Integrals Transforms and Fourier Series Containing Some Product of Special Functions

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ABSTRACT
In this paper, first we present three integrals pertaining to some special functions like Aleph-function, generalization of M-series, multivariable aleph-function and the general class of polynomials. Later, we give three Fourier series for the product of some special functions like Aleph-function, generalized M-series, multivariable aleph-function and the general class of polynomials which have been obtained by the application of the integrals derived in first time.

Keywords : Aleph-function, general polynomials, multivariable Aleph-function, integrals, Fourier series, generalization of M-series.

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1. Introduction

Srivastava ([7],p. 1, Eq. 1) has defined the general class of polynomials

$$S_{N_1}^{M_1}(x)=\sum_{K_1=0}^{[N_1/M_1]}\frac{(-N_1)_{M_1K_1}}{K_1!}A_{N_1,K_1}x^{K_1} \tag{1.1}$$

On suitably specializing the coefficients A_{N_1,K_1}, $S_{N_1}^{M_1}(x)$ yields some known polynomials, these include the Jacobi polynomials, Laguerre polynomials, and others polynomials ([10],p. 158-161).

We shall note $a_1=\dfrac{(-N_1)_{M_1K_1}}{K_1!}A_{N_1,K_1}$

The Aleph- function , introduced by Südland et al. [11,12], however the notation and complete definition is presented here in the following manner in terms of the Mellin-Barnes type integral :

$$\aleph(z)=\aleph_{P_i,Q_i,c_i;r'}^{M,N}\left(\mathrm{z}\ \middle|\ \begin{matrix}(\mathrm{a}_j,A_j)_{1,\mathfrak{n}},[c_i(a_{ji},A_{ji})]_{\mathfrak{n}+1,p_i;r'}\\ (\mathrm{b}_j,B_j)_{1,m},[c_i(b_{ji},B_{ji})]_{m+1,q_i;r'}\end{matrix}\right)=\frac{1}{2\pi\omega}\int_L\ \Omega_{P_i,Q_i,c_i;r'}^{M,N}(s)z^{-s}\mathrm{d}s \tag{1.2}$$

for all z different to 0 and

$$\Omega_{P_i,Q_i,c_i;r'}^{M,N}(s)=\frac{\prod_{j=1}^{M}\Gamma(b_j+B_js)\prod_{j=1}^{N}\Gamma(1-a_j-A_js)}{\sum_{i=1}^{r'}c_i\prod_{j=N+1}^{P_i}\Gamma(a_{ji}+A_{ji}s)\prod_{j=M+1}^{Q_i}\Gamma(1-b_{ji}-B_{ji}s)} \tag{1.3}$$

With $|argz|<\frac{1}{2}\pi\Omega$ where $\Omega=\sum_{j=1}^{M}B_j+\sum_{j=1}^{N}A_j-c_i\left(\sum_{j=M+1}^{Q_i}B_{ji}+\sum_{j=N+1}^{P_i}A_{ji}\right)>0, i=1,\cdots,r'$

For convergence conditions and other details of Aleph-function , see Südland et al [11,12]. The serie representation of Aleph-function is given by Chaurasia and Singh [2].

$$\aleph_{P_i,Q_i,c_i;r'}^{M,N}(z)=\sum_{G=1}^{M}\sum_{g=0}^{\infty}\frac{(-)^g\Omega_{P_i,Q_i,c_i,r'}^{M,N}(s)}{B_Gg!}z^{-s} \tag{1.4}$$

With $s = \eta_{G,g} = \dfrac{b_G + g}{B_G}, P_i < Q_i, |z| < 1$ and $\Omega^{M,N}_{P_i,Q_i,c_i;r'}(s)$ is given in (1.2) (1.5)

