New Operators over the Generalized Intuitionistic Fuzzy Sets

D.Sadhanaa[1], P.Prabakaran[2]
[1]Department of Mathematics, Sri Vasavi College, Erode, Tamilnadu, India.
[2]Full-Time Research scholar, Department of Mathematics, Sri Vasavi College, Erode, Tamilnadu, India.

Abstract: *In this paper, we have introduced new level operators over generalized intuitionistic fuzzy sets. Some of the basic properties of the new level operators are discussed.*

Keywords: *Fuzzy set, Intuitionistic Fuzzy set, Generalized Intuitionistic Fuzzy set, Level operators.*

I.INTRODUCTION

The concept of fuzzy set was introduced by Lotfi.A.Zadeh[1]. It is an extension of the classical sets. Many authors extended the idea of fuzzy set in different directions. In [2] and [3], Atanassov introduced the concept of intuitionistic fuzzy set(IFS), using a degree of membership and a degree of non-membership, under the constraint that the sum of the two degrees does not exceed one. IFS is one of the most successful extension of fuzzy set used for handling the uncertainties in the data. Modal operators, topological operators, level operators, negation operators and aggregation operators are different groups of operators over the IFS due to Atanassov[2]. In [4], T.K.Mondal and S.K.Samanta introduced the concept of generalized intuitionistic fuzzy set (GIFS)$S = \{\langle x, \mu_S(x), \nu_S(x)\rangle : x \in E\}$ where $\mu_S: E \to I$ and $\nu_S: E \to I$ satisfy the condition $\mu_S(x) \wedge \nu_S(x) \leq 0.5, \forall\, x \in E$.E.Baloui Jamkhaneh and S.Nadarajah[5] defined anextension of generalized intuitionistic fuzzy set and introduced new operators $D_\alpha, F_{\alpha,\beta}$ and $G_{\alpha,\beta}$ over GIFS.In [6], E.Baloui Jamkhaneh and H.Garg defined some new arithmetic and geometric mean operations over GIFS and found the relation between these operations. E.Baloui Jamkhaneh and S.Nadarajah[7] defined new modal types of operators over an extended generalized intuitionistic fuzzy set and some of the basic properties of the new operators.In this paper, we introduce new operators $G^*_{\alpha,\beta}$ over GIFS. In section 2, we give some basic definitions and in section 3, we introduce the new level operators on GIFS and derive their properties.

II. PRELIMINARIES

Definition 2.1[1]Let E be an universal set.A Fuzzy Set S in a universal set E is defined by,

$$S = \{\langle x, \mu_S(x)\rangle : x \in E\}$$

Where the function $\mu_S: E \to [0,1]$ defines the degree of membership function of the element $x \in E$ and satisfying $0 \leq \mu_S(x) \leq 1$ for each $x \in E$.

Definition 2.2[2]Let E be a non-empty set. An Intuitionistic Fuzzy set S in a universal set E is defined by$S = \{\langle x, \mu_S(x), \nu_S(x)\rangle : x \in E\}$,

Where $\mu_S: E \to [0,1]$ and $\nu_S: E \to [0,1]$ define the degree of membership and degree of non-membership functions of the element $x \in E$ respectively andsatisfying $0 \leq \mu_S(x) + \nu_S(x) \leq 1$.

Definition 2.3[5]Let E be a non-empty set. A Generalized Intuitionistic Fuzzy Set (GIFS) S in E, is defined by$S = \{\langle x, \mu_S(x), \nu_S(x)\rangle : x \in E\}$

Where the functions $\mu_S: E \to [0,1]$ and $\nu_S: E \to [0,1]$ define the degree of membership and degree of non-membership functions of the element $x \in E$ respectively and satisfying $0 \leq \mu_S(x)^\delta + \nu_S(x)^\delta \leq 1$ for each $x \in E$ where $\delta = n\, or\, \frac{1}{n}$, n is a natural number greater than 0.

