

which means that f is $KD-(k_0,k_1)$ continuous map at the point x_1.

b. Assume that x_1 is a pure closed point and $f(x_1)$ is a pure open point. Then owing to the $K(k_0,k_1)$ continuity of f at the point x_1, there is a smallest open set containing the point x_1, namely 0_{x_1} such that $0_{x_1} \subset N_{k_0}^*(x,r)$ for some $r \in N$. According to the k_0-adjacency of X_{n_0,k_0} and further $N_{k_1}^*(f(x_1),1) = N_{k_1}(f(x_1),1)$ because $\{f(x_1)\} \in T_Y^{n_1}$. Due to the $K(k_0,k_1)$ continuity of f at the point x_1 and $f(0_{x_1}) = \{f(x_1)\}$ and $f(N_{k_0}^*(x,r)) = N_{k_1}^*(f(x_1),1)$. Thus $f(x_1)$ and $f(x_2)$ are equal to each other or k_1-adjacent, where $x_2 \in N_{k_0}(x_1,1)$. Therefore

$$f(N_{k_0}(x_1, 1)) \subset f(N^*_{k_0}(x_1, r)) \subset N^*_{k_1}(f(x_1), 1) = N_{k_1}(f(x_1), 1)$$

for any k_0 adjacency of X_{n_0,k_0} which implies that f is $KD(k_0, k_1)$ continuous at the point x.

c. Assume that x_1 is a mixed point and $f(x_1)$ is a pure open point. Due to the K-(k_0, k_1) continuity of f, there is $N^*_{k_0}(x_1, r)$ such that

$$f(N_{k_0}(x_1, 1)) \subset f(N^*_{k_0}(x_1, r)) \subset N^*_{k_1}(f(x_1), 1) = N_{k_1}(f(x_1), 1)$$

because $\{f(x_1)\} \in T^{n_1}_Y$ where the number r is given by Definition 2.4. Thus the map f is KD-(k_0, k_1) continuous at the point x_1 .

d. Assume that x_1 is a pure open point and $f(x_1)$ is a pure closed point. By the hypothesis of $K(k_0, k_1)$ continuity of f at the point x_1, take $N^*_{k_1}(f(x_1), s) \subset Y$ where s is given in 2.4. Since the singleton $\{x_1\}$ is an open set, there is $N^*_{k_0}(x_1, 1) = N_{k_0}(x_1, 1) \subset X$ such that

$$f(N^*_{k_0}(x_1, 1)) \subset N^*_{k_1}(f(x_1), s).$$

Hence K-(k_0, k_1) continuity of f at any point $x_1 \in X$ implies KD-(k_0, k_1) continuity of f at any point x_1.

$\square$

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