# Kober Fractional Q-Integral of Basic Analogue of Multivariable H-Function and Basic Analogue of Multivariable Meijer-Function

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#### ABSTRACT

The Kober fractional q-integral of the basic analogue of multivariable H-function and basic analogue of multivariable Meijer variables have been evaluated in this paper. At the end we shall give particular cases.

Keywords : Kober fractional q-integral operator, basic integration, basic analogue of multivariable H-function, basic analogue of multivariable Meijer function.

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#### 1. Introduction and preliminaries.

Recently the authors, see ([8],[11]-[14]) have been investigated the applications of the Riemann-Liouville and Kober fractional q-integral operators to various basic hypergeometric function of one variable including the basic analogue of Fox's H-function. These results are a new contribution to the theory of q-fractional calculus. Motivated by these works, and a possible scope for their application in evaluation and solution of the q-integral equations, we further explore the possibility of evaluation of Kober type fractional q-integrals involving basic analogue of multivariable H-function and.

Also, the basic analogue of the Kober fractional integral operator, see Agarwal [1] is defined by

$$I_{q}^{\eta,\alpha}\{f(x)\} = \frac{x^{-\eta-\alpha}}{\Gamma_{q}(\alpha)} \int_{0}^{x} (x-tq)_{\alpha-1} t^{\eta} f(t) d_{q} t \quad (Re(\alpha) > 0, \eta \in \mathbb{R}, |q| < 1).$$
(1.1)

where  $\alpha$  is an arbitrary order of integration with  $Re(\alpha) > 0$  and  $\eta$  being real or complex quantity.

Following Agarwal [1], Al-Salam [2] and Jackson [4] the basic integration is defined as

$$\int_{0}^{t} f(x)d(x;q) = t(1-q)\sum_{k=0}^{\infty} q^{k}f(tq^{k})$$
(1.2)

By virtue of the result (2), the operator (1) can be expressed as :

$$I_q^{\eta,\alpha} = \frac{(1-q)}{\Gamma_q(\alpha)} \sum_{k=0}^{\infty} q^{h(1+\eta)} (1-q^{k+1})_{\alpha-1} f(tq^k)$$
(1.3)

where  $Re(\alpha) > 0$  and  $\eta$  being real or complex quantity.

In the theory of q-series, for real or complex *a* and |q| < 1, the *q*-shifted factorial is defined as :

$$(a;q)_n = \prod_{i=1}^{n-1} (1 - aq^i) = \frac{(a;q)_\infty}{(aq^n;q)_\infty} \qquad (n \in \mathbb{N})$$
(1.4)

so that  $(a; q)_0 = 1$ ,

or equivalently

$$(a,q)_n = \frac{\Gamma_q(a+n)(1-q)^n}{\Gamma_q(a)} \qquad (a \neq 0, -1, -2, \cdots).$$
(1.5)

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The q-gamma function [4] is given by

$$\Gamma_q(\alpha) = \frac{(q;q)_{\infty}(1-q)^{\alpha-1}}{(q^{\alpha}q)_{\infty}} = \frac{[1-q]_{\alpha-1}}{(1-q)^{\alpha-1}} = \frac{(q;q)_{\alpha-1}}{(1-q)^{\alpha-1}} (\alpha \neq 0, -1, -2, \cdots).$$
(1.6)

also

$$[x-y]_{\nu} = x^{\nu} \prod_{n=0}^{\infty} \left[ \frac{1 - (y/x)q^n}{1 - (y/x)q^{n+\nu}} \right]$$
(1.7)

## 2.Basic analogue of multivariable H-function.

In this section, we introduce the basic analogue of multivariable H-function defined by Srivastava and Panda [9,10]