Definition 2.5[5]Let S and T be two GIFS suchthat $S = \{\langle x, \mu_S(x), \nu_S(x)\rangle : x \in E\}$ and $T = \{\langle x, \mu_T(x), \nu_T(x)\rangle : x \in E\}$,define the following relations and operations on S and T

(i) $S \subset T$ if and only if $\mu_S(x) \leq \mu_T(x)$ and $\nu_S(x) \geq \nu_T(x), \forall x \in E$,

(ii) $S = T$ if and only if $\mu_S(x) = \mu_T(x)$ and $\nu_S(x) = \nu_T(x), \forall x \in E$,

(iii) $S \cup T = \left\{\langle x, \max\left(\mu_S(x), \mu_T(x)\right), \min\left(\nu_S(x), \nu_T(x)\right)\rangle : x \in E\right\}$,

(iv) $S \cap T = \left\{\langle x, \min\left(\mu_S(x), \mu_T(x)\right), \max\left(\nu_S(x), \nu_T(x)\right)\rangle : x \in E\right\}$,

(v) $S + T = \left\{\langle x, \mu_S(x)^\delta + \mu_T(x)^\delta - \mu_S(x)^\delta.\mu_T(x)^\delta, \nu_S(x)^\delta \nu_T(x)^\delta\rangle : x \in E\right\}$,

(vi) $S.T = \left\{\langle x, \mu_S(x)^\delta.\mu_T(x)^\delta, \nu_S(x)^\delta + \nu_T(x)^\delta - \nu_S(x)^\delta.\nu_T(x)^\delta\rangle : x \in E\right\}$,

(vii) $\bar{S} = \{\langle x, \nu_S(x), \mu_S(x)\rangle : x \in E\}$,

(viii) $S@T = \left\{\langle x, \frac{\mu_S(x)+\mu_T(x)}{2}, \frac{\nu_S(x)+\nu_T(x)}{2}\rangle : x \in E\right\}$,

(ix) $S\$T = \{\langle x, \sqrt{\mu_S(x)\mu_T(x)}, \sqrt{\nu_S(x)\nu_T(x)}\rangle : x \in E\}$.

Definition 2.6 Let E be a non-empty finite set, and $S \in GIFS$,as$S = \{\langle x, \mu_S(x), \nu_S(x)\rangle : x \in E\}$.Baloui Jamkhaneh and Nadarajah[5] introduced the following operators over $GIFS$:

(i) $\square S = \left\{\langle x, \mu_S(x), \left(1-\mu_S(x)^{\delta}\right)^{\frac{1}{\delta}}\rangle : x \in E\right\}(modal\ logic: the\ necessity\ measure)$,

(ii) $\lozenge\, S = \left\{\langle x, \left(1-\nu_S(x)^{\delta}\right)^{\frac{1}{\delta}}, \nu_S(x)\rangle : x \in E\right\}(modal\ logic: the\ possibility\ measure)$,

(iii) $C(S) = \{\langle x, K, L\rangle : x \in E\}$, where $K = \max_{y\in E}\mu_S(y)$, $L = \min_{y\in E}\nu_S(y)$

(iv) $I(S) = \{\langle x, k, l\rangle : x \in E\}$, where $k = \min_{y\in E}\mu_S(y), l = \max_{y\in E}\nu_S(y)$

(v) $G_{\alpha,\beta}(S) = \left\{\langle x, \alpha^{\frac{1}{\delta}}\mu_S(x), \beta^{\frac{1}{\delta}}\nu_S(x)\rangle : x \in E\right\}$, where$\alpha + \beta \le 1$.

(vi) $G_{\alpha,\beta}(\square S) = \left\{\langle x, \alpha^{\frac{1}{\delta}}\mu_S(x), \beta^{\frac{1}{\delta}}\left(1-\mu_S(x)^{\delta}\right)^{\frac{1}{\delta}}\rangle : x \in E\right\}$

(vii) $G_{\alpha,\beta}(\lozenge\, S) = \left\{\langle x, \alpha^{\frac{1}{\delta}}\left(1-\nu_S(x)^{\delta}\right)^{\frac{1}{\delta}}, \beta^{\frac{1}{\delta}}\nu_S(x)\rangle : x \in E\right\}$

Remark : [7]For every $GIFSS$, the following results are valid

(i) $\square\square S = \square S$,

(ii) $\lozenge\lozenge\, S = \lozenge\, S$,

(iii) $\square\,\lozenge\, S = \lozenge\, S$,

(iv) $\lozenge\,\square S = \square S$.

III. MAIN RESULTS

In this section, we will introduce new level operators over $GIFS$and discuss its properties.

