Images of Multivariable Gimel-Function and Special Functions Pertaining to Multiple Erdélyi-Kober Operator of Weyl Type

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ABSTRACT
The aim in this paper is to establish the images of the product of certain special functions and multivariable Gimel function with $zt^h(t^\mu+c^\mu)^{-\rho}$ as an argument pertaining to the multiple Erdélyi-Saigo operator due to Galué et al. The results encompass several cases of interest for Riemann-Liouville operators, Erdélyi-Kober operator and Saigo operators et cetera involving the product of certain special functions of general arguments.

Keywords :Multivariable Gimel-function, multiple Erdélyi-Kober operator of Weyl type, General class of polynomials.

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1.Introduction and definitions

The multiple Erdélyi-Kober operator of Weyl type introduced by Galué et al. [4] is defined as :

$$K^{(\eta_w),(\zeta_w)}_{(\tau_w),(\lambda_w),r'}f(x)=\int_1^\infty H^{r',0}_{r',r'}\left[\frac{1}{y}\left|\begin{matrix}(\eta_w+\zeta_w+\frac{1}{\tau_w};\frac{1}{\tau_w})_{1,r'}\\(\eta_w+\frac{1}{\lambda_w};\frac{1}{\lambda_w})_{1,r'}\end{matrix}\right.\right]f(xy)\mathrm{d}y,\text{ if }\sum_{w=1}^{r}\zeta_w>0 \tag{1.1}$$

$$=f(x), if \zeta_w=0,\lambda_w=\tau_w,w=1,\cdots,r',\text{ else}$$

where $\sum_{w=1}^{r'}\frac{1}{\lambda_w}\geqslant\sum_{w=1}^{r'}\frac{1}{\tau_w}$ and $f(x)\in C^*_\beta$

The class C^*_β is defined in the form ([3],page.56) .

$$C^*_\beta=\left\{f(x)=x^q\bar{f};q<\beta^*,\bar{f}\in C(0,\infty),|\bar{f}(x)|<A_{\bar{f}}\right\}\text{ and }\beta^*\leqslant max(\lambda_w,\eta_w) \tag{1.2}$$

Galué et al. ([3],p.56) have shown that :

$$K^{(\eta_w),(\zeta_w)}_{(\tau_w),(\lambda_w),r'}x^\rho=\prod_{w=1}^{r'}\frac{\Gamma\left(\eta_w-\frac{\rho}{\lambda_w}\right)}{\Gamma\left(\eta_w+\zeta_w-\frac{\rho}{\lambda_w}\right)}x^\rho \tag{1.3}$$

Srivastava and Garg [8] introduced and defined a general class of multivariable polynomials as follows

$$S_n^{w_1,\cdots,w_s}[x_1,\cdots,x_s]=\sum_{k_1,\cdots,k_s=0}^{w_1k_1+\cdots w_sk_s\leqslant n}(-n)_{w_1k_1+\cdots+w_sk_s}A(n;k_1,\cdots,k_s)\frac{x_1^{k_1}\cdots x_s^{k_s}}{k_1!\cdots k_s!} \tag{1.4}$$

$n,w_1,\cdots,w_s$ are integers and the coefficients $A(n;k_1,\cdots,k_s)$ are arbitrary constants real or complex.

For $s=1$ the polynomials (1.4) reduces to e general class of polynomials due to Srivastava [7]

$$S_n^w(x)=\sum_{k=0}^{[n/w]}\frac{(-n)_{wk}}{k!}A_{n,k}x^k, n\in\mathbb{N} \tag{1.5}$$

where w is an arbitrary positive integer, the coefficients $A_{n,k}$ $(n,k\in\mathbb{N})$ are arbitrary constants real or complex.

a) Since $H_n(x)=\sum_{k=0}^{[n/2]}\frac{(-)^k n!}{k!(n-2k)!}(2x)^{n-2k}$ (1.6)

defines Hermite polynomials therefore in this case, if we take

$$w=2, A_{n,k}=(-)^k, S_n^2(x)\rightarrow x^{n/2}H_n(1/2\sqrt{x}) \tag{1.7}$$

b) On setting $w=1$, $A_{n,k}=\binom{\alpha+n}{n}\frac{(\alpha+\beta+n+1)_k}{(\alpha+1)_k}$, S_n^1 reduces to the Jacobi polynomials $P_n^{(\alpha,\beta)}(1-2x)$

defined by Szegö ([11] ,p. 68, eq . (4.3.2)),

$$P_n^{\alpha,\beta}(x)=\sum_{k=0}^{n}\binom{n+\alpha}{n-k}\binom{n+\beta}{k}\left(\frac{x-1}{2}\right)^k\left(\frac{x+1}{2}\right)^{n-k} \tag{1.8}$$

The following series representation of the H-function given in [3] will be required in the proof.

$$H_{R,S}^{K,L}(z)=H_{R,S}^{K,L}\left[\frac{1}{z}\left|\begin{matrix}(\mathrm{e}_R,E_R)\\(\mathrm{f}_S,F_S)\end{matrix}\right.\right]=\sum_{h=1}^{L}\sum_{v_1=0}^{\infty}\frac{(-)^{v_1}}{v_1!}\frac{\eta(\xi)}{E_h}\left(\frac{1}{z}\right)^{\xi} \tag{1.9}$$

where $\xi=\frac{e_h-1-v_1}{E_h}$, and $h=1,\cdots,L$ (1.10)

we note $\eta(\xi)=\frac{\prod_{j=1}^{K}\Gamma(f_j+\xi F_j)\prod_{j=1,j\neq h}^{L}\Gamma(1-e_j+E_j\xi)}{\prod_{j=K+1}^{S}\Gamma(1-f_j+F_j\xi)\prod_{j=L+1}^{R}\Gamma(e_j+E_j\xi)}$ (1.11)

which exits for $z\neq 0$ if $\mu<0$ and for $|z|>\beta$ if $\mu=0$

$\mu=\sum_{j=1}^{S}F_j-\sum_{j=1}^{R}E_j$ and $\beta=\prod_{j=1}^{R}(E_j)^{E_j}\prod_{j=1}^{S}(F_j)^{-F_j}$

Throughout this paper, let $\mathbb{C},\mathbb{R}$ and $\mathbb{N}$ be set of complex numbers, real numbers and positive integers respectively. Also $\mathbb{N}_0=\mathbb{N}\cup\{0\}$.