We note

$$G(q^a) = \left[\prod_{n=0}^{\infty} (1-q^{a+n})\right]^{-1} = \frac{1}{(q^a;q)_{\infty}}$$
(2.1)

$$H(z_{1}, \cdots, z_{r}; q) = H_{p,q':p_{1},q_{1}; \cdots; p_{r},q_{r}}^{0,n:m_{1},n_{1};\cdots;m_{r},n_{r}} \begin{pmatrix} z_{1} \\ \cdot \\ \cdot \\ z_{r} \end{pmatrix} \begin{pmatrix} (a_{j}; \alpha_{j}', \cdots, \alpha_{j}^{(r)})_{1,p} : (c_{j}', \gamma')_{1,p_{1}}, \cdots, (c_{j}^{(r)}, \gamma_{j}^{(r)})_{1,p_{r}} \\ \cdot \\ \cdot \\ \cdot \\ \vdots \\ (b_{j}; \beta_{j}', \cdots, \beta_{j}^{(r)})_{1,q'} : (d_{j}', \delta')_{1,q_{1}}, \cdots, (d_{j}^{(r)}, \delta_{j}^{(r)})_{1,q_{r}} \end{pmatrix}$$

$$\frac{1}{(2\pi\omega)^r} \int_{L_1} \cdots \int_{L_r} \pi^r \phi(s_1, \cdots, s_r; q) \prod_{i=1}^r \theta_i(s_i; q) x_1^{s_1} \cdots x_r^{s_r} d_q s_1 \cdots d_q s_r$$
(2.2)

where

$$\phi(s_1, \cdots, s_r; q) = \frac{\prod_{j=1}^n G(q^{1-a_j + \sum_{i=1}^r \alpha_j^{(i)} s_i})}{\prod_{j=n+1}^p G(q^{a_j - \sum_{i=1}^r \alpha_j^{(i)} s_i}) \prod_{j=1}^{q'} G(q^{1-b_j + \sum_{i=1}^r \beta_j^{(i)} s_i})}$$
(2.3)

$$\theta_i(s_i;q) = \frac{\prod_{j=1}^{m_i} G(q^{d_j^{(i)} - \delta_j^{(i)} s_i}) \prod_{j=1}^{n_i} G(q^{1 - c_j^{(i)} + \gamma_j^{(i)} s_i})}{\prod_{j=m_i+1}^{q_i} G(q^{1 - d_j^{(i)} + \delta_j^{(i)} s_i}) \prod_{j=n_i+1}^{p_i} G(q^{c_j^{(i)} - \gamma_j^{(i)} s_i}) G(q^{1 - s_i}) \sin \pi s_i}$$
(2.4)

$$i=1,\cdots,r$$

where the integers  $n, p, q, m_i, n_i, p_i, q_i$  are constrained by the inequalities  $0 \le n \le p, 0 \le q', 1 \le m_i \le q_i$  and  $0 \le n_i \le p_i, i = 1, \dots, r$ . The poles of the integrand are assumed to be simple. The quantities,  $a_j, j = 1, \dots, p; c_j^{(i)}, j = 1, \dots, p_i; b_j, j = 1, \dots, q'; d_j^{(i)}, j = 1, \dots, q_i, i = 1, \dots, r$  are complex numbers and the following quantities  $\alpha_j^{(i)}, j = 1, \dots, p; \gamma_j^{(i)}, j = 1, \dots, p_i; \beta_j^{(i)}, j = 1, \dots, q'; \delta_j^{(i)}, j = 1, \dots, q_i, i = 1, \dots, q'; \delta_j^{(i)}, j = 1, \dots, q_i, i = 1, \dots, q'; \delta_j^{(i)}, j = 1, \dots, q_i$ ,  $i = 1, \dots, r$  are positive real numbers.

The contour  $L_i$  in the complex  $s_i$ -plane is of the Mellin-Barnes type which runs from  $-\omega\infty$  to  $\omega\infty$  with indentations, if necessary to ensure that all the poles of  $G(q^{d_j^{(i)}+\delta_j^{(i)}s_i}), j=1,\cdots,m_i$  are separated from those of  $G(q^{1-c_j^{(i)}+\gamma_j^{(i)}s_i}), i=1,\cdots,n_i, G(q^{1-a_j+\sum_{i=1}^r \alpha_j^{(i)}s_i}), j=1,\cdots,n, i=1,\cdots,r$ . For large values of  $|s_i|$  the integrals converge if  $Re(slog(z_i) - log \sin \pi s_i) < 0, i=1,\cdots,r$ .