Definition 3.1 Letting $\alpha, \beta \in [0,1]$, where $\alpha + \beta \le 1$ and $S \in GIFS$, we define the operator as follows

$$G^*_{\alpha,\beta}(S) = \left\{\langle x, (1-\alpha)^{\frac{1}{\delta}}\mu_S(x), (1-\beta)^{\frac{1}{\delta}}\nu_S(x)\rangle : x \in E\right\}$$

Definition 3.2 Let E be a non-empty set. Let S and T be two $GIFS$ in E, we define the following relations and operations on S and T

$(i).\, G^*_{\alpha,\beta}(S) \subset G^*_{\alpha,\beta}(T) \Rightarrow$iff $(1-\alpha)^{\frac{1}{\delta}}\mu_S(x) \le (1-\alpha)^{\frac{1}{\delta}}\mu_T(x)$ and $(1-\beta)^{\frac{1}{\delta}}\nu_S(x) \ge (1-\beta)^{\frac{1}{\delta}}\nu_T(x)\ \ \forall\, x \in E$,

$(ii).\, G^*_{\alpha,\beta}(S) = G^*_{\alpha,\beta}(T)$iff $(1-\alpha)^{\frac{1}{\delta}}\mu_S(x) = (1-\alpha)^{\frac{1}{\delta}}\mu_T(x)$ and $(1-\beta)^{\frac{1}{\delta}}\nu_S(x) = (1-\beta)^{\frac{1}{\delta}}\nu_T(x) \forall\, x \in E$,

$(iii).\, G^*_{\alpha,\beta}(\bar{S}) = \left\{\langle x, (1-\alpha)^{\frac{1}{\delta}}\nu_S(x), (1-\beta)^{\frac{1}{\delta}}\mu_S(x)\rangle : x \in E\right\}$

$(iv).\, G^*_{\alpha,\beta}(S \cup T) = \left\{\langle x, (1-\alpha)^{\frac{1}{\delta}}\max\left(\mu_S(x), \mu_T(x)\right), (1-\beta)^{\frac{1}{\delta}}\min\left(\nu_S(x), \nu_T(x)\right)\rangle : x \in E\right\}$

$(v).\, G^*_{\alpha,\beta}(S \cap T) = \left\{\langle x, (1-\alpha)^{\frac{1}{\delta}}\min\left(\mu_S(x), \mu_T(x)\right), (1-\beta)^{\frac{1}{\delta}}\max(\nu_S(x), \nu_T(x))\rangle : x \in E\right\}$

$(vi).\, G^*_{\alpha,\beta}(S@T) = \left\{\langle x, (1-\alpha)^{\frac{1}{\delta}}\left(\frac{\mu_S(x)+\mu_T(x)}{2}\right), (1-\beta)^{\frac{1}{\delta}}\left(\frac{\nu_S(x)+\nu_T(x)}{2}\right)\rangle : x \in E\right\}$

$(vii).\, G^*_{\alpha,\beta}(S\$T) = \left\{\langle x, (1-\alpha)^{\frac{1}{\delta}}\sqrt{\mu_S(x)\mu_T(x)}, (1-\beta)^{\frac{1}{\delta}}\sqrt{\nu_S(x)\nu_T(x)}\rangle : x \in E\right\}$

$(viii).\, G^*_{\alpha,\beta}\left(C(S)\right) = \left\{\langle x, (1-\alpha)^{\frac{1}{\delta}}\, max_{y\in E}\,\mu_S(y), (1-\beta)^{\frac{1}{\delta}}\, min_{y\in E}\,\nu_S(y)\rangle : x \in E\right\}$

$(ix).\, G^*_{\alpha,\beta}\left(I(S)\right) = \left\{\langle x, (1-\alpha)^{\frac{1}{\delta}}\min_{y\in E}\mu_S(y), (1-\beta)^{\frac{1}{\delta}}\max_{y\in E}\nu_S(y)\rangle : x \in E\right\}$

Theorem 3.1 For every $S, T \in GIFS$ and $\alpha, \beta \in [0,1]$, where $\alpha + \beta \le 1$, we have

(i) $G^*_{\alpha,\beta}(S \cup T) = G^*_{\alpha,\beta}(S) \cup G^*_{\alpha,\beta}(T)$,

(ii) $G^*_{\alpha,\beta}(S \cap T) = G^*_{\alpha,\beta}(S) \cap G^*_{\alpha,\beta}(T)$,

(iii) $G^*_{\alpha,\beta}(S@T) = G^*_{\alpha,\beta}(S)@G^*_{\alpha,\beta}(T)$.