We will use the contracted form about the multivariable aleph-fonction $\aleph(z_1, \cdots, z_u)$ by :

$$\aleph(z_1,\cdots,z_r) = \aleph^{0,\mathrm{n}:V}_{p_i,q_i,\tau_i;R:W}\left(\begin{array}{c|c} \mathrm{z}_1 & \mathrm{A}\ ;\ \mathrm{C} \\ \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \mathrm{B};\ \mathrm{D} \\ \mathrm{z}_r & \end{array}\right) = \frac{1}{(2\pi\omega)^u}\int_{L_1}\cdots\int_{L_u}\psi'(s_1,\cdots,s_u)\prod_{k=1}^{u}\theta'_k(s_k)z_k^{s_k}\mathrm{d}s_1\cdots\mathrm{d}s_u \tag{1.6}$$

with $\omega = \sqrt{-1}$

See Ayant [1], concerning the definition of the following quantities $V, W, \psi'(s_1, \cdots, s_r), A, B, C, D$ and $\theta'_k(s_k)$.

The complex numbers z_i are not zero. Throughout this document , we assume the existence and absolute convergence conditions of the multivariable Aleph-function.

A new generalization of M-series was introduced and developped by Salim et al. [4] as

$$\underset{p,q}{}\overset{v,\mu}{M}_{m,n}(x) = \sum_{k=0}^{\infty}\frac{(a_1)_{km}\cdots(a_p)_{km}}{(b_1)_{kn}\cdots(b_q)_{kn}}\frac{x^k}{\Gamma(vk+\mu)} \tag{1.7}$$

where $\eta, v, \mu \in \mathbb{C}, Re(v) > 0$ and m, n are non-negative real numbers. The series in (1.7) is absolutely convergent for all values of η provided that $pm < qn + Re(v)$, moreover if $pm = qn + Re(v)$, the series converge for $|\eta| < \delta = v^v$
If $m = n = 1$, the above series reduce in M-series defined by Sharma and Jain [5]. For more details, see Salim et al. [4].

We shall note $b_k = \dfrac{(a_1)_{km}\cdots(a_p)_{km}}{(b_1)_{kn}\cdots(b_q)_{kn}}$

2. Required integrals.

In this section, we shall see three integrals.

Lemma 1 (Sneddon, [6], p.41)

$$\int_0^{\frac{\pi}{2}} cos2u\theta(\sin\theta)^v\mathrm{d}\theta = \frac{\Gamma(v+1)\Gamma\left(\frac{1}{2}\pm u\right)}{\Gamma\left(\frac{v}{2}\pm u+1\right)} \tag{2.1}$$

where $Re(v) > 0$.

Lemma 2

$$\int_0^{\frac{\pi}{2}} cosu\theta(\cos\theta)^v\mathrm{d}\theta = \frac{\pi\Gamma(v+1)}{\Gamma\left(\frac{v}{2}\pm\frac{u}{2}+1\right)} \tag{2.2}$$

where $Re(v) > 0$.

Lemma 3 (Honn [3], eq. (2.3), p.73)

$$\int_0^{\frac{\pi}{2}} (\sin y)^{v-1}(\cos y)^{u-1}e^{i(v+u)y}\mathrm{d}y = \frac{\Gamma(v)\Gamma(u)}{\Gamma(v+u)}e^{v\pi/2} \tag{2.3}$$

where, $Re(u), Re(v) > 0$.

2. Main integrals.

In this section, we shall establish three integrals pertaining to some special functions like generalization of M-series, Aleph-function, multivariable aleph-function and the general class of polynomials.