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If the quantities  $\alpha_j^{(i)}(j = 1, \dots, p) = \gamma_j^{(i)}(j = 1, \dots, p_i) = \beta_j^{(i)}(j = 1, \dots, q') = (\delta_j^{(i)}, j = 1, \dots, q_i), (i = 1, \dots, r) = 1$ , then the basic analogue of multivariable H-function reduces in basic analogue of multivariable Meijer function defined by Khadia and Goyal [5], we obtain

$$G(z_{1}, \cdots, z_{r}; q) = G_{p,q':p_{1},q_{1}; \cdots; p_{r},q_{r}}^{0,n:m_{1},n_{1}; \cdots; m_{r},n_{r}} \begin{pmatrix} z_{1} \\ \vdots \\ \vdots \\ z_{r} \end{pmatrix} \begin{pmatrix} (a_{j})_{p} : (c_{j}')_{p_{1}}, \cdots, (c_{j}^{(r)})_{p_{r}} \\ \vdots \\ (b_{j})_{q'} : (d_{j}')_{q_{1}}, \cdots, (d_{j}^{(r)})_{q_{r}} \end{pmatrix}$$

$$\frac{1}{(2\pi\omega)^{r}} \int_{L_{1}} \cdots \int_{L_{r}} \pi^{r} \psi(s_{1}, \cdots, s_{r}; q) \prod_{i=1}^{r} \psi_{i}(s_{i}; q) x_{1}^{s_{1}} \cdots x_{r}^{s_{r}} d_{q} s_{1} \cdots d_{q} s_{r}$$

$$(2.5)$$

where

$$\psi(s_1, \cdots, s_r; q) = \frac{\prod_{j=1}^n G(q^{1-a_j + \sum_{i=1}^r s_i})}{\prod_{j=n+1}^p G(q^{a_j - \sum_{i=1}^r s_i}) \prod_{j=1}^{q'} G(q^{1-b_j + \sum_{i=1}^r s_i})}$$
(2.6)

$$\psi_i(s_i;q) = \frac{\prod_{j=1}^{m_i} G(q^{d_j^{(i)}-s_i}) \prod_{j=1}^{n_i} G(q^{1-c_j^{(i)}+s_i})}{\prod_{j=m_i+1}^{q_i} G(q^{1-d_j^{(i)}+s_i}) \prod_{j=n_i+1}^{p_i} G(q^{c_j^{(i)}-s_i}) G(q^{1-s_i}) \sin \pi s_i}$$
(2.7)

 $i = 1, \cdots, r$ 

where the integers  $n, p, q, m_i, n_i, p_i, q_i$  are constrained by the inequalities  $0 \le n \le p, 0 \le q', 1 \le m_i \le q_i$  and  $0 \le n_i \le p_i, i = 1, \dots, r$ . The poles of the integrand are assumed to be simple. The quantities,  $a_j, j = 1, \dots, p; c_j^{(i)}, j = 1, \dots, p_i; b_j, j = 1, \dots, q'; d_j^{(i)}, j = 1, \dots, q_i, i = 1, \dots, r$  are complex numbers.

The contour  $L_i$  in the complex  $s_i$ -plane is of the Mellin-Barnes type which runs from  $-\omega\infty$  to  $\omega\infty$  with indentations, if necessary to ensure that all the poles of  $G(q^{d_j^{(i)}+s_i}), j = 1, \cdots, m_i$  are separated from those of  $G(q^{1-c_j^{(i)}+s_i}), i = 1, \cdots, n_i, G(q^{1-a_j+\sum_{i=1}^r s_i}), j = 1, \cdots, n, i = 1, \cdots, r$ . For large values of  $|s_i|$  the integrals converge if  $Re(slog(z_i) - log \sin \pi s_i) < 0, i = 1, \cdots, r$ .