Proof:

(i) $G^*_{\alpha,\beta}(S \cup T) = \left\{\langle x, (1-\alpha)^{\frac{1}{\delta}}\, max\left(\mu_S(x), \mu_T(x)\right), (1-\beta)^{\frac{1}{\delta}}\, min\left(\nu_S(x), \nu_T(x)\right)\rangle : x \in E\right\}$

$$= \{\langle x, max\left((1-\alpha)^{\frac{1}{\delta}}\mu_S(x), (1-\alpha)^{\frac{1}{\delta}}\mu_T(x)\right), min\left((1-\beta)^{\frac{1}{\delta}}\nu_S(x), (1-\beta)^{\frac{1}{\delta}}\nu_T(x)\right)\rangle : x \in E\}$$

$$= \left\{\langle x, (1-\alpha)^{\frac{1}{\delta}}\mu_S(x), (1-\beta)^{\frac{1}{\delta}}\nu_S(x)\rangle : x \in E\right\} \cup \left\{\langle x, (1-\alpha)^{\frac{1}{\delta}}\mu_T(x), (1-\beta)^{\frac{1}{\delta}}\nu_T(x)\rangle : x \in E\right\}$$

$$= G^*_{\alpha,\beta}(S) \cup G^*_{\alpha,\beta}(T)$$

Hence $G^*_{\alpha,\beta}(S \cup T) = G^*_{\alpha,\beta}(S) \cup G^*_{\alpha,\beta}(T)$.

(ii) $G^*_{\alpha,\beta}(S \cap T) = \left\{ \langle x, (1-\alpha)^{\frac{1}{\delta}} \min(\mu_S(x), \mu_T(x)), (1-\beta)^{\frac{1}{\delta}} \max(\nu_S(x), \nu_T(x)) \rangle : x \in E \right\}$

$= \{\langle x, \min\left((1-\alpha)^{\frac{1}{\delta}}\mu_S(x), (1-\alpha)^{\frac{1}{\delta}}\mu_T(x) \right), \max\left((1-\beta)^{\frac{1}{\delta}}\nu_S(x), (1-\beta)^{\frac{1}{\delta}}\nu_T(x) \right) \rangle : x \in E\}$

$= \left\{ \langle x, (1-\alpha)^{\frac{1}{\delta}}\mu_S(x), (1-\beta)^{\frac{1}{\delta}}\nu_S(x) \rangle : x \in E \right\} \cap \left\{ \langle x, (1-\alpha)^{\frac{1}{\delta}}\mu_T(x), (1-\beta)^{\frac{1}{\delta}}\nu_T(x) \rangle : x \in E \right\}$

$= G^*_{\alpha,\beta}(S) \cap G^*_{\alpha,\beta}(T)$

Hence $G^*_{\alpha,\beta}(S \cap T) = G^*_{\alpha,\beta}(S) \cap G^*_{\alpha,\beta}(T)$.

(iii) $G^*_{\alpha,\beta}(S@T) = \left\{ \langle x, (1-\alpha)^{\frac{1}{\delta}} \left(\frac{\mu_S(x)+\mu_T(x)}{2} \right), (1-\beta)^{\frac{1}{\delta}} \left(\frac{\nu_S(x)+\nu_T(x)}{2} \right) \rangle : x \in E \right\}$

$$= \left\{ \langle x, \frac{(1-\alpha)^{\frac{1}{\delta}}\mu_S(x) + (1-\alpha)^{\frac{1}{\delta}}\mu_T(x)}{2}, \frac{(1-\beta)^{\frac{1}{\delta}}\nu_S(x) + (1-\beta)^{\frac{1}{\delta}}\nu_T(x)}{2} \rangle : x \in E \right\}$$

$$= \left\{ \langle x, (1-\alpha)^{\frac{1}{\delta}}\mu_S(x), (1-\beta)^{\frac{1}{\delta}}\nu_S(x) \rangle : x \in E \right\} @ \left\{ \langle x, (1-\alpha)^{\frac{1}{\delta}}\mu_T(x), (1-\beta)^{\frac{1}{\delta}}\nu_T(x) \rangle : x \in E \right\}$$

$= G^*_{\alpha,\beta}(S) @ G^*_{\alpha,\beta}(T)$

Hence $G^*_{\alpha,\beta}(S@T) = G^*_{\alpha,\beta}(S) @ G^*_{\alpha,\beta}(T)$. □

Theorem 3.2 For every $S \in GIFS$ and $\alpha, \beta \in [0,1]$, where $\alpha + \beta \leq 1$, we have

(i) $G^*_{\alpha,\beta}(C(S)) = C\left(G^*_{\alpha,\beta}(S)\right)$,

(ii) $G^*_{\alpha,\beta}(I(S)) = I\left(G^*_{\alpha,\beta}(S)\right)$,

(iii) $\overline{G^*_{\alpha,\beta}(\bar{S})} = G^*_{\beta,\alpha}(S)$.

