Properties of Type -2 Fuzzy Boolean Algebra

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Abstract:
In this paper I study the Type 2 Fuzzy Boolean Algebra And It's properties by using Type 2 Fuzzy Set Theory . I divided this paper into three section namely Classical set theory , fuzzy set and Type 2 Fuzzy Set .

Key Word: *Boolean Algebra, Fuzzy Set, Type-2 Fuzzy Set.*

SECTION I
CLASSICAL SET THEORY

A. Introduction

Boolean algebra, as developed in 1854 by George Boole [1] . In nineteenth century Jevons, Schroder, Huntington did further development in Boolean algebra . In 1936, M. H. Stone proved that every Boolean algebra is isomorphic to a field of set .

B. Partial Ordered Relation

Let X be any nonempty set , the relation '$\leq$' on X which is reflexive, antisymmetric , transitive called as partial ordered relation .

i) Reflexive : $x \leq x$, $\forall x \in X$.

ii) Ant symmetric : $x \leq y$, $y \leq x \leftrightarrow x = y$, $\forall x , y \in X$.

iii) Transitive : $x \leq y$, $y \leq z \leftrightarrow x \leq z$ $\forall x , y , z \in X$.

1. Lattice

A poset L with minimal element (0) and maximal element (1) and two binary operation meet ($\wedge$) and join ($\vee$) defined as $x \wedge y = g.l.b (x , y)$, and $x \vee y = l.u.b (x , y) \forall x , y \in L$. is lattice .

2. Distributive Lattice

A lattice (L, $\vee$, $\wedge$, 0 ,1) is distributive if join is distributive over meet and vice versa

i) $x \vee (y \wedge z) = (x \vee y) \wedge (x \vee z)$ $\forall x , y , z \in L$.

ii) $x \wedge (y \vee z) = (x \wedge y) \vee (x \wedge z)$ $\forall x , y , z \in L$.

3. Complemented Lattice

A lattice (L, $\vee$, $\wedge$, 0 ,1) is said to be complemented if $\forall x \in L$. there exist $x' \in L$ such that $x \wedge x' = 0$, $x \vee x' = 1$.

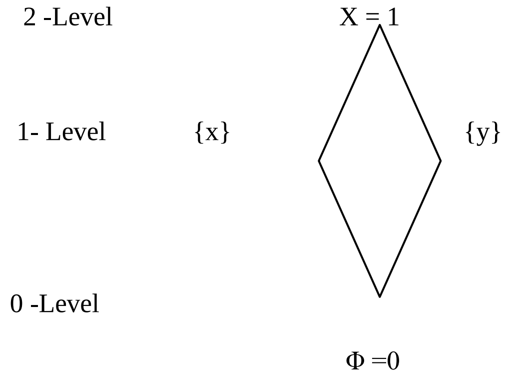
Note : $0' = 1$ and $1' = 0$.

C. Boolean Algebra

A lattices (B, $\vee$, $\wedge$, - , 0 ,1) which is distributive and complemented called as Boolean lattice and algebraic system on B with above two binary operation and one unary operation called as Boolean algebra.

Example :

1. Let X= { x , y } be any set and B = Þ (X)= { Φ. {x}, {y}, X } be Boolean algebra on partial order relation ' subsets ' with Φ = 0 and X = 1,with Hasse diagram and level of elements are



- Note : Every Boolean algebra on finite set with n elements is isomorphic to B_2^n and in Hasse diagram having ${}^nC_r$ elements in each r levels. r = 0 , 1 ,…..n.

Theorem : Let (B, ∨ , ∧ , - , 0 ,1) be Boolean algebra , the following properties are always holds

i) $x \vee (y \vee z) = (x \vee y) \vee z$
$x \wedge (y \wedge z) = (x \wedge y) \wedge z \quad \forall\ x , y , z \in B.$
(Associative law)

ii) $x \vee y = y \vee x , x \wedge y = y \wedge x , \quad \forall\ x , y \in B$
(commutative law)

iii) $x \vee 0 = x,\ x \wedge 1 = x \quad \forall\ x \in B$
(identity law)

iv) $x \vee (x \wedge y) = x , x \wedge (x \vee y) = x \ \forall x , y \in B$
(absorption law)

v) $x \vee x = x , x \wedge x = x \quad \forall x \in B$

$x \vee x' = 1 , x \wedge x' = 0 \quad \forall x \in B$ (idempotent law)

vi) $(x \vee y)' = x' \wedge y' , (x \wedge y)' = x' \vee y' \quad \forall\ x , y \in B$ (De Morgan's law)

SECTION II
FUZZY SET THEORY

A. Introduction

L. A. Zadeh introduced fuzzy sets in 1965 and many fuzzy mathematical systems [3,4] . Mamon Dhar define fuzzy set towards forming Boolean algebra except some properties [2] . In this section we study about fuzzy Boolean algebra and try to defined different structure of fuzzy Boolean algebra .

