

# On a Fractional Differential Operator Involving Multivariable Gimel-Function and Product of General Class of Polynomials

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ABSTRACT

In the present paper we use a fractional differential operator  $D_{k,\alpha,x}^n$  and product of general class of multivariable polynomials, Riemann Zeta function and multivariable Gimel-function. On account of the kernel, due to the general class of multivariable polynomials and multivariable Gimel-function our findings provide interesting unification and extension of a number of results. Some new special cases of our main result are mentioned briefly.

KEYWORDS : Multivariable Gimel-function, multiple integral contours, fractional differential operator, general class of polynomials, Riemann Zeta function.

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## 1. Introduction and preliminaries.

Throughout this paper, let  $\mathbb{C}, \mathbb{R}$  and  $\mathbb{N}$  be set of complex numbers, real numbers and positive integers respectively. Also  $\mathbb{N}_0 = \mathbb{N} \cup \{0\}$ . We define a generalized transcendental function of several complex variables.

$$\mathfrak{J}(z_1, \dots, z_r) = \mathfrak{J}_{p_{i_2, q_{i_2}, \tau_{i_2}}^{0, n_2}; 0, n_3; \dots; 0, n_r; m^{(1)}, n^{(1)}; m^{(2)}, n^{(2)}; \dots; m^{(r)}, n^{(r)}} \left( \begin{matrix} z_1 \\ \cdot \\ \cdot \\ \cdot \\ z_r \end{matrix} \right)$$

$$[(a_{2j}; \alpha_{2j}^{(1)}, \alpha_{2j}^{(2)}; A_{2j})]_{1, n_2}, [\tau_{i_2}(a_{2ji_2}; \alpha_{2ji_2}^{(1)}, \alpha_{2ji_2}^{(2)}; A_{2ji_2})]_{n_2+1, p_{i_2}}; [(a_{3j}; \alpha_{3j}^{(1)}, \alpha_{3j}^{(2)}, \alpha_{3j}^{(3)}; A_{3j})]_{1, n_3},$$

$$[\tau_{i_2}(b_{2ji_2}; \beta_{2ji_2}^{(1)}, \beta_{2ji_2}^{(2)}; B_{2ji_2})]_{1, q_{i_2}};$$

$$[\tau_{i_3}(a_{3ji_3}; \alpha_{3ji_3}^{(1)}, \alpha_{3ji_3}^{(2)}, \alpha_{3ji_3}^{(3)}; A_{3ji_3})]_{n_3+1, p_{i_3}}; \dots; [(a_{rj}; \alpha_{rj}^{(1)}, \dots, \alpha_{rj}^{(r)}; A_{rj})]_{1, n_r},$$

$$[\tau_{i_3}(b_{3ji_3}; \beta_{3ji_3}^{(1)}, \beta_{3ji_3}^{(2)}, \beta_{3ji_3}^{(3)}; B_{3ji_3})]_{1, q_{i_3}}; \dots$$

$$[\tau_{i_r}(a_{rji_r}; \alpha_{rji_r}^{(1)}, \dots, \alpha_{rji_r}^{(r)}; A_{rji_r})]_{n_r+1, p_r} : [(c_j^{(1)}, \gamma_j^{(1)}; C_j^{(1)})]_{1, n^{(1)}}, [\tau_{i^{(1)}}(c_{ji^{(1)}}^{(1)}, \gamma_{ji^{(1)}}^{(1)}; C_{ji^{(1)}}^{(1)})]_{n^{(1)}+1, p_i^{(1)}}]$$

$$[\tau_{i_r}(b_{rji_r}; \beta_{rji_r}^{(1)}, \dots, \beta_{rji_r}^{(r)}; B_{rji_r})]_{1, q_r} : [(d_j^{(1)}, \delta_j^{(1)}; D_j^{(1)})]_{1, m^{(1)}}, [\tau_{i^{(1)}}(d_{ji^{(1)}}^{(1)}, \delta_{ji^{(1)}}^{(1)}; D_{ji^{(1)}}^{(1)})]_{m^{(1)}+1, q_i^{(1)}}]$$

$$\left. \begin{matrix} \dots; [(c_j^{(r)}, \gamma_j^{(r)}; C_j^{(r)})]_{1, m^{(r)}}, [\tau_{i^{(r)}}(c_{ji^{(r)}}^{(r)}, \gamma_{ji^{(r)}}^{(r)}; C_{ji^{(r)}}^{(r)})]_{m^{(r)}+1, p_i^{(r)}}] \\ \dots; [(d_j^{(r)}, \delta_j^{(r)}; D_j^{(r)})]_{1, n^{(r)}}, [\tau_{i^{(r)}}(d_{ji^{(r)}}^{(r)}, \delta_{ji^{(r)}}^{(r)}; D_{ji^{(r)}}^{(r)})]_{n^{(r)}+1, q_i^{(r)}}] \end{matrix} \right)$$