Proof:

(i) $G^*_{\alpha,\beta}(C(S)) = \left\{ \langle x, (1-\alpha)^{\frac{1}{\delta}} max_{y\in E}\, \mu_S(y), (1-\beta)^{\frac{1}{\delta}} min_{y\in E}\, \nu_S(y) \rangle : x \in E \right\}$

$$= \left\{ \langle x, \max_{y\in E}(1-\alpha)^{\frac{1}{\delta}}\mu_S(y), \min_{y\in E}(1-\beta)^{\frac{1}{\delta}}\nu_S(y) \rangle : x \in E \right\} = C(G^*_{\alpha,\beta}(S))$$

Hence$G^*_{\alpha,\beta}(C(S)) = C\left(G^*_{\alpha,\beta}(S)\right)$.

(ii) $G^*_{\alpha,\beta}(I(S)) = \left\{ \langle x, (1-\alpha)^{\frac{1}{\delta}} min_{y\in E}\, \mu_S(y), (1-\beta)^{\frac{1}{\delta}} max_{y\in E}\, \nu_S(y) \rangle : x \in E \right\}$

$$= \left\{ \langle x, \min_{y\in E}(1-\alpha)^{\frac{1}{\delta}}\mu_S(y), \max_{y\in E}(1-\beta)^{\frac{1}{\delta}}\nu_S(y) \rangle : x \in E \right\} = I(G^*_{\alpha,\beta}(S))$$

Hence $G^*_{\alpha,\beta}(I(S)) = I\left(G^*_{\alpha,\beta}(S)\right)$.

(iii) Since $S = \{\langle x, \mu_S(x), \nu_S(x) \rangle : x \in E\}$and $\bar{S} = \{\langle x, \nu_S(x), \mu_S(x) \rangle : x \in E\}$

$$G^*_{\alpha,\beta}(\bar{S}) = \left\{ \langle x, (1-\alpha)^{\frac{1}{\delta}}\nu_S(x), (1-\beta)^{\frac{1}{\delta}}\mu_S(x) \rangle : x \in E \right\}$$

$\overline{G^*_{\alpha,\beta}(\bar{S})} = \left\{ \langle x, (1-\beta)^{\frac{1}{\delta}}\mu_S(x), (1-\alpha)^{\frac{1}{\delta}}\nu_S(x) \rangle : x \in E \right\}$ ....(I)

$G^*_{\beta,\alpha}(S) = \left\{ \langle x, (1-\beta)^{\frac{1}{\delta}}\mu_S(x), (1-\alpha)^{\frac{1}{\delta}}\nu_S(x) \rangle : x \in E \right\}$ ....(II)

From (I) and (II), we have$\overline{G^*_{\alpha,\beta}(\bar{S})} = G^*_{\beta,\alpha}(S)$. □

Theorem 3.3 For every two $GIFSS$ and T, and $\alpha, \beta \in [0,1]$ where $\alpha + \beta \leq 1$, we have

(i) $G^*_{\alpha,\beta}(S@T) = G^*_{\alpha,\beta}(T@S)$,

(ii) $G^*_{\alpha,\beta}(S\$T) = G^*_{\alpha,\beta}(T\$S)$,

(iii) $G^*_{\alpha,\beta}(S@S) = G^*_{\alpha,\beta}(S)$,

(iv) $G^*_{1-\alpha,1-\beta}(S@S) = G_{\alpha,\beta}(S)$.

Proof:

(i) $G^*_{\alpha,\beta}(S@T) = \left\{ \langle x, (1-\alpha)^{\frac{1}{\delta}} \left(\frac{\mu_S(x)+\mu_T(x)}{2} \right), (1-\beta)^{\frac{1}{\delta}} \left(\frac{\nu_S(x)+\nu_T(x)}{2} \right) \rangle : x \in E \right\}$

$$= \left\{ \langle x, (1-\alpha)^{\frac{1}{\delta}} \left(\frac{\mu_T(x)+\mu_S(x)}{2} \right), (1-\beta)^{\frac{1}{\delta}} \left(\frac{\nu_T(x)+\nu_S(x)}{2} \right) \rangle : x \in E \right\} = G^*_{\alpha,\beta}(T@S)$$

Hence $G^*_{\alpha,\beta}(S@T) = G^*_{\alpha,\beta}(T@S)$.