Theorem 1.

$$\int_0^{\frac{\pi}{2}} \cos 2u\theta (sin\theta)^{v} \aleph^{M,N}_{P_i,Q_i,c_i;r'}(z(\sin\theta)^{2\rho'_1}) S^{M_1}_{N_1}[y_1(\sin\theta)^{2\rho}]\, {}_{p,q}\overset{v,\mu}{M}_{m,n}(y(\sin\theta)^{2\rho'_2})$$

$$\aleph[z_1(\sin\theta)^{2\sigma_1},\cdots,z_r(sin\theta)^{2\sigma_r}]\mathrm{d}\theta =$$

$$\sum_{G=1}^{M}\sum_{g=0}^{\infty}\sum_{k=0}^{\infty}\sum_{K_1=0}^{[N_1/M_1]} \frac{(-)^g\Omega^{M,N}_{P_i,Q_i,c_i,r'}(s)}{B_G g!} z^{-s} \frac{y^k}{\Gamma(vk+\mu)}$$

$$\frac{(-)^s\Gamma\left(\frac{1}{2}\pm u\right)}{2^{v+2\rho'_1\eta_{G;g}+2\rho'_2k+2\rho_1K_1+1}} a_1 b_k y_1^{K_1}\, \aleph^{0,\mathfrak{n}+1:V}_{p_i+1,q_i+2,\tau_i;R:W}$$

$$\left(\begin{array}{c|c} 4^{\sigma_1}x_1 & (\text{-v-}2\rho'_1\eta_{G,g}-2\rho'_2k+\rho K_1 : 2\sigma_1,\cdots,2\sigma_r), A; C \\ \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ 4^{\sigma_r}x_r & \mathrm{B}, \left(-\frac{v}{2}-\rho'_1\eta_{G,g}-\rho'_2k-\frac{1}{2}\rho K_1 \pm u : \sigma_1,\cdots,\sigma_r\right), D \end{array}\right) \tag{2.1}$$

provided that

$\sigma_i > 0, i = 1,\cdots,r$ and $Re(v+2\rho'_1\eta_{G,g}+2\rho'_2k)+2\sum_{i=1}^{r}\sigma_i \min_{1\leqslant j\leqslant m_i} Re\left[\left(\frac{d_j^{(i)}}{\delta_j^{(i)}}\right)\right]+1>0$

$|argz(\sin\theta)^{2\rho'_1}| < \frac{1}{2}\pi\Omega$ where $\Omega = \sum_{j=1}^{M}B_j + \sum_{j=1}^{N}A_j - c_i\left(\sum_{j=M+1}^{Q_i}B_{ji} + \sum_{j=N+1}^{P_i}A_{ji}\right) > 0, i = 1,\cdots,r'$

$\left|\arg(z_i(\sin\theta)^{2\sigma_i})\right| < \frac{1}{2}A_i^{(k)}\pi$ where $A_i^{(k)}$ is defined by Ayant [1], $pm < qn + Re(v)$

Proof

Express , the Aleph-function, the generalization of M-serie and the class of polynomial in series with the help (1.4), (1,7) and (1.1) respectively and expressing the multivariable Aleph-function in multiple Mellin-Barnes integrals. Interchange the series and Mellin-Barnes integrals and interchanging the multiple Mellin-Barnes integrals and θ-integral due to absolute convergence of the series and integrals involved in the process. Now evaluate the inner θ-integral with the help of the lemma 1 and interpret the expression in multivariable Aleph-function, after algebraic manipulations, we obtain the result (2.1) with the help of the above integral.

Theorem 2.

$$\int_0^{\frac{\pi}{2}} \cos 2u\theta (cos\theta)^{v} \aleph^{M,N}_{P_i,Q_i,c_i;r'}(z(\cos\theta)^{2\rho'_1}) S^{M_1}_{N_1}[y_1(\cos\theta)^{2\rho}]_{p,q}\overset{v,\mu}{M}_{m,n}(y(\cos\theta)^{2\rho'_2})$$

$$\aleph[z_1(\cos\theta)^{2\sigma_1},\cdots,z_r(cos\theta)^{2\sigma_r}]\mathrm{d}\theta =$$

$$\sum_{G=1}^{M}\sum_{g=0}^{\infty}\sum_{k=0}^{\infty}\sum_{K_1=0}^{[N_1/M_1]} \frac{(-)^g\Omega^{M,N}_{P_i,Q_i,c_i,r'}(s)}{B_G g!} z^{-s} \frac{y^k}{\Gamma(vk+\mu)}$$