$$= \frac{1}{(2\pi\omega)^r} \int_{L_1} \dots \int_{L_r} \psi(s_1, \dots, s_r) \prod_{k=1}^r \theta_k(s_k) z_k^{s_k} ds_1 \dots ds_r \tag{1.1}$$

with  $\omega = \sqrt{-1}$

$$\psi(s_1, \dots, s_r) = \frac{\prod_{j=1}^{n_2} \Gamma^{A_{2j}} (1 - a_{2j} + \sum_{k=1}^2 \alpha_{2j}^{(k)} s_k)}{\sum_{i_2=1}^{R_2} [\tau_{i_2} \prod_{j=n_2+1}^{p_{i_2}} \Gamma^{A_{2ji_2}} (a_{2ji_2} - \sum_{k=1}^2 \alpha_{2ji_2}^{(k)} s_k) \prod_{j=1}^{q_{i_2}} \Gamma^{B_{2ji_2}} (1 - b_{2ji_2} + \sum_{k=1}^2 \beta_{2ji_2}^{(k)} s_k)]}$$

$$\frac{\prod_{j=1}^{n_3} \Gamma^{A_{3j}} (1 - a_{3j} + \sum_{k=1}^3 \alpha_{3j}^{(k)} s_k)}{\sum_{i_3=1}^{R_3} [\tau_{i_3} \prod_{j=n_3+1}^{p_{i_3}} \Gamma^{A_{3ji_3}} (a_{3ji_3} - \sum_{k=1}^3 \alpha_{3ji_3}^{(k)} s_k) \prod_{j=1}^{q_{i_3}} \Gamma^{B_{3ji_3}} (1 - b_{3ji_3} + \sum_{k=1}^3 \beta_{3ji_3}^{(k)} s_k)]}$$

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$$\frac{\prod_{j=1}^{n_r} \Gamma^{A_{rj}} (1 - a_{rj} + \sum_{k=1}^r \alpha_{rj}^{(k)} s_k)}{\sum_{i_r=1}^{R_r} [\tau_{i_r} \prod_{j=n_r+1}^{p_{i_r}} \Gamma^{A_{rji_r}} (a_{rji_r} - \sum_{k=1}^r \alpha_{rji_r}^{(k)} s_k) \prod_{j=1}^{q_{i_r}} \Gamma^{B_{rji_r}} (1 - b_{rji_r} + \sum_{k=1}^r \beta_{rji_r}^{(k)} s_k)]} \tag{1.2}$$

and

$$\theta_k(s_k) = \frac{\prod_{j=1}^{m^{(k)}} \Gamma^{D_j^{(k)}} (d_j^{(k)} - \delta_j^{(k)} s_k) \prod_{j=1}^{n^{(k)}} \Gamma^{C_j^{(k)}} (1 - c_j^{(k)} + \gamma_j^{(k)} s_k)}{\sum_{i^{(k)}=1}^{R^{(k)}} [\tau_{i^{(k)}} \prod_{j=m^{(k)}+1}^{q_{i^{(k)}}} \Gamma^{D_{ji^{(k)}}^{(k)}} (1 - d_{ji^{(k)}}^{(k)} + \delta_{ji^{(k)}}^{(k)} s_k) \prod_{j=n^{(k)}+1}^{p_{i^{(k)}}} \Gamma^{C_{ji^{(k)}}^{(k)}} (c_{ji^{(k)}}^{(k)} - \gamma_{ji^{(k)}}^{(k)} s_k)]} \tag{1.3}$$

1)  $[(c_j^{(1)}; \gamma_j^{(1)})]_{1, n_1}$  stands for  $(c_1^{(1)}; \gamma_1^{(1)}), \dots, (c_{n_1}^{(1)}; \gamma_{n_1}^{(1)})$ .

2)  $n_2, \dots, n_r, m^{(1)}, n^{(1)}, \dots, m^{(r)}, n^{(r)}, p_{i_2}, q_{i_2}, R_2, \tau_{i_2}, \dots, p_{i_r}, q_{i_r}, R_r, \tau_{i_r}, p_{i^{(r)}}, q_{i^{(r)}}, \tau_{i^{(r)}}, R^{(r)} \in \mathbb{N}$  and verify :

$$0 \leq m_2, \dots, 0 \leq m_r, 0 \leq n_2 \leq p_{i_2}, \dots, 0 \leq n_r \leq p_{i_r}, 0 \leq m^{(1)} \leq q_{i^{(1)}}, \dots, 0 \leq m^{(r)} \leq q_{i^{(r)}}$$

$$0 \leq n^{(1)} \leq p_{i^{(1)}}, \dots, 0 \leq n^{(r)} \leq p_{i^{(r)}}.$$

3)  $\tau_{i_2} (i_2 = 1, \dots, R_2) \in \mathbb{R}^+; \tau_{i_r} \in \mathbb{R}^+ (i_r = 1, \dots, R_r); \tau_{i^{(k)}} \in \mathbb{R}^+ (i = 1, \dots, R^{(k)}), (k = 1, \dots, r)$ .