We define a generalized transcendental function of several complex variables noted $\gimel$.

$$\gimel(z_1,\cdots,z_r)=\gimel_{p_{i_2},q_{i_2},\tau_{i_2};R_2;p_{i_3},q_{i_3},\tau_{i_3};R_3;\cdots;p_{i_r},q_{i_r},\tau_{i_r}:R_r:p_{i^{(1)}},q_{i^{(1)}},\tau_{i^{(1)}};R^{(1)};\cdots;p_{i^{(r)}},q_{i^{(r)}};\tau_{i^{(r)}};R^{(r)}}^{0,n_2;0,n_3;\cdots;0,n_r:m^{(1)},n^{(1)};m^{(2)},n^{(2)};\cdots;m^{(r)},n^{(r)}}\left(\begin{matrix}\mathrm{z}_1\\ \cdot\\ \cdot\\ \cdot\\ \mathrm{z}_r\end{matrix}\right|$$

$$[(\mathrm{a}_{2j};\alpha_{2j}^{(1)},\alpha_{2j}^{(2)};A_{2j})]_{1,n_2},[\tau_{i_2}(a_{2ji_2};\alpha_{2ji_2}^{(1)},\alpha_{2ji_2}^{(2)};A_{2ji_2})]_{n_2+1,p_{i_2}};[(a_{3j};\alpha_{3j}^{(1)},\alpha_{3j}^{(2)},\alpha_{3j}^{(3)};A_{3j})]_{1,n_3},$$
$$[\tau_{i_2}(b_{2ji_2};\beta_{2ji_2}^{(1)},\beta_{2ji_2}^{(2)};B_{2ji_2})]_{1,q_{i_2}};$$

$$[\tau_{i_3}(a_{3ji_3};\alpha_{3ji_3}^{(1)},\alpha_{3ji_3}^{(2)},\alpha_{3ji_3}^{(3)};A_{3ji_3})]_{n_3+1,p_{i_3}};\cdots;[(\mathrm{a}_{rj};\alpha_{rj}^{(1)},\cdots,\alpha_{rj}^{(r)};A_{rj})_{1,n_r}],$$
$$[\tau_{i_3}(b_{3ji_3};\beta_{3ji_3}^{(1)},\beta_{3ji_3}^{(2)},\beta_{3ji_3}^{(3)};B_{3ji_3})]_{1,q_{i_3}};\cdots;\quad \cdots$$

$$[\tau_{i_r}(a_{rji_r};\alpha_{rji_r}^{(1)},\cdots,\alpha_{rji_r}^{(r)};A_{rji_r})_{n_r+1,p_r}]:\ [(\mathrm{c}_j^{(1)},\gamma_j^{(1)};C_j^{(1)})_{1,n^{(1)}}],[\tau_{i^{(1)}}(c_{ji^{(1)}}^{(1)},\gamma_{ji^{(1)}}^{(1)};C_{ji^{(1)}}^{(1)})_{n^{(1)}+1,p_i^{(1)}}]$$
$$[\tau_{i_r}(b_{rji_r};\beta_{rji_r}^{(1)},\cdots,\beta_{rji_r}^{(r)};B_{rji_r})_{1,q_r}]:\ [(\mathrm{d}_j^{(1)}),\delta_j^{(1)};D_j^{(1)})_{1,m^{(1)}}],[\tau_{i^{(1)}}(d_{ji^{(1)}}^{(1)},\delta_{ji^{(1)}}^{(1)};D_{ji^{(1)}}^{(1)})_{m^{(1)}+1,q_i^{(1)}}]$$

$$\left.\begin{matrix};\cdots;[(c_j^{(r)},\gamma_j^{(r)};C_j^{(r)})_{1,m^{(r)}}],[\tau_{i^{(r)}}(c_{ji^{(r)}}^{(r)},\gamma_{ji^{(r)}}^{(r)};C_{ji^{(r)}}^{(r)})_{m^{(r)}+1,p_i^{(r)}}]\\;\cdots;[(d_j^{(r)},\delta_j^{(r)};D_j^{(r)})_{1,n^{(r)}}],[\tau_{i^{(r)}}(d_{ji^{(r)}}^{(r)},\delta_{ji^{(r)}}^{(r)};D_{ji^{(r)}}^{(r)})_{n^{(r)}+1,q_i^{(r)}}]\end{matrix}\right)$$

$$=\frac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r}\psi(s_1,\cdots,s_r)\prod_{k=1}^{r}\theta_k(s_k)z_k^{s_k}\,\mathrm{d}s_1\cdots\mathrm{d}s_r \tag{1.12}$$

with $\omega=\sqrt{-1}$

$$\psi(s_1,\cdots,s_r)=\frac{\prod_{j=1}^{n_2}\Gamma^{A_{2j}}(1-a_{2j}+\sum_{k=1}^{2}\alpha_{2j}^{(k)}s_k)}{\sum_{i_2=1}^{R_2}[\tau_{i_2}\prod_{j=n_2+1}^{p_{i_2}}\Gamma^{A_{2ji_2}}(a_{2ji_2}-\sum_{k=1}^{2}\alpha_{2ji_2}^{(k)}s_k)\prod_{j=1}^{q_{i_2}}\Gamma^{B_{2ji_2}}(1-b_{2ji2}+\sum_{k=1}^{2}\beta_{2ji2}^{(k)}s_k)]}$$

$$\frac{\prod_{j=1}^{n_3}\Gamma^{A_{3j}}(1-a_{3j}+\sum_{k=1}^{3}\alpha_{3j}^{(k)}s_k)}{\sum_{i_3=1}^{R_3}[\tau_{i_3}\prod_{j=n_3+1}^{p_{i_3}}\Gamma^{A_{3ji_3}}(a_{3ji_3}-\sum_{k=1}^{3}\alpha_{3ji_3}^{(k)}s_k)\prod_{j=1}^{q_{i_3}}\Gamma^{B_{3ji_3}}(1-b_{3ji3}+\sum_{k=1}^{3}\beta_{3ji3}^{(k)}s_k)]}$$