$$\frac{(-)^s\pi}{2^{v+2\rho'_1\eta_{G;g}+2\rho'_2k+2\rho_1K_1+1}} a_1 b_k y_1^{K_1}\, \aleph^{0,\mathfrak{n}+1:V}_{p_i+1,q_i+2,\tau_i;R:W}$$

$$\left(\begin{array}{c|c} 4^{\sigma_1}x_1 & (\text{-v-}2\rho_1'\eta_{G,g}-2\rho_2'k-\rho K_1:2\sigma_1,\cdots,2\sigma_r),A;C \\ \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ 4^{\sigma_r}x_r & \text{B},\left(-\frac{v}{2}-\rho_1'\eta_{G,g}-\rho_2'k-\frac{1}{2}\rho K_1\pm\frac{u}{2}:\sigma_1,\cdots,\sigma_r\right),D \end{array}\right) \tag{2.2}$$

provided that

$\sigma_i>0, i=1,\cdots,r$ and $Re(v+2\rho_1'\eta_{G,g}+2\rho_2'k)+2\sum_{i=1}^{r}\sigma_i \min_{1\leqslant j\leqslant m_i} Re\left[\left(\frac{d_j^{(i)}}{\delta_j^{(i)}}\right)\right]+1>0$

$|argz(\sin\theta)^{2\rho_1'}|<\frac{1}{2}\pi\Omega$ where $\Omega=\sum_{j=1}^{M}B_j+\sum_{j=1}^{N}A_j-c_i\left(\sum_{j=M+1}^{Q_i}B_{ji}+\sum_{j=N+1}^{P_i}A_{ji}\right)>0, i=1,\cdots,r'$

$\left|\arg(z_i(\sin\theta)^{2\sigma_i})\right|<\frac{1}{2}A_i^{(k)}\pi$, $pm<qn+Re(v)$

To prove the integral 2, we use the same method that above but to evaluate the inner θ-integral, we use the lemma 2.

Theorem 3.

$$\int_0^{\frac{\pi}{2}}(\sin y)^{u-1}(\cos y)^{v-1}e^{i(v+u)y}\aleph_{P_i,Q_i,c_i;r'}^{M,N}(z(\sin y)^h(\cos y)^k e^{w(h+k)})S_{N_1}^{M_1}[x(\cos y)^{k'}(\sin y)^{h'}e^{w(h'+k')}]$$

$${}_{p,q}\overset{v,\mu}{M}_{m,n}(y(\cos y)^{k''}(\sin y)^{h''}e^{w(h''+k'')})$$

$$\aleph[z_1(\cos y)^{k_1}(\sin y)^{k_1}e^{(h_1+k_1)y},\cdots,z_r(\cos y)^{k_r}(\sin y)^{k_r}e^{(h_r+k_r)y}]\mathrm{d}y=$$

$$\sum_{G=1}^{M}\sum_{g=0}^{\infty}\sum_{k=0}^{\infty}\sum_{K_1=0}^{[N_1/M_1]}\frac{(-)^g\Omega_{P_i,Q_i,c_i,r'}^{M,N}(s)}{B_G g!}z^{-s}\frac{y^k}{\Gamma(vk+\mu)}e^{w(u+h\eta_{G,g}\pi/2)}$$

$$a_1 b_k x^{K_1}\aleph_{p_2+1,q_i+2,\tau_i;R:W}^{0,\mathfrak{n}+2:V}$$

$$\left(\begin{array}{c|c} 4^{\sigma_1}x_1e^{iwh_1\pi/2} & (\text{1-u-h}\eta_{G,g}-k''k-k'K_1:h_1,\cdots,h_r),(1-v-k\eta_{G,g}-h''k-h'K_1:k_1,\cdots,k_r)A;C \\ \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ 4^{\sigma_r}x_re^{iwh_r\pi/2} & \text{B},(\text{1-u-v-(h+k)}\eta_{G,g}-(k''+h'')k-(k'+h')K_1:h_1+k_1,\cdots,h_r+k_r),D \end{array}\right) \tag{3.3}$$

provided that

$h_i,k_i>0, i=1,\cdots,r$ and $Re(v+h\eta_{G,g}+k''g)+2\sum_{i=1}^{r}h_i \min_{1\leqslant j\leqslant m_i} Re\left[\left(\frac{d_j^{(i)}}{\delta_j^{(i)}}\right)\right]>0$