4)  $\gamma_j^{(k)}, C_j^{(k)} \in \mathbb{R}^+; (j = 1, \dots, n^{(k)}); (k = 1, \dots, r); \delta_j^{(k)}, D_j^{(k)} \in \mathbb{R}^+; (j = 1, \dots, m^{(k)}); (k = 1, \dots, r)$ .

$C_{ji^{(k)}}^{(k)} \in \mathbb{R}^+, (j = m^{(k)} + 1, \dots, p^{(k)}); (k = 1, \dots, r);$

$D_{ji^{(k)}}^{(k)} \in \mathbb{R}^+, (j = n^{(k)} + 1, \dots, q^{(k)}); (k = 1, \dots, r)$ .

$\alpha_{kj}^{(l)}, A_{kj} \in \mathbb{R}^+; (j = 1, \dots, n_k); (k = 2, \dots, r); (l = 1, \dots, k)$ .

$\alpha_{kji_k}^{(l)}, A_{kji_k} \in \mathbb{R}^+; (j = n_k + 1, \dots, p_{i_k}); (k = 2, \dots, r); (l = 1, \dots, k)$ .

$\beta_{kji_k}^{(l)}, B_{kji_k} \in \mathbb{R}^+; (j = m_k + 1, \dots, q_{i_k}); (k = 2, \dots, r); (l = 1, \dots, k)$ .

$\delta_{ji^{(k)}}^{(k)} \in \mathbb{R}^+; (i = 1, \dots, R^{(k)}); (j = m^{(k)} + 1, \dots, q_{i^{(k)}}); (k = 1, \dots, r)$ .

$\gamma_{ji^{(k)}}^{(k)} \in \mathbb{R}^+; (i = 1, \dots, R^{(k)}); (j = n^{(k)} + 1, \dots, p_{i^{(k)}}); (k = 1, \dots, r)$ .

5)  $c_j^{(k)} \in \mathbb{C}; (j = 1, \dots, n^{(k)}); (k = 1, \dots, r); d_j^{(k)} \in \mathbb{C}; (j = 1, \dots, m^{(k)}); (k = 1, \dots, r)$ .

$$a_{kj i_k} \in \mathbb{C}; (j = n_k + 1, \dots, p_{i_k}); (k = 2, \dots, r).$$

$$b_{kj i_k} \in \mathbb{C}; (j = 1, \dots, q_{i_k}); (k = 2, \dots, r).$$

$$d_{j i^{(k)}}^{(k)} \in \mathbb{C}; (i = 1, \dots, R^{(k)}); (j = m^{(k)} + 1, \dots, q_{i^{(k)}}); (k = 1, \dots, r).$$

$$\gamma_{j i^{(k)}}^{(k)} \in \mathbb{C}; (i = 1, \dots, R^{(k)}); (j = n^{(k)} + 1, \dots, p_{i^{(k)}}); (k = 1, \dots, r).$$

The contour  $L_k$  is in the  $s_k (k = 1, \dots, r)$ - plane and run from  $\sigma - i\infty$  to  $\sigma + i\infty$  where  $\sigma$  if is a real number with loop, if necessary to ensure that the poles of  $\Gamma^{A_{2j}} \left( 1 - a_{2j} + \sum_{k=1}^2 \alpha_{2j}^{(k)} s_k \right) (j = 1, \dots, n_2), \Gamma^{A_{3j}} \left( 1 - a_{3j} + \sum_{k=1}^3 \alpha_{3j}^{(k)} s_k \right) (j = 1, \dots, n_3), \dots, \Gamma^{A_{rj}} \left( 1 - a_{rj} + \sum_{i=1}^r \alpha_{rj}^{(i)} \right) (j = 1, \dots, n_r), \Gamma^{C_j^{(k)}} \left( 1 - c_j^{(k)} + \gamma_j^{(k)} s_k \right) (j = 1, \dots, n^{(k)}) (k = 1, \dots, r)$  to the right of the contour  $L_k$  and the poles of  $\Gamma^{D_j^{(k)}} \left( d_j^{(k)} - \delta_j^{(k)} s_k \right) (j = 1, \dots, m^{(k)}) (k = 1, \dots, r)$  lie to the left of the contour  $L_k$ . The condition for absolute convergence of multiple Mellin-Barnes type contour (1.1) can be obtained of the corresponding conditions for multivariable H-function given by as :