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$$\frac{\prod_{j=1}^{n_r}\Gamma^{A_{rj}}(1-a_{rj}+\sum_{k=1}^{r}\alpha_{rj}^{(k)}s_k)}{\sum_{i_r=1}^{R_r}[\tau_{i_r}\prod_{j=n_r+1}^{p_{i_r}}\Gamma^{A_{rji_r}}(a_{rji_r}-\sum_{k=1}^{r}\alpha_{rji_r}^{(k)}s_k)\prod_{j=1}^{q_{i_r}}\Gamma^{B_{rji_r}}(1-b_{rjir}+\sum_{k=1}^{r}\beta_{rjir}^{(k)}s_k)]} \tag{1.13}$$

and

$$\theta_k(s_k)=\frac{\prod_{j=1}^{m^{(k)}}\Gamma^{D_j^{(k)}}(d_j^{(k)}-\delta_j^{(k)}s_k)\prod_{j=1}^{n^{(k)}}\Gamma^{C_j^{(k)}}(1-c_j^{(k)}+\gamma_j^{(k)}s_k)}{\sum_{i^{(k)}=1}^{R^{(k)}}[\tau_{i^{(k)}}\prod_{j=m^{(k)}+1}^{q_{i^{(k)}}}\Gamma^{D_{ji^{(k)}}^{(k)}}(1-d_{ji^{(k)}}^{(k)}+\delta_{ji^{(k)}}^{(k)}s_k)\prod_{j=n^{(k)}+1}^{p_{i^{(k)}}}\Gamma^{C_{ji^{(k)}}^{(k)}}(c_{ji^{(k)}}^{(k)}-\gamma_{ji^{(k)}}^{(k)}s_k)]} \tag{1.14}$$

1) $[(c_j^{(1)};\gamma_j^{(1)}]_{1,n_1}$ stands for $(c_1^{(1)};\gamma_1^{(1)}),\cdots,(c_{n_1}^{(1)};\gamma_{n_1}^{(1)})$.

2) $n_2,\cdots,n_r,m^{(1)},n^{(1)},\cdots,m^{(r)},n^{(r)},p_{i_2},q_{i_2},R_2,\tau_{i_2},\cdots,p_{i_r},q_{i_r},R_r,\tau_{i_r},p_{i^{(r)}},q_{i^{(r)}},\tau_{i^{(r)}},R^{(r)}\in\mathbb{N}$ and verify :

$0\leqslant m_2,\cdots,0\leqslant m_r,0\leqslant n_2\leqslant p_{i_2},\cdots,0\leqslant n_r\leqslant p_{i_r},0\leqslant m^{(1)}\leqslant q_{i^{(1)}},\cdots,0\leqslant m^{(r)}\leqslant q_{i^{(r)}}$

$0 \leqslant n^{(1)} \leqslant p_{i^{(1)}}, \cdots, 0 \leqslant n^{(r)} \leqslant p_{i^{(r)}}$.

3) $\tau_{i_2}(i_2 = 1, \cdots, R_2) \in \mathbb{R}^+; \tau_{i_r} \in \mathbb{R}^+(i_r = 1, \cdots, R_r); \tau_{i^{(k)}} \in \mathbb{R}^+(i = 1, \cdots, R^{(k)}), (k = 1, \cdots, r)$.

4) $\gamma_j^{(k)}, C_j^{(k)} \in \mathbb{R}^+; (j = 1, \cdots, n^{(k)}); (k = 1, \cdots, r); \delta_j^{(k)}, D_j^{(k)} \in \mathbb{R}^+; (j = 1, \cdots, m^{(k)}); (k = 1, \cdots, r)$.

$\mathrm{C}_{ji^{(k)}}^{(k)} \in \mathbb{R}^+, (j = m^{(k)} + 1, \cdots, p^{(k)}); (k = 1, \cdots, r)$;

$\mathrm{D}_{ji^{(k)}}^{(k)} \in \mathbb{R}^+, (j = n^{(k)} + 1, \cdots, q^{(k)}); (k = 1, \cdots, r)$.

$\alpha_{kj}^{(l)}, A_{kj} \in \mathbb{R}^+; (j = 1, \cdots, n_k); (k = 2, \cdots, r); (l = 1, \cdots, k)$.

$\alpha_{kji_k}^{(l)}, A_{kji_k} \in \mathbb{R}^+; (j = n_k + 1, \cdots, p_{i_k}); (k = 2, \cdots, r); (l = 1, \cdots, k)$.

$\beta_{kji_k}^{(l)}, B_{kji_k} \in \mathbb{R}^+; (j = m_k + 1, \cdots, q_{i_k}); (k = 2, \cdots, r); (l = 1, \cdots, k)$.

$\delta_{ji^{(k)}}^{(k)} \in \mathbb{R}^+; (i = 1, \cdots, R^{(k)}); (j = m^{(k)} + 1, \cdots, q_{i^{(k)}}); (k = 1, \cdots, r)$.

$\gamma_{ji^{(k)}}^{(k)} \in \mathbb{R}^+; (i = 1, \cdots, R^{(k)}); (j = n^{(k)} + 1, \cdots, p_{i^{(k)}}); (k = 1, \cdots, r)$.

5) $c_j^{(k)} \in \mathbb{C}; (j = 1, \cdots, n^{(k)}); (k = 1, \cdots, r); d_j^{(k)} \in \mathbb{C}; (j = 1, \cdots, m^{(k)}); (k = 1, \cdots, r)$.

$a_{kji_k} \in \mathbb{C}; (j = n_k + 1, \cdots, p_{i_k}); (k = 2, \cdots, r)$.

$b_{kji_k} \in \mathbb{C}; (j = 1, \cdots, q_{i_k}); (k = 2, \cdots, r)$.

$d_{ji^{(k)}}^{(k)} \in \mathbb{C}; (i = 1, \cdots, R^{(k)}); (j = m^{(k)} + 1, \cdots, q_{i^{(k)}}); (k = 1, \cdots, r)$.

$\gamma_{ji^{(k)}}^{(k)} \in \mathbb{C}; (i = 1, \cdots, R^{(k)}); (j = n^{(k)} + 1, \cdots, p_{i^{(k)}}); (k = 1, \cdots, r)$.