(ii) $G^*_{\alpha,\beta}(S\$T) = \left\{ \langle x, (1-\alpha)^{\frac{1}{\delta}}\sqrt{\mu_S(x)\mu_T(x)}, (1-\beta)^{\frac{1}{\delta}}\sqrt{\nu_S(x)\nu_T(x)} \rangle : x \in E \right\}$

$$= \left\{ \langle x, (1-\alpha)^{\frac{1}{\delta}}\sqrt{\mu_T(x)\mu_S(x)}, (1-\beta)^{\frac{1}{\delta}}\sqrt{\nu_T(x)\nu_S(x)} \rangle : x \in E \right\} = G^*_{\alpha,\beta}(T\$S)$$

Hence $G^*_{\alpha,\beta}(S\$T) = G^*_{\alpha,\beta}(T\$S)$.

(iii) $G^*_{\alpha,\beta}(S@S) = G^*_{\alpha,\beta}\left\{\langle x, \frac{\mu_S(x)+\mu_S(x)}{2}, \frac{\nu_S(x)+\nu_S(x)}{2}\rangle : x \in E\right\}$

$= G^*_{\alpha,\beta}\{\langle x, \mu_S(x), \nu_S(x)\rangle : x \in E\} = \left\{\langle x, (1-\alpha)^{\frac{1}{\delta}}\mu_S(x), (1-\beta)^{\frac{1}{\delta}}\nu_S(x)\rangle : x \in E\right\} = G^*_{\alpha,\beta}(S)$

Hence $G^*_{\alpha,\beta}(S@S) = G^*_{\alpha,\beta}(S)$.

(iv) $G^*_{1-\alpha,1-\beta}(S@S) = G^*_{1-\alpha,1-\beta}\left\{\langle x, \frac{\mu_S(x)+\mu_S(x)}{2}, \frac{\nu_S(x)+\nu_S(x)}{2}\rangle : x \in E\right\}$

$= G^*_{1-\alpha,1-\beta}\{\langle x, \mu_S(x), \nu_S(x)\rangle : x \in E\}$

$= \left\{\langle x, (1-(1-\alpha))^{\frac{1}{\delta}}\mu_S(x), (1-(1-\beta))^{\frac{1}{\delta}}\nu_S(x)\rangle : x \in E\right\}$

$= \left\{\langle x, \alpha^{\frac{1}{\delta}}\mu_S(x), \beta^{\frac{1}{\delta}}\nu_S(x)\rangle : x \in E\right\} = G_{\alpha,\beta}(S)$

Hence $G^*_{1-\alpha,1-\beta}(S@S) = G_{\alpha,\beta}(S)$. □

Theorem 3.4 For every $S, T \in GIFS$ and $\alpha, \beta \in [0,1]$ where $\alpha + \beta \leq 1$, we have

(i) $G^*_{1-\alpha,1-\beta}(\Box\Box S) = G_{\alpha,\beta}(\Box S)$,

(ii) $G^*_{1-\alpha,1-\beta}(\Box \lozenge S) = G_{\alpha,\beta}(\lozenge S)$,

(iii) $G^*_{1-\alpha,1-\beta}(\lozenge\lozenge S) = G_{\alpha,\beta}(\lozenge S)$,

(iv) $G^*_{1-\alpha,1-\beta}(\lozenge \Box S) = G_{\alpha,\beta}(\Box S)$,

(v) $G^*_{1-\alpha,1-\beta}(S)@G^*_{1-\alpha,1-\beta}(T) = G_{\alpha,\beta}(S@T)$.

Proof:

Since $G^*_{\alpha,\beta}(S) = \left\{\langle x, (1-\alpha)^{\frac{1}{\delta}}\mu_S(x), (1-\beta)^{\frac{1}{\delta}}\nu_S(x)\rangle : x \in E\right\}$

(i) $G^*_{1-\alpha,1-\beta}(\Box\Box S) = G^*_{1-\alpha,1-\beta}\Box\left\{\langle x, \mu_S(x), (1-\mu_S(x)^\delta)^{\frac{1}{\delta}}\rangle : x \in E\right\}$

$= G^*_{1-\alpha,1-\beta}\left\{\langle x, \mu_S(x), (1-\mu_S(x)^\delta)^{\frac{1}{\delta}}\rangle : x \in E\right\}$

$= \left\{\langle x, (1-(1-\alpha))^{\frac{1}{\delta}}\mu_S(x), (1-(1-\beta))^{\frac{1}{\delta}}(1-\mu_S(x)^\delta)^{\frac{1}{\delta}}\rangle : x \in E\right\}$

$= \left\{\langle x, \alpha^{\frac{1}{\delta}}\mu_S(x), \beta^{\frac{1}{\delta}}(1-\mu_S(x)^\delta)^{\frac{1}{\delta}}\rangle : x \in E\right\} = G_{\alpha,\beta}(\Box S)$

Therefore $G^*_{1-\alpha,1-\beta}(\Box\Box S) = G_{\alpha,\beta}(\Box S)$.