$$|arg(z_k)| < \frac{1}{2} A_i^{(k)} \pi \text{ where}$$

$$A_i^{(k)} = \sum_{j=1}^{m^{(k)}} D_j^{(k)} \delta_j^{(k)} + \sum_{j=1}^{n^{(k)}} C_j^{(k)} \gamma_j^{(k)} - \tau_{i^{(k)}} \left( \sum_{j=m^{(k)}+1}^{q_{i^{(k)}}} D_{j i^{(k)}}^{(k)} \delta_{j i^{(k)}}^{(k)} + \sum_{j=n^{(k)}+1}^{p_{i^{(k)}}} C_{j i^{(k)}}^{(k)} \gamma_{j i^{(k)}}^{(k)} \right) - \tau_{i_2} \left( \sum_{j=n_2+1}^{p_{i_2}} A_{2j i_2} \alpha_{2j i_2}^{(k)} + \sum_{j=1}^{q_{i_2}} B_{2j i_2} \beta_{2j i_2}^{(k)} \right) - \dots - \tau_{i_r} \left( \sum_{j=n_r+1}^{p_{i_r}} A_{rj i_r} \alpha_{rj i_r}^{(k)} + \sum_{j=1}^{q_{i_r}} B_{rj i_r} \beta_{rj i_r}^{(k)} \right) \tag{1.4}$$

Following the lines of Braaksma ([2] p. 278), we may establish the the asymptotic expansion in the following convenient form :

$$\aleph(z_1, \dots, z_r) = 0 (|z_1|^{\alpha_1}, \dots, |z_r|^{\alpha_r}), \max(|z_1|, \dots, |z_r|) \rightarrow 0$$

$$\aleph(z_1, \dots, z_r) = 0 (|z_1|^{\beta_1}, \dots, |z_r|^{\beta_r}), \min(|z_1|, \dots, |z_r|) \rightarrow \infty \text{ where } i = 1, \dots, r :$$

$$\alpha_i = \min_{1 \leq j \leq m^{(i)}} \operatorname{Re} \left[ D_j^{(i)} \left( \frac{d_j^{(i)}}{\delta_j^{(i)}} \right) \right] \text{ and } \beta_i = \max_{1 \leq j \leq n^{(i)}} \operatorname{Re} \left[ C_j^{(i)} \left( \frac{c_j^{(i)} - 1}{\gamma_j^{(i)}} \right) \right]$$

**Remark 1.**

If  $n_2 = \dots = n_{r-1} = p_{i_2} = q_{i_2} = \dots = p_{i_{r-1}} = q_{i_{r-1}} = 0$  and  $A_{2j} = A_{2j i_2} = B_{2j i_2} = \dots = A_{rj} = A_{rj i_r} = B_{rj i_r} = 1$   $A_{rj} = A_{rj i_r} = B_{rj i_r} = 1$ , then the multivariable Gimel-function reduces in the multivariable Aleph- function defined by Ayant [1].

**Remark 2.**

If  $n_2 = \dots = n_r = p_{i_2} = q_{i_2} = \dots = p_{i_r} = q_{i_r} = 0$  and  $\tau_{i_2} = \dots = \tau_{i_r} = \tau_{i^{(1)}} = \dots = \tau_{i^{(r)}} = R_2 = \dots = R_r = R^{(1)} = \dots = R^{(r)} = 1$ , then the multivariable Gimel-function reduces in a multivariable I-function defined by Prathima et al. [7].

**Remark 3.**

If  $A_{2j} = A_{2j i_2} = B_{2j i_2} = \dots = A_{rj} = A_{rj i_r} = B_{rj i_r} = 1$  and  $\tau_{i_2} = \dots = \tau_{i_r} = \tau_{i^{(1)}} = \dots = \tau_{i^{(r)}} = R_2 = \dots = R_r = R^{(1)} = \dots = R^{(r)} = 1$ , then the generalized multivariable Gimel-function reduces in multivariable I-function defined by Prasad [6].

**Remark 4.**

If the three above conditions are satisfied at the same time, then the generalized multivariable Gimel-function reduces in the multivariable H-function defined by Srivastava and Panda [10,11].

In your investigation, we shall use the following notations.

$$\mathbb{A} = [(a_{2j}; \alpha_{2j}^{(1)}, \alpha_{2j}^{(2)}; A_{2j})]_{1, n_2}, [\tau_{i_2}(a_{2ji_2}; \alpha_{2ji_2}^{(1)}, \alpha_{2ji_2}^{(2)}; A_{2ji_2})]_{n_2+1, p_{i_2}}, [(a_{3j}; \alpha_{3j}^{(1)}, \alpha_{3j}^{(2)}, \alpha_{3j}^{(3)}; A_{3j})]_{1, n_3},$$