The contour L_k is in the $s_k(k = 1, \cdots, r)$- plane and run from $\sigma - i\infty$ to $\sigma + i\infty$ where σ if is a real number with loop, if necessary to ensure that the poles of $\Gamma^{A_{2j}}\left(1 - a_{2j} + \sum_{k=1}^{2} \alpha_{2j}^{(k)} s_k\right) (j = 1, \cdots, n_2), \Gamma^{A_{3j}}\left(1 - a_{3j} + \sum_{k=1}^{3} \alpha_{3j}^{(k)} s_k\right)$ $(j = 1, \cdots, n_3), \cdots, \Gamma^{A_{rj}}\left(1 - a_{rj} + \sum_{i=1}^{r} \alpha_{rj}^{(i)}\right) (j = 1, \cdots, n_r)$, $\Gamma^{C_j^{(k)}}\left(1 - c_j^{(k)} + \gamma_j^{(k)} s_k\right) (j = 1, \cdots, n^{(k)})(k = 1, \cdots, r)$to the right of the contour L_k and the poles of $\Gamma^{D_j^{(k)}}\left(d_j^{(k)} - \delta_j^{(k)} s_k\right) (j = 1, \cdots, m^{(k)})(k = 1, \cdots, r)$ lie to the left of the contour L_k. The condition for absolute convergence of multiple Mellin-Barnes type contour (1.1) can be obtained of the corresponding conditions for multivariable H-function given by as :

$|arg(z_k)| < \frac{1}{2} A_i^{(k)} \pi$ where

$$A_i^{(k)} = \sum_{j=1}^{m^{(k)}} D_j^{(k)} \delta_j^{(k)} + \sum_{j=1}^{n^{(k)}} C_j^{(k)} \gamma_j^{(k)} - \tau_{i^{(k)}} \left(\sum_{j=m^{(k)}+1}^{q_i^{(k)}} D_{ji^{(k)}}^{(k)} \delta_{ji^{(k)}}^{(k)} + \sum_{j=n^{(k)}+1}^{p_i^{(k)}} C_{ji^{(k)}}^{(k)} \gamma_{ji^{(k)}}^{(k)} \right) +$$

$$-\tau_{i_2} \left(\sum_{j=n_2+1}^{p_{i_2}} A_{2ji_2} \alpha_{2ji_2}^{(k)} + \sum_{j=1}^{q_{i_2}} B_{2ji_2} \beta_{2ji_2}^{(k)} \right) - \cdots - \tau_{i_r} \left(\sum_{j=n_r+1}^{p_{i_r}} A_{rji_r} \alpha_{rji_r}^{(k)} + \sum_{j=1}^{q_{i_r}} B_{rji_r} \beta_{rji_r}^{(k)} \right) \tag{1.15}$$

Following the lines of Braaksma ([2] p. 278), we may establish the the asymptotic expansion in the following convenient form :

$$\gimel(z_1,\cdots,z_r)=0(\,|z_1|^{\alpha_1},\cdots,|z_r|^{\alpha_r}\,)\,,\quad max(\,|z_1|,\cdots,|z_r|\,)\to 0$$

$$\gimel(z_1,\cdots,z_r)=0(\,|z_1|^{\beta_1},\cdots,|z_r|^{\beta_r}\,)\,,min(\,|z_1|,\cdots,|z_r|\,)\to\infty \text{ where } i=1,\cdots,r:$$

$$\alpha_i=\min_{1\leqslant j\leqslant m^{(i)}} Re\left[D_j^{(i)}\left(\frac{d_j^{(i)}}{\delta_j^{(i)}}\right)\right] \text{ and } \beta_i=\max_{1\leqslant j\leqslant n^{(i)}} Re\left[C_j^{(i)}\left(\frac{c_j^{(i)}-1}{\gamma_j^{(i)}}\right)\right]$$

Remark 1.
If $n_2=\cdots=n_{r-1}=p_{i_2}=q_{i_2}=\cdots=p_{i_{r-1}}=q_{i_{r-1}}=0$ and $A_{2j}=A_{2ji_2}=B_{2ji_2}=\cdots=A_{rj}=A_{rji_r}=B_{rji_r}=1$ $A_{rj}=A_{rji_r}=B_{rji_r}=1$, then the multivariable Gimel-function reduces in the multivariable Aleph- function defined by Ayant [1].

Remark 2.
If $n_2=\cdots=n_r=p_{i_2}=q_{i_2}=\cdots=p_{i_r}=q_{i_r}=0$ and $\tau_{i_2}=\cdots=\tau_{i_r}=\tau_{i^{(1)}}=\cdots=\tau_{i^{(r)}}=R_2=\cdots=R_r=R^{(1)}=\cdots=R^{(r)}=1$, then the multivariable Gimel-function reduces in a multivariable I-function defined by Prathima et al. [6].

Remark 3.
If$A_{2j}=A_{2ji_2}=B_{2ji_2}=\cdots=A_{rj}=A_{rji_r}=B_{rji_r}=1$and $\tau_{i_2}=\cdots=\tau_{i_r}=\tau_{i^{(1)}}=\cdots=\tau_{i^{(r)}}=R_2=\cdots=R_r=R^{(1)}=\cdots=R^{(r)}=1$, then the generalized multivariable Gimel-function reduces in multivariable I-function defined by Prasad [5].

Remark 4.
If the three above conditions are satisfied at the same time, then the generalized multivariable Gimel-function reduces in the multivariable H-function defined by Srivastava and Panda [9,10].

In your investigation, we shall use the following notations.

$$\mathbb{A}=[(\mathrm{a}_{2j};\alpha_{2j}^{(1)},\alpha_{2j}^{(2)};A_{2j})]_{1,n_2},[\tau_{i_2}(a_{2ji_2};\alpha_{2ji_2}^{(1)},\alpha_{2ji_2}^{(2)};A_{2ji_2})]_{n_2+1,p_{i_2}},[(a_{3j};\alpha_{3j}^{(1)},\alpha_{3j}^{(2)},\alpha_{3j}^{(3)};A_{3j})]_{1,n_3},$$

$$[\tau_{i_3}(a_{3ji_3};\alpha_{3ji_3}^{(1)},\alpha_{3ji_3}^{(2)},\alpha_{3ji_3}^{(3)};A_{3ji_3})]_{n_3+1,p_{i_3}};\cdots;[(\mathrm{a}_{(r-1)j};\alpha_{(r-1)j}^{(1)},\cdots,\alpha_{(r-1)j}^{(r-1)};A_{(r-1)j})_{1,n_{r-1}}],$$

$$[\tau_{i_{r-1}}(a_{(r-1)ji_{r-1}};\alpha_{(r-1)ji_{r-1}}^{(1)},\cdots,\alpha_{(r-1)ji_{r-1}}^{(r-1)};A_{(r-1)ji_{r-1}})_{n_{r-1}+1,p_{i_{r-1}}}] \tag{1.16}$$