$Re(v+k\eta_{G,g}+h''g)+2\sum_{i=1}^{r}k_i \min_{1\leqslant j\leqslant m_i} Re\left[\left(\frac{d_j^{(i)}}{\delta_j^{(i)}}\right)\right]>0$

$$\left|arg(z(\sin y)^{h}(\cos y)^{k}e^{w(h+k)})\right|< \frac{1}{2}\pi\Omega \quad \text{where } \Omega=\sum_{j=1}^{M}B_j+\sum_{j=1}^{N}A_j-c_i\left(\sum_{j=M+1}^{Q_i}B_{ji}+\sum_{j=N+1}^{P_i}A_{ji}\right)>0,$$

$i=1,\cdots,r'$

$$\left|arg(z_i(\sin y)^{h_i}(\cos y)^{k_i}e^{w(h_i+k_i)})\right|< \frac{1}{2}A_i^{(k)}\pi,\ pm<qn+Re(v)$$

To prove the integral 3, we use the same method that above but to evaluate the inner θ-integral, we use the lemma 3.

3. Fourier series

The following Fourier series expansion formulae have been established in this section by making use of the result derived in the proceeding section.

Theorem 4.

$$(sin\theta)^{v}\aleph_{P_i,Q_i,c_i;r'}^{M,N}(z(\sin\theta)^{2\rho_1'})S_{N_1}^{M_1}[y_1(\sin\theta)^{2\rho}]_{p,q}\overset{v,\mu}{M}_{m,n}(y(\sin\theta)^{2\rho_2'})\,\aleph[z_1(\sin\theta)^{2\sigma_1},\cdots,z_r(sin\theta)^{2\sigma_r}]=$$

$$\sum_{t=0}^{\infty}\sum_{G=1}^{M}\sum_{g=0}^{\infty}\sum_{k=0}^{\infty}\sum_{K_1=0}^{[N_1/M_1]}\frac{(-)^g\Omega_{P_i,Q_i,c_i,r'}^{M,N}(s)}{B_G g!}z^{-s}\frac{y^k}{\Gamma(vk+\mu)}$$

$$\frac{(-)^s\Gamma\left(\frac{1}{2}\pm v\right)\cos 2t\theta}{\pi 2^{v+2\rho_1'\eta_{G;g}+2\rho_2'k+2\rho_1K_1+1}}a_1b_ky_1^{K_1}\aleph_{p_i+1,q_i+2,\tau_i;R:W}^{0,\mathfrak{n}+1:V}$$

$$\left(\begin{array}{c|c} 4^{\sigma_1}x_1 & (\text{-v-}2\rho_1'\eta_{G,g}-2\rho_2'k+\rho K_1:2\sigma_1,\cdots,2\sigma_r),A;C \\ \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ 4^{\sigma_r}x_r & \text{B},\left(-\frac{v}{2}-\rho_1'\eta_{G,g}-\rho_2'k-\frac{1}{2}\rho K_1\pm u:\sigma_1,\cdots,\sigma_r\right),D \end{array}\right) \tag{3.1}$$

under the same condtitions that theorem 1.