$$[\tau_{i_3}(a_{3ji_3}; \alpha_{3ji_3}^{(1)}, \alpha_{3ji_3}^{(2)}, \alpha_{3ji_3}^{(3)}; A_{3ji_3})]_{n_3+1, p_{i_3}}; \dots; [(a_{(r-1)j}; \alpha_{(r-1)j}^{(1)}, \dots, \alpha_{(r-1)j}^{(r-1)}; A_{(r-1)j})]_{1, n_{r-1}},$$

$$[\tau_{i_{r-1}}(a_{(r-1)ji_{r-1}}; \alpha_{(r-1)ji_{r-1}}^{(1)}, \dots, \alpha_{(r-1)ji_{r-1}}^{(r-1)}; A_{(r-1)ji_{r-1}})]_{n_{r-1}+1, p_{i_{r-1}}} \tag{1.5}$$

$$\mathbf{A} = [(a_{rj}; \alpha_{rj}^{(1)}, \dots, \alpha_{rj}^{(r)}; A_{rj})]_{1, n_r}, [\tau_{i_r}(a_{rji_r}; \alpha_{rji_r}^{(1)}, \dots, \alpha_{rji_r}^{(r)}; A_{rji_r})]_{n+1, p_{i_r}} \tag{1.6}$$

$$A = [(c_j^{(1)}, \gamma_j^{(1)}; C_j^{(1)})]_{1, n^{(1)}}, [\tau_{i^{(1)}}(c_{ji^{(1)}}^{(1)}, \gamma_{ji^{(1)}}^{(1)}; C_{ji^{(1)}}^{(1)})]_{m^{(1)}+1, p_i^{(1)}}; \dots;$$

$$[(c_j^{(r)}, \gamma_j^{(r)}; C_j^{(r)})]_{1, m^{(r)}}, [\tau_{i^{(r)}}(c_{ji^{(r)}}^{(r)}, \gamma_{ji^{(r)}}^{(r)}; C_{ji^{(r)}}^{(r)})]_{m^{(r)}+1, p_i^{(r)}} \tag{1.7}$$

$$\mathbb{B} = [\tau_{i_2}(b_{2ji_2}; \beta_{2ji_2}^{(1)}, \beta_{2ji_2}^{(2)}; B_{2ji_2})]_{1, q_{i_2}}, [\tau_{i_3}(b_{3ji_3}; \beta_{3ji_3}^{(1)}, \beta_{3ji_3}^{(2)}, \beta_{3ji_3}^{(3)}; B_{3ji_3})]_{1, q_{i_3}}; \dots;$$

$$[\tau_{i_{r-1}}(b_{(r-1)ji_{r-1}}; \beta_{(r-1)ji_{r-1}}^{(1)}, \dots, \beta_{(r-1)ji_{r-1}}^{(r-1)}; B_{(r-1)ji_{r-1}})]_{1, q_{i_{r-1}}} \tag{1.8}$$

$$\mathbf{B} = [\tau_{i_r}(b_{rji_r}; \beta_{rji_r}^{(1)}, \dots, \beta_{rji_r}^{(r)}; B_{rji_r})]_{1, q_{i_r}} \tag{1.9}$$

$$B = [(d_j^{(1)}, \delta_j^{(1)}; D_j^{(1)})]_{1, m^{(1)}}, [\tau_{i^{(1)}}(d_{ji^{(1)}}^{(1)}, \delta_{ji^{(1)}}^{(1)}; D_{ji^{(1)}}^{(1)})]_{m^{(1)}+1, q_i^{(1)}}; \dots;$$

$$[(d_j^{(r)}, \delta_j^{(r)}; D_j^{(r)})]_{1, m^{(r)}}, [\tau_{i^{(r)}}(d_{ji^{(r)}}^{(r)}, \delta_{ji^{(r)}}^{(r)}; D_{ji^{(r)}}^{(r)})]_{m^{(r)}+1, q_i^{(r)}} \tag{1.10}$$

$$U = 0, n_2; 0, n_3; \dots; 0, n_{r-1}; V = m^{(1)}, n^{(1)}; m^{(2)}, n^{(2)}; \dots; m^{(r)}, n^{(r)} \tag{1.11}$$

$$X = p_{i_2}, q_{i_2}, \tau_{i_2}; R_2; \dots; p_{i_{r-1}}, q_{i_{r-1}}, \tau_{i_{r-1}}; R_{r-1}; Y = p_{i^{(1)}}, q_{i^{(1)}}, \tau_{i^{(1)}}; R^{(1)}; \dots; p_{i^{(r)}}, q_{i^{(r)}}, \tau_{i^{(r)}}; R^{(r)} \tag{1.12}$$

The generalized polynomials defined by Srivastava ([9], p. 251, Eq. (C.1)), is given in the following manner :

$$S_{N_1, \dots, N_s}^{M_1, \dots, M_s} [y_1, \dots, y_s] = \sum_{K_1=0}^{[N_1/M_1]} \dots \sum_{K_s=0}^{[N_s/M_s]} \frac{(-N_1)_{M_1 K_1}}{K_1!} \dots \frac{(-N_s)_{M_s K_s}}{K_s!} A[N_1, K_1; \dots; N_s, K_s] y_1^{K_1} \dots y_s^{K_s} \tag{1.13}$$

Where  $M_1, \dots, M_s$  are arbitrary positive integers and the coefficients  $A[N_1, K_1; \dots; N_s, K_s]$  are arbitrary constants, real or complex. I we take  $s = 1$  in the (1.13) and denote  $A[N, K]$  thus obtained by  $A_{N, K}$ , we arrive at general class of polynomials  $S_N^M(x)$  study by Srivastava ([8], p. 1, Eq. 1).