Definition : (Zadeh) Let X be a set of finite object , fuzzy set A on X is defined as

$$A = \{ (x, \mu_A (x) / \mu_A (x) \in [0,1] , \forall x \in X \}$$ [4]

Definition : (Xuhua Liu and Ju Ye) Let X be a set , and L be complemented and bounded lattice denoted by (L ,+, . , -, 0,1). Then a fuzzy set A of X is characterized by a function

$\mu_A : X \to L$ from X to L , called the membership function [8].

Definition : Let X is finite set with n elements and B be a finite Boolean algebra denoted by (B, ∨ , ∧ , - , 0 ,1). Then fuzzy set A is defined on B as $\boldsymbol{\mu_A} : B \to [0, 1]$

$$A = \{ (x , \mu_A(x) / \mu_A(x) \in [0,1] ,\forall\ x \in B \}$$

where $\mu_A(x) = \frac{h}{n}$, h = level of element .

also $\mu_A(x) \wedge \mu_A(y) = \min(\mu_A(x), \mu_A(y))$ if $x \le y$

$\mu_A(x) \wedge \mu_A(y) = \min(\mu_A(x), \mu_A(y)) - 1/n$ if $x \lneqq y$

$\mu_A(x) \vee \mu_A(y) = \max(\mu_A(x), \mu_A(y))$ if $x \le y$

$\mu_A(x) \vee \mu_A(y) = \max(\mu_A(x), \mu_A(y)) + 1/n$ if $x \lneqq y$

$\forall\, x, y \in B$.

Example 1: Let X= { a , b } finite set and B ={ Φ. {a}, { b}, X } be Boolean algebra .

Then fuzzy Boolean algebra on X is described as

$A = \{ (\Phi, 0), (\{a\}, \frac{1}{2}), (\{b\}, \frac{1}{2}), (X, 1) \}$.

Example 2: Let X= { a , b , c } is finite set and

B = { Φ. {a}, { b }, {c} , { a, b }, { a, c } , { b , c }, X } . be Boolean algebra on X. Then fuzzy Boolean algebra on X described as

$A = \{ (\Phi, 0), (\{a\}, \frac{1}{3}), (\{b\}, \frac{1}{3}), (\{c\}, \frac{1}{3}), (\{a, b\}, \frac{2}{3}), (\{a, c\}, \frac{2}{3}) (\{b, c\}, \frac{2}{3}) (X, 1) \}$.

Theorem : Let (B, $\vee$, $\wedge$, - , 0 ,1). Be fuzzy Boolean algebra ,then following properties are always hold

i) $\mu_A(x) \wedge \mu_A(y) = \mu_A(y) \wedge \mu_A(x)$, $\forall\, x \in B$

$\mu_A(x) \vee \mu_A(y) = \mu_A(y) \vee \mu_A(x)$, (commutative law)

ii) $\mu_A(x) \wedge 1 = \mu_A(x)$, $\mu_A(x) \vee 0 = \mu_A(x)$, $\forall\, x \in B$ (identity law)

iii) $\mu_A(x) \wedge (\mu_A(x) \vee \mu_A(y)) = \mu_A(x)$,

$\mu_A(x) \vee (\mu_A(x) \wedge \mu_A(y)) = \mu_A(x)$, $\forall\, x \in B$ (absorption law)

iv) $\mu_A(x) \wedge \mu_A(x) = \mu_A(x)$,

$\mu_A(x) \vee (\mu_A(x) = \mu_A(x)$, $\forall\, x \in B$ (Idempotent law)

v) $\mu_A(x) \wedge \mu_A(x') = 0$, $\mu_A(x) \vee (\mu_A(x')) = 1$.

vi) $(\mu_A(x) \wedge \mu_A(y))' = (\mu_A(x')) \vee (\mu_A(y'))$

$(\mu_A(x) \vee \mu_A(y))' = (\mu_A(x')) \wedge (\mu_A(y'))$ (De Morgan's law)

SECTION III
TYPE – 2 FUZZY SET

A. Introduction

The concept of fuzzy sets of type 2 has been defined by L . A . Zadeh as extension of type 1 fuzzy set [3] . The type 2 fuzzy set can be characterize by fuzzy fuzzy membership function . Since from 2001. Jerry Mendel with his researcher team done lot of work in type-2 fuzzy set [5 , 6 ,7], Definition : (Mendel and Karnik) Let X be a set of finite object , type -2 fuzzy set Å X is defined as [8]

$\mathring{A} = \{ ((x, u), \mu_{\mathring{A}}(x, u)) / u \in \mu_A(x) \in [0,1],$

$\mu_{\mathring{A}}(x, u) \in [0, 1], \forall x \in X \}$

Definition : Let X be a finite set with n elements and B be a finite Boolean algebra denoted by (B, ˅ , ˄ , - , 0 ,1). Then type-2 fuzzy set Å is a set on B is defined as $\mu_{\mathring{A}} : B \to [0, 1]$.