$$\mathbf{A}=\ [(\mathrm{a}_{rj};\alpha_{rj}^{(1)},\cdots,\alpha_{rj}^{(r)};A_{rj})_{1,n_r}],[\tau_{i_r}(a_{rji_r};\alpha_{rji_r}^{(1)},\cdots,\alpha_{rji_r}^{(r)};A_{rji_r})_{\mathfrak{n}+1,p_{i_r}}] \tag{1.17}$$

$$A=[(\mathrm{c}_j^{(1)},\gamma_j^{(1)};C_j^{(1)})_{1,n^{(1)}}],[\tau_{i^{(1)}}(c_{ji^{(1)}}^{(1)},\gamma_{ji^{(1)}}^{(1)};C_{ji^{(1)}}^{(1)})_{n^{(1)}+1,p_i^{(1)}}];\cdots;$$

$$[(c_j^{(r)},\gamma_j^{(r)};C_j^{(r)})_{1,m^{(r)}}],[\tau_{i^{(r)}}(c_{ji^{(r)}}^{(r)},\gamma_{ji^{(r)}}^{(r)};C_{ji^{(r)}}^{(r)})_{m^{(r)}+1,p_i^{(r)}}] \tag{1.18}$$

$$\mathbb{B}=[\tau_{i_2}(b_{2ji_2};\beta_{2ji_2}^{(1)},\beta_{2ji_2}^{(2)};B_{2ji_2})]_{1,q_{i_2}},[\tau_{i_3}(b_{3ji_3};\beta_{3ji_3}^{(1)},\beta_{3ji_3}^{(2)},\beta_{3ji_3}^{(3)};B_{3ji_3})]_{1,q_{i_3}};\cdots;$$

$$[\tau_{i_{r-1}}(b_{(r-1)ji_{r-1}};\beta_{(r-1)ji_{r-1}}^{(1)},\cdots,\beta_{(r-1)ji_{r-1}}^{(r-1)};B_{(r-1)ji_{r-1}})_{1,q_{i_{r-1}}}] \tag{1.19}$$

$$\mathbf{B}=\ [\tau_{i_r}(b_{rji_r};\beta_{rji_r}^{(1)},\cdots,\beta_{rji_r}^{(r)};B_{rji_r})_{1,q_{i_r}}] \tag{1.20}$$

$$\mathrm{B}=[(\mathrm{d}_j^{(1)},\delta_j^{(1)};D_j^{(1)})_{1,m^{(1)}}],[\tau_{i^{(1)}}(d_{ji^{(1)}}^{(1)},\delta_{ji^{(1)}}^{(1)};D_{ji^{(1)}}^{(1)})_{m^{(1)}+1,q_i^{(1)}}];\cdots;$$

$$[(\mathrm{d}_j^{(r)},\delta_j^{(r)};D_j^{(r)})_{1,m^{(r)}}],[\tau_{i^{(r)}}(d_{ji^{(r)}}^{(r)},\delta_{ji^{(r)}}^{(r)};D_{ji^{(r)}}^{(r)})_{m^{(r)}+1,q_i^{(r)}}] \tag{1.21}$$

$$U = 0, n_2; 0, n_3; \cdots; 0, n_{r-1}; V = m^{(1)}, n^{(1)}; m^{(2)}, n^{(2)}; \cdots; m^{(r)}, n^{(r)} \tag{1.22}$$

$$X = p_{i_2}, q_{i_2}, \tau_{i_2}; R_2; \cdots; p_{i_{r-1}}, q_{i_{r-1}}, \tau_{i_{r-1}} : R_{r-1}; Y = p_{i^{(1)}}, q_{i^{(1)}}, \tau_{i^{(1)}}; R^{(1)}; \cdots; p_{i^{(r)}}, q_{i^{(r)}}; \tau_{i^{(r)}}; R^{(r)} \tag{1.23}$$

For the sake of brevity, and an empty product is interpreted as unity We shall note.

$$A_1 = Re\left(t + \mu\eta Re \min_{1\leqslant i'\leqslant M}\left(\frac{f_{i'}}{F_{i'}}\right)\right) + \sum_{i=1}^{r}(h_i + \mu\rho_i) \min_{1\leqslant j\leqslant m^{(i)}} Re\left[D_j^{(i)}\left(\frac{d_j^{(i)}}{\delta_j^{(i)}}\right)\right] > 0 \tag{1.24}$$

2. Images under multiple Erdélyi-Kober operator.

We shall consider the Gimel-function of r-variables.

Theorem.

If will be shown here that , if

$$f(x) = x^{\rho}(x^{\mu}+c^{\mu})^{-\sigma} \gimel\begin{pmatrix} \mathrm{z}_1 x^{-h_1}(x^{\mu}+c^{\mu})^{-\rho_1} \\ \cdot \\ \cdot \\ \mathrm{z}_r x^{-h_r}(x^{\mu}+c^{\mu})^{-\rho_r}\end{pmatrix} S_n^{w_1,\cdots,w_s}\begin{pmatrix} \mathrm{x}^{p_1}(x^{\mu}+c^{\mu})^{-q_1} \\ \cdot \\ \cdot \\ \mathrm{x}^{p_r}(x^{\mu}+c^{\mu})^{-q_r}\end{pmatrix}$$

$$H_{R,S}^{K,L}\left[zx^{-\ddot{t}}(x^{\mu}+c^{\mu})^{-\eta}\middle| \begin{matrix} (\mathrm{e}_R, E_R) \\ (\mathrm{f}_S, F_S)\end{matrix}\right] \tag{2.1}$$

with $Re(-A_1 + \min_{1\leqslant k\leqslant r'}(\lambda_k\gamma_k)) > 0$, $\sum_{i=1}^{r'}\frac{1}{\lambda_i} \geqslant \sum_{i=1}^{r'}\frac{1}{\tau_i}$ and $\eta, \rho, \sigma, h_i, \rho_i(i = 1, \cdots, r), p_i, q_i(i = 1, \cdots, s) > 0$ and