We shall note

$$B_1 = \frac{(-N_1)_{M_1 K_1}}{K_1!} \dots \frac{(-N_s)_{M_s K_s}}{K_s!} A[N_1, K_1; \dots; N_s, K_s] \tag{1.14}$$

Goyal and Laddha ([4], p. 99-108, Vol 11(2)) introduced an extension to the generalized Riemann Zeta function defined in the following slightly modified form

$$\phi_h(Z, \tau, d, n) = \sum_{n=0}^{\infty} (h)_n (d+n)^{-\tau} \frac{Z^n}{n!}, h \geq 1, Re(d) > 0, Re(\tau) > 0 \tag{1.15}$$

where

$$Z = yu^v(ax^\mu + b)^\xi, |Z| < 1. \tag{1.16}$$

2. Required results.

We use the fractional derivative operator defined in the following manner :

**Lemma 1.** ([5], Mishra).

$$D_{k,\alpha,x}^n(x^u) = \prod_{t=0}^{n-1} \left[ \frac{\Gamma(u + tk + 1)}{\Gamma(u + tk - \alpha + 1)} \right] x^{u+nk} \tag{2.1}$$

where  $\alpha \neq u + 1$  and  $\alpha$  and  $k$  are not necessarily integers.

We use the binomial expansion in the following manner :

**Lemma 2.**

$$(ax^u + b)^\lambda = b^\lambda \sum_{l=0}^{\infty} \binom{\lambda}{l} \left( \frac{ax^u}{b} \right)^l \tag{2.2}$$

where  $\left| \frac{ax^u}{b} \right| < 1$  and  $\left| \arg \left( \frac{ax^u}{b} \right) \right| < \pi$

3. Main result.

**Theorem.**

$$D_{k,\alpha,x}^n \left[ x^\mu (ax^v + b)^\lambda S_{N_1, \dots, N_s}^{M_1, \dots, M_s} (y_1 x^{u_1} (ax^v + b)^{\eta_1}, \dots, y_s x^{u_s} (ax^v + b)^{\eta_s}) S_{N'_1, \dots, N'_s}^{M'_1, \dots, M'_s} (y'_1 x^{u'_1} (ax^v + b)^{\eta'_1}, \dots, y'_s x^{u'_s} (ax^v + b)^{\eta'_s}) \phi_h(Z, \tau, d, \epsilon) \mathfrak{I} \left( Z_1 x^{\lambda_1} (ax^v + b)^{\sigma_1}, \dots, Z_r x^{\lambda_r} (ax^v + b)^{\sigma_r} \right) \right]$$

$$= \sum_{t_1=1}^{[N_1/M_1]} \dots \sum_{t_s=1}^{[N_s/M_s]} \sum_{t'_1=1}^{[N'_1/M'_1]} \dots \sum_{t'_s=1}^{[N'_s/M'_s]} \sum_{\epsilon,l=0}^{\infty} B_1 B'_1 y_1^{t_1} \dots y_s^{t_s} y'_1{}^{t'_1} \dots y'_s{}^{t'_s} (h)_\epsilon (d + \epsilon)^{-\tau} \left( \frac{a}{b} \right)^l \frac{1}{(2\pi\omega)^r l!}$$

$$b^{\lambda+\epsilon\xi+\sum_{i=1}^s(\eta_i t_i + \eta'_i t'_i)} x^{\mu+vl+\epsilon\epsilon+\sum_{i=1}^s(u_i t_i + u'_i t'_i)+nk}$$

$$\mathfrak{I}_{X;p_i,r+n+1,q_i,r+n+1,\tau_i,r;R_r;Y}^{U;0,n_r+n+1;V} \left( \begin{array}{c} Z_1 b^{\sigma_1} x^{\lambda_1} \\ \vdots \\ Z_r b^{\sigma_r} x^{\lambda_r} \end{array} \middle| \begin{array}{l} \mathbb{A}; (-u-vh-\epsilon\epsilon - tk - \sum_{j=1}^s(u_j t_j + u'_j t'_j); \lambda_1, \dots, \lambda_r; 1)_{t=0,n-1}, \\ \mathbb{B}; \mathbf{B}, (a-u-vh-\epsilon\epsilon - tk - \sum_{j=1}^s(u_j t_j + u'_j t'_j); \lambda_1, \dots, \lambda_r; 1)_{t=0,n-1}, \end{array} \right)$$