Proof
To prove the equation (3.1), let we consider a function

$$f(\theta)=(sin\theta)^{v}\aleph_{P_i,Q_i,c_i;r'}^{M,N}(z(\sin\theta)^{2\rho_1'})S_{N_1}^{M_1}[y_1(\sin\theta)^{2\rho}]_{p,q}\overset{v,\mu}{M}_{m,n}(y(\sin\theta)^{2\rho_2'})\,\aleph[z_1(\sin\theta)^{2\sigma_1},\cdots,z_r(sin\theta)^{2\sigma_r}]=$$

$$=\sum_{t=0}^{\infty}C_t\cos 2t\theta(0<\theta<\pi/2) \tag{3.2}$$

The above equation is valid since $f(\theta)$ is continuous and bounded variation in the open interval $\left(0,\frac{\pi}{2}\right)$. Now multiplying both sides of (3.2) by $\cos 2u\theta$ and integrating with respect to θ from 0 to $\pi/2$ and changing the order of integrations and summation (which is permitted) on the right hand side, we get

$$\int_0^{\frac{\pi}{2}}\cos 2u\theta(sin\theta)^{v}\aleph_{P_i,Q_i,c_i;r'}^{M,N}(z(\sin\theta)^{2\rho_1'})S_{N_1}^{M_1}[y_1(\sin\theta)^{2\rho}]_{p,q}\overset{v,\mu}{M}_{m,n}(y(\sin\theta)^{2\rho_2'})$$

$$\aleph[z_1(\sin\theta)^{2\sigma_1},\cdots,z_r(sin\theta)^{2\sigma_r}]\mathrm{d}\theta=\sum_{t=0}^{\infty}C_t\int_0^{\pi/2}\cos 2t\theta\cos 2u\theta\mathrm{d}\theta \tag{3.3}$$

Now applying the orthogonal association for cosine function in the right hand side and the result obtained in the equation (2.1) on the left hand side of the equation (3.3), we get

$$C_t=\sum_{G=1}^{M}\sum_{g=0}^{\infty}\sum_{k=0}^{\infty}\sum_{K_1=0}^{[N_1/M_1]}\frac{(-)^g\Omega_{P_i,Q_i,c_i,r'}^{M,N}(s)}{B_G g!}z^{-s}\frac{y^k}{\Gamma(\upsilon k+\mu)}$$

$$\frac{(-)^s\Gamma\left(\frac{1}{2}\pm\alpha\right)}{\pi 2^{v+2\rho_1'\eta_{G;g}+2\rho_2'k+2\rho_1 K_1+1}}a_1 b_k y_1^{K_1}\aleph_{p_i+1,q_i+2,\tau_i;R:W}^{0,\mathfrak{n}+1:V}$$

$$\left(\begin{array}{c|c} 4^{\sigma_1}x_1 & \text{(-v-}2\rho_1'\eta_{G,g}-2\rho_2'k+\rho K_1:2\sigma_1,\cdots,2\sigma_r),A;C \\ \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ 4^{\sigma_r}x_r & \text{B},\left(-\frac{v}{2}-\rho_1'\eta_{G,g}-\rho_2'k-\frac{1}{2}\rho K_1\pm u:\sigma_1,\cdots,\sigma_r\right),D \end{array}\right) \tag{3.4}$$

The Fourier series of the equation (3.1) is found now by using equations (3.2) and (3.4).

Theorem 5

$$(cos\theta)^v\aleph_{P_i,Q_i,c_i;r'}^{M,N}(z(\cos\theta)^{2\rho_1'})S_{N_1}^{M_1}[y_1(\cos\theta)^{2\rho}]_{p,q}\overset{v,\mu}{M}_{m,n}(y(\cos\theta)^{2\rho_2'})\aleph[z_1(\cos\theta)^{2\sigma_1},\cdots,z_r(cos\theta)^{2\sigma_r}]$$

$$=\sum_{t=0}^{\infty}\sum_{G=1}^{M}\sum_{g=0}^{\infty}\sum_{k=0}^{\infty}\sum_{K_1=0}^{[N_1/M_1]}\frac{(-)^g\Omega_{P_i,Q_i,c_i,r'}^{M,N}(s)}{B_G g!}z^{-s}\frac{y^k}{\Gamma(\upsilon k+\mu)}$$