$|argz_k| < \frac{1}{2}A_i^{(k)}\pi$,see (1.15), then there holds the following formula

$$K_{(\tau_w),(\lambda_w),r'}^{(\eta_w),(\zeta_w)}[f(x)] = x^{\rho}c^{-\mu\sigma}\sum_{k_1,\cdots,k_s=0}^{w_1k_1+\cdots w_sk_s\leqslant n}(-n)_{w_1k_1+\cdots+w_sk_s}A(n;k_1,\cdots,k_s)\frac{c^{-\mu\sum_{i=1}^{s}q_ik_i}}{k_1!\cdots k_s!}$$

$$\sum_{l=0}^{\infty}\frac{(-)^l x^{\mu l}}{l! c^{\mu l}} H_{R,S}^{K,L}\left(zx^{-\ddot{t}}c^{-\mu\eta}\right)\gimel_{X;p_{i_r}+r'+1,q_{i_r}+r'+1,\tau_{i_r}:R_r:Y}^{U;0,n_r+r'+1:V}\left(\begin{matrix}\frac{z_1}{x^{h_1}c^{\mu\rho_1}} \\ \cdot \\ \cdot \\ \frac{z_r}{x^{h_r}c^{\mu\rho_r}}\end{matrix}\middle|\begin{matrix}\mathbb{A}:\ [1\text{-}\eta_j + E; \frac{h_1}{\lambda_j}, \cdots, \frac{h_r}{\lambda_j}; 1]_{1,r'}, \\ \cdots \\ \mathbb{B}:\ \mathbf{B}, [1\text{-}\eta_j - \zeta_j + E; \frac{h_1}{\lambda_j}, \cdots, \frac{h_r}{\lambda_j}; 1]_{1,r'}\end{matrix}\right.$$

$$\left.\begin{matrix}, [1\text{-}\Delta - l; \rho_1, \cdots, \rho_r; 1], \mathbf{A} : A \\ \cdots \\ , [1\text{-}\Delta; \rho_1, \cdots, \rho_r; 1] : B\end{matrix}\right) \tag{2.2}$$

where

$$E = \frac{\rho + \sum_{i=1}^{s} p_ik_i + \mu l - \ddot{t}\xi}{\lambda_j}, \Delta = \sigma + \sum_{i=1}^{s} q_ik_i - \eta\xi \tag{2.3}$$

and the serie defined by the equation (2.2) is convergent.

Proof : Let $M = \dfrac{1}{(2\pi\omega)^r}\displaystyle\int_{L_1}\cdots\int_{L_r}\psi(s_1,\cdots,s_r)\prod_{k=1}^{r}\theta_k(s_k)z_k^{\xi_k}$ (2.4)

and $f(x) = x^\rho(x^\mu+c^\mu)^{-\sigma}\,\gimel\begin{pmatrix} z_1x^{-h_1}(x^\mu+c^\mu)^{-\rho_1}\\ \cdot\\ \cdot\\ z_rx^{-h_r}(x^\mu+c^\mu)^{-\rho_r}\end{pmatrix} S_n^{w_1,\cdots,w_s}\begin{pmatrix} x^{p_1}(x^\mu+c^\mu)^{-q_1}\\ \cdot\\ \cdot\\ x^{p_s}(x^\mu+c^\mu)^{-q_s}\end{pmatrix}$

$$H_{R,S}^{K,L}\left[zx^{-\ddot{t}}(x^\mu+c^\mu)^{-\eta}\left|\begin{matrix}(e_R,E_R)\\(f_S,F_S)\end{matrix}\right.\right] \tag{2.5}$$

First, we express the multivariable Gimel-function , general class of polynomials and H-function by using equations (1.12), (1.4) and (1.9) respectively.

We have : $f(x) = x^\rho(x^\mu+c^\mu)^{-\sigma}\,M\left\{\left[x^{-h_1}(x^\mu+c^\mu)^{-\rho_1}\right]^{-\xi_1}\cdots\left[x^{-h_r}(x^\mu+c^\mu)^{-\rho_r}\right]^{-\xi_r}\right]$

$$\sum_{k_1,\cdots,k_s=0}^{w_1k_1+\cdots w_sk_s\leqslant n}(-n)_{w_1k_1+\cdots+w_sk_s}A(n;k_1,\cdots,k_s)\frac{[x^{p_1}(x^\mu+c^\mu)^{-q_1}]^{k_1}}{k_1!}\cdots\frac{[x^{p_s}(x^\mu+c^\mu)^{-q_s}]^{k_s}}{k_s!}$$

$$\sum_{h=1}^{L}\sum_{v_1=0}^{\infty}\frac{(-)^{v_1}}{v_1!}\frac{\eta(\xi)}{E_h}\left(zx^{-\ddot{t}}(x^\mu+c^\mu)^{-\eta}\right)^{\xi}\Bigg\}\,\mathrm{d}\xi_1\cdots\mathrm{d}\xi_r \tag{2.6}$$

Now, change the order of integration ans summation (which is easily seen to be justified due to the absolute convergence of the integral and the summations involved in the process), we thus find that

$$f(x) = x^\rho\sum_{k_1,\cdots,k_s=0}^{w_1k_1+\cdots w_sk_s\leqslant n}\sum_{h=1}^{L}\sum_{v_1=0}^{\infty}(-n)_{w_1k_1+\cdots+w_sk_s}\frac{A(n;k_1,\cdots,k_s)}{k_1!\cdots k_s!}\frac{(-)^{v_1}}{v_1!}\frac{\eta(\xi)}{E_h}z^{-\xi}$$

$$M\left\{x^{\rho-\ddot{t}\xi+\sum_{i=1}^{r}h_i\xi_i+\sum_{i=1}^{s}p_ik_i}(x^\mu+c^\mu)^{-\sigma+\eta\xi+\sum_{i=1}^{r}\rho_i\xi_i-\sum_{i=1}^{s}q_ik_i}\right\}\mathrm{d}s_1\cdots\mathrm{d}s_r \tag{2.7}$$

with algebraic manipulations, we obtain

$$f(x) = \sum_{k_1,\cdots,k_s=0}^{w_1k_1+\cdots w_sk_s\leqslant n}\sum_{h=1}^{L}\sum_{v_1=0}^{\infty}(-n)_{w_1k_1+\cdots+w_sk_s}\frac{A(n;k_1,\cdots,k_s)}{k_1!\cdots k_s!}\frac{(-)^{v_1}}{v_1!}\frac{\eta(\xi)}{E_h}z^{-\xi}$$

$$M\left\{x^{\rho-\ddot{t}\xi+\sum_{i=1}^{r}h_i\xi_i+\sum_{i=1}^{s}p_ik_i}\left(1+\frac{x^\mu}{c^\mu}\right)^{-\sigma+\eta\xi+\sum_{i=1}^{r}\rho_i\xi_i-\sum_{i=1}^{s}q_ik_i}c^{\mu(-\sigma+\eta\xi+\sum_{i=1}^{r}\rho_i\xi_i-\sum_{i=1}^{s}q_ik_i}\right\}$$