$$\left( \begin{array}{c} (-\lambda - \xi\epsilon - \sum_{j=1}^s(\eta_j t_j + \eta'_j t'_j); \sigma_1, \dots, \sigma_r; 1), \mathbf{A} : A \\ \vdots \\ (\lambda - \xi\epsilon - rk - \sum_{j=1}^s(u_j t_j + u'_j t'_j); \lambda_1, \dots, \lambda_r; 1) : B \end{array} \right) \tag{3.1}$$

provided that

$u_j, u'_j, \eta_j, \eta'_j > 0 (j = 1, \dots, s); \lambda_i, \sigma_i > 0 (i = 1, \dots, r), b \neq 0, |\arg(ax^v + b)| < \pi$  and

$$Re(v) + \sum_{i=1}^r \lambda_i \min_{1 \leq j \leq m^{(i)}} Re \left[ D_j^{(i)} \left( \frac{d_j^{(i)}}{\delta_j^{(i)}} \right) \right] > -1.$$

$$|arg(Z_i x^{\lambda_i} (ax^v + b)^{\sigma_i})| < \frac{1}{2} A_i^{(k)} \pi \text{ where } A_i^{(k)} \text{ is defined by (1.4).}$$

**Proof**

We first express the multivariable gmel-function by this multiple integrals contour with the help of (1.1), express the general class of multivariable polynomials occurring on the left-hand side of (3.1) in the series forms with the help of the equation (1.13) and the extension to the generalized Riemann Zeta function by (1.15). Interchange the order of summations and integrations, which is justified under the conditions mentioned above, we have (say I).

$$I = \sum_{t_1=1}^{[N_1/M_1]} \cdots \sum_{t_s=1}^{[N_s/M_s]} \sum_{t'_1=1}^{[N'_1/M'_1]} \cdots \sum_{t'_s=1}^{[N'_s/M'_s]} \sum_{\epsilon, l=0}^{\infty} \frac{1}{(2\pi\omega)^r} \int_{L_1} \cdots \int_{L_r} \psi(s_1, \dots, s_r) \prod_{k=1}^r \theta_k(s_k) z_k^{s_k} \\ D_{k,\alpha,x}^n \left[ x^{\mu+\epsilon s + \sum_{j=1}^s (u_j t_j + u'_j t'_j) + \sum_{i=1}^r \lambda_i s_i} (ax^v + b)^{\lambda + \epsilon \xi + \sum_{j=1}^s (\eta_j t_j + \eta'_j t'_j) + \sum_{i=1}^r \sigma_i s_i} \right] ds_1 \cdots ds_r \tag{3.2}$$

Now we use the lemmae 1 and 2 and we interpret the resulting multiple integrals contour with the help of (1.1) about the gmel-function of r-variables, we obtain the desired theorem 1.

#### 4. Special cases.

Our main results provides unifications and extensions of various results on fractional differential operators. For the sake of illustration, we mention the following special cases.

Taking  $\tau = 1, d = 1, \epsilon = 0$  and  $\sigma_i = 0 (i = 1, \dots, r)$  and  $M'_1 = 1, (s = 1 \text{ in } M'_s, N'_s \text{ and } t'_s), N'_1 = \dots = N'_s = 0, t'_1 = 0$  and  $A'_{0,0} = 1$ , then we obtain the following result

**Corollary 1.**

$$D_{k,\alpha,x}^n \left[ x^\mu (ax^v + b)^\lambda S_{N_1, \dots, N_s}^{M_1, \dots, M_s} (y_1 x^{u_1} (ax^v + b)^{\eta_1}, \dots, y_s x^{u_s} (ax^v + b)^{\eta_s}) \mathfrak{J} (Z_1 x^{\lambda_1}, \dots, Z_r x^{\lambda_r}) \right] \\ = \sum_{t_1=1}^{[N_1/M_1]} \cdots \sum_{t_s=1}^{[N_s/M_s]} \sum_{l=0}^{\infty} B_1 y_1^{t_1} \cdots y_s^{t_s} \left(\frac{a}{b}\right)^l \frac{(\lambda + \sum_{j=1}^s \eta_j t_j)!}{l!(\lambda - l + \sum_{j=1}^s \eta_j t_j)} \\ b^{\lambda + \sum_{i=1}^s \eta_i t_i} x^{\mu + vl + \epsilon c + \sum_{i=1}^s u_i t_i + nk} \\ \mathfrak{J}_{X; p_{i_r} + n, q_{i_r} + n, \tau_{i_r}; R_r; Y}^{U; 0, n_r + n; V} \left( \begin{matrix} Z_1 x^{\lambda_1} & \mathbb{A}; (-u-vh-tk - \sum_{j=1}^s u_j t_j; \lambda_1, \dots, \lambda_r; 1)_{t=0, n-1}, \mathbf{A} : A \\ \vdots & \vdots \\ Z_r x^{\lambda_r} & \mathbb{B}; \mathbf{B}, (a-u-vh-tk - \sum_{j=1}^s u_j t_j; \lambda_1, \dots, \lambda_r; 1)_{t=0, n-1} : B \end{matrix} \right) \tag{4.1}$$