$\mathring{A} = \{ ((x, u), \mu_{\mathring{A}}(x, u)) / u \in \mu_A(x) \in [0,1], \mu_{\mathring{A}}(x, u) \in [0, 1], \forall x \in B \}$ where $\mu_{\mathring{A}}(x) = 1 - \frac{h}{n}$

$\mu_{\mathring{A}}(x, u) \wedge \mu_{\mathring{A}}(y, u) = \max(\mu_{\mathring{A}}(x, u), \mu_{\mathring{A}}(y, u))$ if $x \le y$

$\mu_{\mathring{A}}(x, u) \wedge \mu_{\mathring{A}}(y, u) = \max(\mu_{\mathring{A}}(x, u), \mu_{\mathring{A}}(y, u)) + 1/n$ if $x \lneqq y$

$\mu_{\mathring{A}}(x, u) \vee \mu_{\mathring{A}}(y, u) = \min(\mu_{\mathring{A}}(x, u), \mu_{\mathring{A}}(y, u))$ if $x \le y$

$\mu_{\mathring{A}}(x, u) \vee \mu_{\mathring{A}}(y, u) = \min(\mu_{\mathring{A}}(x, u), (\mu_{\mathring{A}}(y, u)) - 1/n$ if $x \lneqq y$

Example 1: Let X= { a , b } be any set . and

B = { Φ. {a}, {b}, X } be Boolean algebra .

The type – 2 fuzzy Boolean algebra on X described as

$\mathring{A} = \{((\Phi, 0), 1), ((\{a\}, \}, \frac{1}{2}) \frac{1}{2}), ((\{b\}, \frac{1}{2}) \frac{1}{2}),$

$((X, 1), 0) \}$.

Example 2: Let X= { a , b , c } be any set and

B = { Φ. {a}, {b}, {c} , { a, b }, { a, c } , { b , c }, X } .

The type – 2 fuzzy Boolean algebra on X described as

$\mathring{A} = \{ ((\Phi, 0), 1), ((\{a\}, \frac{1}{3}), \frac{2}{3}), ((\{b\}, \frac{1}{3}). \frac{2}{3}),$

$((\{c\}, \frac{1}{3}, \frac{2}{3}), ((\{a, b\}, \frac{2}{3}), \frac{1}{3}), ((\{a, c\}, \frac{2}{3}), \frac{1}{3}), ((\{b, c\}, \frac{2}{3} \frac{1}{3}), ((X, 1), 0) \}$

Theorem: Let (B, ˅ , ˄ , - , 0 ,1). Be fuzzy Boolean algebra, then following properties are always

i) $\mu_{\mathring{A}}(x, u) \wedge \mu_{\mathring{A}}(y, u) = \mu_{\mathring{A}}(y, u) \wedge \mu_{\mathring{A}}(x, u)$,

$\mu_{\mathring{A}}(x, u) \vee \mu_{\mathring{A}}(y, u) = \mu_{\mathring{A}}(y, u) \vee \mu_{\mathring{A}}(x, u)$, (commutative law)

ii) $\mu_{\mathring{A}}(x, u) \vee 1 = \mu_{\mathring{A}}(x, u)$, $\mu_{\mathring{A}}(x, u) \wedge 0 = \mu_{\mathring{A}}(x, u)$ $\forall x \varepsilon B$ (identity law)

iii) $\mu_{\mathring{A}}(x, u) \wedge (\mu_{\mathring{A}}(x, u) \vee \mu_{\mathring{A}}(y, u)) = \mu_{\mathring{A}}(x, u)$,

$\mu_{\mathring{A}}(x, u) \vee (\mu_{\mathring{A}}(x, u) \wedge \mu_{\mathring{A}}(y, u)) = \mu_{\mathring{A}}(x, u) \; \forall x \varepsilon B$ (absorption law)

iv) $\mu_{\mathring{A}}(x, u) \wedge \mu_{\mathring{A}}(x, u) = \mu_{\mathring{A}}(x, u)$,

$\mu_{\mathring{A}}(x, u) \vee (\mu_{\mathring{A}}(x, u)) = \mu_{\mathring{A}}(x, u)$ $\forall x \varepsilon B$ (Idempotent law)

v) $\mu_{\mathring{A}}(x, u) \wedge \mu_{\mathring{A}}(x', u) = 1$, $\mu_{\mathring{A}}(x, u) \vee \mu_{\mathring{A}}(x', u) = 0$, vi) $(\mu_{\dot{A}} (x ,u) \wedge \mu_{\dot{A}} (y ,u))' = (\mu_{\dot{A}} (x' ,u)) \vee (\mu_{\dot{A}} (y' ,u))$

$(\mu_{\dot{A}}(x ,u) \vee \mu_{\dot{A}} (y ,u))' = (\mu_{\dot{A}} (x' ,u)) \wedge \mu_{\dot{A}} (y' ,u))$ (De Morgan's laws)

IV. CONCLUSION

In this paper author gives definition of fuzzy Boolean algebra and all properties of fuzzy Boolean algebra . In last section gives definition of Type-2 fuzzy Boolean algebra and all properties of Type-2 fuzzy Boolean algebra . By given definition Type-2 fuzzy Boolean algebra acts as dual of fuzzy Boolean Algebra .

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