$$\frac{(-)^s\pi}{2^{v+2\rho_1'\eta_{G;g}+2\rho_2'k+2\rho_1 K_1+1}}a_1 b_k y_1^{K_1}\aleph_{p_i+1,q_i+2,\tau_i;R:W}^{0,\mathfrak{n}+1:V}$$

$$\left(\begin{array}{c|c} 4^{\sigma_1}x_1 & \text{(-v-}2\rho_1'\eta_{G,g}-2\rho_2'k-\rho K_1:2\sigma_1,\cdots,2\sigma_r),A;C \\ \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ 4^{\sigma_r}x_r & \text{B},\left(-\frac{v}{2}-\rho_1'\eta_{G,g}-\rho_2'k-\frac{1}{2}\rho K_1\pm\frac{u}{2}:\sigma_1,\cdots,\sigma_r\right),D \end{array}\right)\cos t\theta \tag{3.5}$$

under the same condtitions that theorem 2.

Theorem 6.

$$(\sin y)^{4u-1}(\cos y)^{4v-1}\aleph_{P_i,Q_i,c_i;r'}^{M,N}(z(\sin y)^h(\cos y)^k e^{w(h+k)})S_{N_1}^{M_1}[x(\cos y)^{k'}(\sin y)^{h'}e^{w(h'+k')}]$$

$${}_{p,q}\overset{v,\mu}{M}_{m,n}(y(\cos y)^{k''}(\sin y)^{h''}e^{w(h''+k'')})$$

$$\aleph[z_1(\cos y)^{k_1}(\sin y)^{k_1}e^{(h_1+k_1)y},\cdots,z_r(\cos y)^{k_r}(\sin y)^{k_r}e^{(h_r+k_r)y}]\mathrm{d}y=$$

$$\frac{4}{\pi\omega}\sum_{G=1}^{M}\sum_{g=0}^{\infty}\sum_{t=0}^{\infty}\sum_{k=0}^{\infty}\sum_{K_1=0}^{[N_1/M_1]}\frac{(-)^g\Omega_{P_i,Q_i,c_i,r'}^{M,N}(s)}{B_G g!}z^{-s}\frac{y^k}{\Gamma(\upsilon k+\mu)}e^{w(u+h\eta_{G,g}\pi/2)}a_1 b_k x^{K_1}\aleph_{p_2+1,q_i+2,\tau_i;R:W}^{0,\mathfrak{n}+2:V}$$

$$\left(\begin{array}{c|c} 4^{\sigma_1}x_1 e^{iwh_1\pi/2} & \text{(1-u-h}\eta_{G,g}-k''k-k'K_1:h_1,\cdots,h_r),(1-v-k\eta_{G,g}-h''k-h'K_1:k_1,\cdots,k_r)A;C \\ \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ 4^{\sigma_r}x_r e^{iwh_r\pi/2} & \text{B,(1-u-v-(h+k)}\eta_{G,g}-(k''+h'')k-(k'+h')K_1:h_1+k_1,\cdots,h_r+k_r),D \end{array}\right)$$

$$\sin 4(u+t)y \tag{3.6}$$

under the same condtitions that theorem 3.

In the same pattern, the other Fourier series expansion method from equations (3.5) and (3.6) can be easily obtained by using the results which are obtained in the theorems 2 and 3.

We obtain the same relation with the multivariable H-function defined by Srivastava and Panda [8,9].

4.Conclusion.

By specializing the various parameters as well as variables in the multivariable Aleph-function, the aleph-function, the generalization of M-serie and the multivariable polynomial, we get a several formulae involving remarkably wide variety of useful functions (or product of such functions) which are expressible in terms of E, F, G, H, I, Aleph-function of one and several variables and simpler special functions of one and several variables. Hence the formulae derived in this paper are most general in character and may prove to be useful in several interesting cases appearing in literature of Pure and Applied Mathematics and Mathematical Physics. We can obtain a large number of integrals and Fourier series.

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