$$\mathrm{d}s_1\cdots\mathrm{d}s_r \tag{2.8}$$

Use the binomial formula, we get

$$f(x)=\sum_{k_1,\cdots,k_s=0}^{w_1k_1+\cdots w_sk_s\leqslant n}\sum_{h=1}^{L}\sum_{v_1=0}^{\infty}(-n)_{w_1k_1+\cdots+w_sk_s}\frac{A(n;k_1,\cdots,k_s)}{k_1!\cdots k_s!}\frac{(-)^{v_1}}{v_1!}\frac{\eta(\xi)}{E_h}z^{-\xi}$$

$$\sum_{l=0}^{\infty}\frac{(-)^l}{l!}\quad M\left\{(\sigma-\eta\xi-\textstyle\sum_{i=1}^{r}\rho_i\xi_i+\sum_{i=1}^{s}q_ik_i\right)_l\ c^{\mu(-\sigma-l+\eta\xi+\sum_{i=1}^{r}\rho_i\xi_i-\sum_{i=1}^{s}q_ik_i)}$$

$$x^{\rho+\mu l-\ddot{\imath}\xi+\sum_{i=1}^{r}h_i\xi_i+\sum_{i=1}^{s}p_ik_i}\Big\}\mathrm{d}s_1\cdots\mathrm{d}s_r \tag{2.9}$$

Now to establish the images under multiple Erdélyi-Kober operator of the function defined in equations (2.1) and (1.3) ,we get :

$$K_{(\tau_w),(\lambda_w),r'}^{(\eta_w),(\zeta_w)}\left[f(x)\right]=\sum_{k_1,\cdots,k_s=0}^{w_1k_1+\cdots w_sk_s\leqslant n}\sum_{h=1}^{L}\sum_{v_1=0}^{\infty}(-n)_{w_1k_1+\cdots+w_sk_s}\frac{A(n;k_1,\cdots,k_s)}{k_1!\cdots k_s!}\frac{(-)^{v_1}}{v_1!}\frac{\eta(\xi)}{E_h}z^{-\xi}$$

$$\sum_{l=0}^{\infty}\frac{(-)^l}{l!}\ M\left\{(\sigma-\eta\xi-\textstyle\sum_{i=1}^{r}\rho_i\xi_i+\sum_{i=1}^{s}q_ik_i\right)_l\quad c^{\mu(-\sigma-l+\eta\xi-\sum_{i=1}^{r}\rho_i\xi_i-\sum_{i=1}^{s}q_ik_i)}$$

$$\prod_{w=1}^{r'}\frac{\Gamma\left[\eta_w-\frac{\rho+\mu l-\ddot{\imath}\xi+\sum_{i=1}^{r}h_i\xi_i+\sum_{i=1}^{s}p_ik_i}{\lambda_w}\right]}{\Gamma\left[\eta_w-\rho+\zeta_w-\frac{\mu l-\ddot{\imath}\xi+\sum_{i=1}^{r}h_i\xi_i+\sum_{i=1}^{s}p_ik_i}{\lambda_w}\right]}x^{\rho+\mu l-\ddot{\imath}\xi+\sum_{i=1}^{r}h_i\xi_i+\sum_{i=1}^{s}p_ik_i}\Big\}\mathrm{d}s_1\cdots\mathrm{d}s_r \tag{2.10}$$

Finally interpreting the result thus obtained with the Mellin-barnes contour integral with the help of (1.12) and use also the equation (1.9), we arrive at the desired result (2.2).

3. Applications.

Taking $s=1$ in the equation (2.2), the polynomial (1.4) will reduce to $S_n^w(x)$ which is defined by the equation (1.5) and consequently, we obtain the following result

Corollary 1.

$$K_{(\tau_w),(\lambda_w),r'}^{(\eta_w),(\zeta_w)}\left[f_1(x)\right]=x^{\rho}c^{-\mu\sigma}\sum_{k=0}^{[n/w]}\frac{A(n;k)x^{pk}}{k!}c^{-q\mu k}\sum_{l=0}^{\infty}\frac{(-)^l x^{\mu l}}{l!c^{\mu l}}\ H_{R,S}^{K,L}\left(zx^{-\ddot{\imath}}c^{-\mu\eta}\right)$$

$$\daleth_{X;p_{i_r}+r'+1,q_{i_r}+r'+1,\tau_{i_r}:R_r:Y}^{U;0,n_r+r'+1:V}\left(\begin{array}{c|c}\frac{z_1}{x^{h_1}c^{\mu\rho_1}} & \mathbb{A}:\left[1\text{-}\eta_j+E^*;\frac{h_1}{\lambda_j},\cdots,\frac{h_r}{\lambda_j};1\right]_{1,r'}\\ \cdot & \cdots\\ \cdot & \\ \frac{z_r}{x^{h_r}c^{\mu\rho_r}} & \mathbb{B}:\ \mathbf{B},\ \left[1\text{-}\eta_j-\zeta_j+E'^*;\frac{h_1}{\lambda_j},\cdots,\frac{h_r}{\lambda_j};1\right]_{1,r'}\end{array}\right.$$

$$\left.\begin{array}{c},\left[1\text{-}\Delta^*-l;\rho_1,\cdots,\rho_r;1\right],\mathbf{A}:A\\ \cdots\\ ,\left[1\text{-}\Delta^*;\rho_1,\cdots,\rho_r;1\right]:B\end{array}\right) \tag{3.1}$$

where $E^* = \dfrac{\rho + pk + \mu l - \ddot{t}\xi}{\lambda_j}, \Delta^* = \sigma + qk - \eta\xi$ (3.2)

and $f_1(x) = x^{\rho}(x^{\mu} + c^{\mu})^{-\sigma} \daleth \begin{pmatrix} z_1 x^{-h_1}(x^{\mu} + c^{\mu})^{-\rho_1} \\ \cdot \\ \cdot \\ z_r x^{-h_r}(x^{\mu} + c^{\mu})^{-\rho_r} \end{pmatrix} S_n^w(x^p(x^{\mu} + c^{\mu})^{-q})$

$$H_{R,S}^{K,L}\left[zx^{-\ddot{t}}(x^{\mu} + c^{\mu})^{-\eta} \middle| \begin{matrix} (e_R, E_R) \\ (f_S, F_S) \end{matrix} \right] \quad (3.3)$$

under the same validity conditions that (2.2)

Setting $s = 1, w = 2, A_{n,k} = (-)^k$ in (2.2), then by vertue of the result (1.7), we have the following result.