(ii) $G^*_{1-\alpha,1-\beta}(\Box \lozenge S) = G^*_{1-\alpha,1-\beta}\Box\left\{\langle x, (1-\nu_S(x)^\delta)^{\frac{1}{\delta}}, \nu_S(x)\rangle : x \in E\right\}$

$= G^*_{1-\alpha,1-\beta}\left\{\langle x, (1-\nu_S(x)^\delta)^{\frac{1}{\delta}}, \nu_S(x)\rangle : x \in E\right\}$

$= \left\{\langle x, (1-(1-\alpha))^{\frac{1}{\delta}}(1-\nu_S(x)^\delta)^{\frac{1}{\delta}}, (1-(1-\beta))^{\frac{1}{\delta}}\nu_S(x)\rangle : x \in E\right\}$

$= \left\{\langle x, \alpha^{\frac{1}{\delta}}(1-\nu_S(x)^\delta)^{\frac{1}{\delta}}, \beta^{\frac{1}{\delta}}\nu_S(x)\rangle : x \in E\right\} = G_{\alpha,\beta}(\lozenge S)$

Therefore $G^*_{1-\alpha,1-\beta}(\Box \lozenge S) = G_{\alpha,\beta}(\lozenge S)$.

(iii) $G^*_{1-\alpha,1-\beta}(\lozenge\lozenge S) = G^*_{1-\alpha,1-\beta}\lozenge\left\{\langle x, (1-\nu_S(x)^\delta)^{\frac{1}{\delta}}, \nu_S(x)\rangle : x \in E\right\}$

$= G^*_{1-\alpha,1-\beta}\left\{\langle x, (1-\nu_S(x)^\delta)^{\frac{1}{\delta}}, \nu_S(x)\rangle : x \in E\right\}$

$= \left\{\langle x, (1-(1-\alpha))^{\frac{1}{\delta}}(1-\nu_S(x)^\delta)^{\frac{1}{\delta}}, (1-(1-\beta))^{\frac{1}{\delta}}\nu_S(x)\rangle : x \in E\right\}$

$= \left\{\langle x, \alpha^{\frac{1}{\delta}}(1-\nu_S(x)^\delta)^{\frac{1}{\delta}}, \beta^{\frac{1}{\delta}}\nu_S(x)\rangle : x \in E\right\} = G_{\alpha,\beta}(\lozenge S)$

Therefore $G^*_{1-\alpha,1-\beta}(\lozenge\lozenge S) = G_{\alpha,\beta}(\lozenge S)$.

(iv) $G^*_{1-\alpha,1-\beta}(\lozenge \Box S) = G^*_{1-\alpha,1-\beta}\lozenge\left\{\langle x, \mu_S(x), (1-\mu_S(x)^\delta)^{\frac{1}{\delta}}\rangle : x \in E\right\}$

$= G^*_{1-\alpha,1-\beta}\left\{\langle x, \mu_S(x), (1-\mu_S(x)^\delta)^{\frac{1}{\delta}}\rangle : x \in E\right\}$

$= \left\{\langle x, (1-(1-\alpha))^{\frac{1}{\delta}}\mu_S(x), (1-(1-\beta))^{\frac{1}{\delta}}(1-\mu_S(x)^\delta)^{\frac{1}{\delta}}\rangle : x \in E\right\}$

$= \left\{\langle x, \alpha^{\frac{1}{\delta}}\mu_S(x), \beta^{\frac{1}{\delta}}(1-\mu_S(x)^\delta)^{\frac{1}{\delta}}\rangle : x \in E\right\} = G_{\alpha,\beta}(\Box S)$

Therefore $G^*_{1-\alpha,1-\beta}(\lozenge \Box S) = G_{\alpha,\beta}(\Box S)$.