provided that

$$u_j, \eta_j > 0 (j = 1, \dots, s); \lambda_i > 0 (i = 1, \dots, r), b \neq 0, |arg(ax^v + b)| < \pi \text{ and}$$

$$Re(v) + \sum_{i=1}^r \lambda_i \min_{1 \leq j \leq m^{(i)}} Re \left[ D_j^{(i)} \left( \frac{d_j^{(i)}}{\delta_j^{(i)}} \right) \right] > -1.$$

The generalized polynomials reduces to general class of polynomials  $S_N^M(x)$  study by Srivastava ([7], p. 1, Eq. 1) and taking  $\eta_1 = 0$ , we obtain

**Corollary 2 .**

$$\begin{aligned}
 & D_{k,\alpha,x}^n [x^\mu (ax^v + b)^\lambda S_N^M(y_1 x^{u_1}) \mathfrak{J} (Z_1 x^{\lambda_1}, \dots, Z_r x^{\lambda_r})] \\
 &= \sum_{t_1=1}^{[N/M]} \sum_{l=0}^{\infty} \frac{(-N)_{Mt_1}}{t_1!} A[N, t] y^{t_1} \left(\frac{a}{b}\right)^l \frac{(\lambda)!}{l!(\lambda-l)!} b^\lambda x^{\mu+vl+ut_1+nk} \\
 & \mathfrak{J}_{X;p_i r+n, q_i r+n, \tau_i r: R_r: Y}^{U; 0, n_r+n: V} \left( \begin{array}{c|c} Z_1 x^{\lambda_1} & \mathbb{A}; (-u-vh-tk-ut_1; \lambda_1, \dots, \lambda_r; 1)_{t=0, n-1}, \mathbf{A} : A \\ \vdots & \vdots \\ Z_r x^{\lambda_r} & \mathbb{B}; \mathbf{B}, (a-u-vh-tk-ut_1; \lambda_1, \dots, \lambda_r; 1)_{t=0, n-1} : B \end{array} \right) \tag{4.2}
 \end{aligned}$$

provided that

$u, \eta > 0 (j = 1, \dots, s); \lambda_i > 0 (i = 1, \dots, r), b \neq 0, |arg(ax^v + b)| < \pi$  and

$$Re(v) + \sum_{i=1}^r \lambda_i \min_{1 \leq j \leq m^{(i)}} Re \left[ D_j^{(i)} \left( \frac{d_j^{(i)}}{\delta_j^{(i)}} \right) \right] > -1.$$

On specializing these coefficients  $A_{N,K}, S_N^M[.]$  yields a number of known polynomials as special cases. These include, among others, Hermite polynomials, Jacobi polynomials, Laguerre polynomials, Bessel polynomials and several others ([12], p. 158-161).

Take  $A_{N,K} = \frac{(\alpha' + 1)_N (\alpha' + \beta' + N + 1)_j}{(\alpha' + 1)_j}$ , the class of polynomials defined above reduces in Jacobi polynomials ([12], p. 158-161).

Take  $A_{N,K} = \frac{\Gamma(\alpha' + 1 + uN) N!}{(\alpha' + 1)_j \Gamma(1 + \alpha' + uN)}$ , the class of polynomials defined above reduces in Konhauser polynomials ([12], p. 158-161).

We get interesting results involving Jacobi polynomials and Konhauser biorthogonal polynomials form our main results.

**Remarks :**

We obtain easily the same relations with the functions defined in the section 1. O.P. Garg et al. [3] have obtained the same relation with the multivariable H-function.

**5. Conclusion.**

The importance of our all the results lies in their manifold generality. By specialising the various parameters as well as variables in the multivariable Gimel-function, we get several fractional differential operators involving remarkably wide variety of useful functions ( or product of such functions) which are expressible in terms of E, F, G, H, I, Aleph-function of one and several variables and simpler special functions of one and several variables. Hence the formulae derived in this paper are most general in character and may prove to be useful in several interesting cases appearing in literature of Pure and Applied Mathematics and Mathematical Physics. Again the class of multivariable polynomials involved in these integrals and multiplication formulae reduces to a large number of polynomials listed by Srivastava and Singh ([11], p.158-161), therefore, from the integrals we can further obtain various integrals involving a number of simpler polynomials.

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