Corollary 2.

$$K^{(\eta_w),(\zeta_w)}_{(\tau_w),(\lambda_w),r'}[f_2(x)] = x^{\rho} c^{-\mu\sigma} \sum_{k=0}^{[n/2]} (-)^k (-n)_{2k} \frac{c^{-kq\mu}}{k!} x^{pk} \sum_{l=0}^{\infty} \frac{(-)^l x^{\mu l}}{l! c^{\mu l}} H_{R,S}^{K,L}(zx^{-\ddot{t}} c^{-\mu\eta})$$

$$\daleth^{U;0,n_r+r'+1:V}_{X;p_{i_r}+r'+1,q_{i_r}+r'+1,\tau_{i_r}:R_r:Y} \left(\begin{matrix} \frac{z_1}{x^{h_1} c^{\mu\rho_1}} \\ \cdot \\ \cdot \\ \frac{z_r}{x^{h_r} c^{\mu\rho_r}} \end{matrix} \middle| \begin{matrix} \mathbb{A}: [1\text{-}\eta_j + E^*; \frac{h_1}{\lambda_j}, \cdots, \frac{h_r}{\lambda_j}; 1]_{1,r'} \\ \cdots \\ \mathbb{B}: \mathbf{B}, [1\text{-}\eta_j - \zeta_j + E'^*; \frac{h_1}{\lambda_j}, \cdots, \frac{h_r}{\lambda_j}; 1]_{1,r'} \end{matrix} \right.$$

$$\left. \begin{matrix} ,[1\text{-}\Delta^* - l; \rho_1, \cdots, \rho_r; 1], \mathbf{A} : A \\ \cdots \\ ,[1\text{-}\Delta^*; \rho_1, \cdots, \rho_r; 1] : B \end{matrix} \right) \quad (3.4)$$

and $f_2(x) = x^{\rho + \frac{np}{2}}(x^{\mu} + c^{\mu})^{-\sigma - \frac{nq}{2}} \daleth \begin{pmatrix} z_1 x^{-h_1}(x^{\mu} + c^{\mu})^{-\rho_1} \\ \cdot \\ \cdot \\ z_r x^{-h_r}(x^{\mu} + c^{\mu})^{-\rho_r} \end{pmatrix} H_n\left[\dfrac{(x^{\mu} + c^{\mu})^{q/2}}{2x^{p/2}}\right]$

$$H_{R,S}^{K,L}\left[zx^{-\ddot{t}}(x^{\mu} + c^{\mu})^{-\eta} \middle| \begin{matrix} (e_R, E_R) \\ (f_S, F_S) \end{matrix} \right] \quad (3.5)$$

Next , if we set $s = 1, w = 1$ and $A_{n,k} = \binom{n+\alpha}{n} \dfrac{(\alpha + \beta + n + 1)_k}{(\alpha + 1)_k}$ in (2.2), then by vertue of the result (1.8), we have the following result under the same validity conditions and notations that (3.1).

Corollary 3.

$$K^{(\eta_w),(\zeta_w)}_{(\tau_w),(\lambda_w),r}\left[f_3(x)\right] = x^{\rho}c^{-\mu\sigma}\sum_{k=0}^{n}\binom{n+\alpha}{n}\frac{(\alpha+\beta+n+1)_k}{(\alpha+1)_k}(-n)_k\frac{c^{-kq\mu}}{k!}x^{pk}\sum_{l=0}^{\infty}\frac{(-)^l x^{\mu l}}{l!c^{\mu l}}\ H^{K,L}_{R,S}\left(zx^{-\ddot{t}}c^{-\mu\eta}\right)$$

$$\gimel^{U;0,n_r+r'+1:V}_{X;p_{i_r}+r'+1,q_{i_r}+r'+1,\tau_{i_r}:R_r:Y}\left(\begin{array}{c|c} \frac{z_1}{x^{h_1}c^{\mu\rho_1}} & \mathbb{A}:\left[1\text{-}\eta_j+E^*;\frac{h_1}{\lambda_j},\cdots,\frac{h_r}{\lambda_j};1\right]_{1,r'} \\ \cdot & \cdots \\ \cdot & \\ \frac{z_r}{x^{h_r}c^{\mu\rho_r}} & \mathbb{B}:\ \mathbf{B},\ \left[1\text{-}\eta_j-\zeta_j+E'^*;\frac{h_1}{\lambda_j},\cdots,\frac{h_r}{\lambda_j};1\right]_{1,r'} \end{array}\right.$$

$$\left.\begin{array}{l} ,\left[1\text{-}\Delta^*-l;\rho_1,\cdots,\rho_r;1\right],\mathbf{A}:A \\ \cdots \\ ,\left[1\text{-}\Delta^*;\rho_1,\cdots,\rho_r;1\right]:B \end{array}\right) \tag{3.6}$$

and $f_3(x) = x^{\rho}(x^{\mu}+c^{\mu})^{-\sigma}\,\gimel\left(\begin{array}{c} z_1x^{-h_1}(x^{\mu}+c^{\mu})^{-\rho_1} \\ \cdot \\ \cdot \\ z_rx^{-h_r}(x^{\mu}+c^{\mu})^{-\rho_r} \end{array}\right)P_n^{(\alpha,\beta)}\left[1-2x^p(x^{\mu}+c^{\mu})^{-q}\right]$

$$H^{K,L}_{R,S}\left[zx^{-\ddot{t}}(x^{\mu}+c^{\mu})^{-\eta}\left|\begin{array}{c} (\mathrm{e}_R,E_R) \\ (\mathrm{f}_S,F_S) \end{array}\right.\right] \tag{3.7}$$

under the same validity conditions and notations that (3.1)

4. Conclusion.

In this paper we have evaluated the images of the product of certain special functions and multivariable Gimel-function , a class of polynomials and Fox's H-function of one variable pertaining to multiple Erdélyi-Kober operator .The images established in this paper is of very general nature as it contains multivariable Gimel-function, which is a general function of several complex variables studied so far. Thus, the integral established in this research work would serve as a key formula from which, upon specializing the parameters, as many as desired results involving the special functions of one and several variables can be obtained.

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