(v) $G^*_{1-\alpha,1-\beta}(S)@G^*_{1-\alpha,1-\beta}(T)$

$= G^*_{1-\alpha,1-\beta}\{\langle x. \mu_S(x), \nu_S(x)\rangle : x \in E\}@G^*_{1-\alpha,1-\beta}\{\langle x, \mu_T(x), \nu_T(x)\rangle : x \in E\}$

$$= \left\{\langle x, \left(1-(1-\alpha)\right)^{\frac{1}{\delta}}\mu_S(x), \left(1-(1-\beta)\right)^{\frac{1}{\delta}}\nu_S(x)\rangle : x \in E\right\} @$$
$$\left\{\langle x, \left(1-(1-\alpha)\right)^{\frac{1}{\delta}}\mu_T(x), \left(1-(1-\beta)\right)^{\frac{1}{\delta}}\nu_T(x)\rangle : x \in E\right\}$$
$$= \left\{\langle x, \alpha^{\frac{1}{\delta}}\mu_S(x), \beta^{\frac{1}{\delta}}\nu_S(x)\rangle : x \in E\right\} @ \left\{\langle x, \alpha^{\frac{1}{\delta}}\mu_T(x), \beta^{\frac{1}{\delta}}\nu_T(x)\rangle : x \in E\right\}$$
$$= \left\{\langle x, \frac{\alpha^{\frac{1}{\delta}}\mu_S(x) + \alpha^{\frac{1}{\delta}}\mu_T(x)}{2}, \frac{\beta^{\frac{1}{\delta}}\nu_S(x) + \beta^{\frac{1}{\delta}}\nu_T(x)}{2}\rangle : x \in E\right\}$$
$$= \left\{\langle x, \alpha^{\frac{1}{\delta}}\left(\frac{\mu_S(x)+\mu_T(x)}{2}\right), \beta^{\frac{1}{\delta}}\left(\frac{\nu_S(x)+\nu_T(x)}{2}\right)\rangle : x \in E\right\} = G_{\alpha,\beta}(S@T).$$

Therefore $G^*_{1-\alpha,1-\beta}S@G^*_{1-\alpha,1-\beta}T = G_{\alpha,\beta}(S@T)$. □

Theorem 3.5 For every two $GIFSS$ and T and for every $\alpha, \beta \in [0,1]$, where $\alpha + \beta \leq 1$ we have

(i) $\alpha \leq \gamma \Rightarrow G^*_{\gamma,\beta}(S) \subset G^*_{\alpha,\beta}(S)$, where$\beta$ is fixed.

(ii) $\beta \leq \gamma \Rightarrow G^*_{\alpha,\beta}(S) \subset G^*_{\alpha,\gamma}(S)$. where α is fixed.

Proof:

(i) Since $G^*_{\alpha,\beta}(S) = \left\{\langle x, (1-\alpha)^{\frac{1}{\delta}}\mu_S(x), (1-\beta)^{\frac{1}{\delta}}\nu_S(x)\rangle : x \in E\right\}$

$$G^*_{\gamma,\beta}(S) = \left\{\langle x, (1-\gamma)^{\frac{1}{\delta}}\mu_S(x), (1-\beta)^{\frac{1}{\delta}}\nu_S(x)\rangle : x \in E\right\}$$

Since $\alpha \leq \gamma$, $\Rightarrow (1-\alpha)^{\frac{1}{\delta}} \geq (1-\gamma)^{\frac{1}{\delta}}$

We have $(1-\alpha)^{\frac{1}{\delta}}\mu_S(x) \geq (1-\gamma)^{\frac{1}{\delta}}\mu_S(x)$ and so $G^*_{\gamma,\beta}(S) \subset G^*_{\alpha,\beta}(S)$.

(ii) Since $G^*_{\alpha,\gamma}(S) = \left\{\langle x, (1-\alpha)^{\frac{1}{\delta}}\mu_S(x), (1-\gamma)^{\frac{1}{\delta}}\nu_S(x)\rangle : x \in E\right\}$

Since $\beta \leq \gamma \Rightarrow (1-\beta)^{\frac{1}{\delta}} \geq (1-\gamma)^{\frac{1}{\delta}}$

We have $(1-\beta)^{\frac{1}{\delta}}\nu_S(x) \geq (1-\gamma)^{\frac{1}{\delta}}\nu_S(x)$ and so $G^*_{\alpha,\beta}(S) \subset G^*_{\alpha,\gamma}(S)$.□

IV. CONCLUSION

In this paper, we have introduced the new level operator over GIFS and established some of its properties. My future study will be on level operators on GIFS and their applications in various fields. This paves way for further research.

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