## 3. Main results.

Let

$$U = m_1, n_1; \cdots; m_r, n_r; V = p_1, q_1; \cdots; p_r, q_r$$
(3.1)

$$A = (a_j; \alpha'_j, \cdots, \alpha^{(r)}_j)_{1,p} : B = (c'_j, \gamma')_{1,p_1}, \cdots, (c^{(r)}_j, \gamma^{(r)}_j)_{1,p_r}$$
(3.2)

$$C = (b_j; \beta'_j, \cdots, \beta^{(r)}_j)_{1,q'} : D = (d'_j, \delta')_{1,q_1}, \cdots, (d^{(r)}_j, \delta^{(r)}_j)_{1,q_r}$$
(3.3)

In this section, the following result involving the Kober fractional q-integral operator [8] of the multiple basic analogue of multivariable H-function

#### Theorem.

$$I_{q}^{\eta,\alpha}\left\{t^{\lambda-1}H_{p,q':V}^{0,n:U}\left(\begin{array}{cc} \mathbf{x}_{1}t^{k_{1}} \\ \cdot \\ \mathbf{x}_{r}t^{k_{r}} \end{array}\right| \mathbf{x}_{r} \mathbf{x}_{r}$$

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$$H_{p+1,q'+1:V}^{0,n+1:U} \begin{pmatrix} x_1 t^{k_1} & (1-\lambda - \eta; k_1, \cdots, k_r), A : B \\ \vdots & \vdots \\ x_r t^{k_r} & \vdots \\ C, (1-\lambda - \eta - \alpha; k_1, \cdots, k_r) : D \end{pmatrix}$$
(3.4)

where  $\operatorname{Re}(\lambda + \alpha) > 0$ ,  $\operatorname{Re}(\operatorname{slog}(z_i) - \log \sin \pi s_i) < 0, i = 1, \cdots, r$ 

## Proof

Replace the basic analogue of multivariable H-function by this integral contour, we obtain by using (1.3) and (2.2)

$$(1-q)^{\mu} \sum_{k=0}^{\infty} \frac{q^{k(1+\alpha)}(q_{u};q)_{k}}{(q;q)_{k}} (tq^{k})^{\lambda-1} \frac{1}{(2\pi\omega)^{r}} \int_{L_{1}} \cdots \int_{L_{r}} \pi^{r} \phi(s_{1},\cdots,s_{r};q) \prod_{i=1}^{r} \theta_{i}(s_{i};q)$$

$$(x_{1}t^{k_{1}}q^{k_{1}})^{s_{1}} \cdots (x_{r}t^{k_{r}}q^{k_{r}})^{s_{r}} d_{q}s_{1} \cdots d_{q}s_{r}$$
(3.4)

On Interchanging the order of integrations and then on summing the inner  $_1\phi_0(.)$  series with the help of the Heine summation theorem, Gasper and Rahman [3] namely

$${}_1\phi_0(\alpha;-q,x) = \frac{(ax;q)_\infty}{(x;q)_\infty}$$
(3.5)

the left hand side, after algebraic manipulations, we obtain

$$(1-q)^{\mu}t^{\lambda-1}\frac{1}{(2\pi\omega)^{r}}\int_{L_{1}}\cdots\int_{L_{r}}\pi^{r}\phi(s_{1},\cdots,s_{r};q)\prod_{i=1}^{r}\theta_{i}(s_{i};q)$$
$$(x_{1}t^{k_{1}})^{s_{1}}\cdots(x_{r}t^{k_{r}})^{s_{r}}\frac{(q^{\lambda+\eta};q)_{m_{1}k_{1}+\cdots+m_{r}k_{r}}}{(q^{\lambda+\eta};q)_{m_{1}k_{1}+\cdots+m_{r}k_{r}}(q^{\lambda+\eta};q)_{\alpha}}d_{q}s_{1}\cdots d_{q}s_{r}$$

Interpreting in view of the definition (2.2), we obtain the result

## **Corollary 1.**

$$I_{q}^{\eta,\alpha} \left\{ t^{\lambda-1} G_{p,q':p_{1},q_{1};\cdots;p_{r},q_{r}}^{0,n:m_{1},n_{1};\cdots;m_{r},n_{r}} \begin{pmatrix} z_{1}t \\ \cdot \\ \cdot \\ z_{r}t \end{pmatrix} \left| \begin{pmatrix} (a_{j})_{p} : (c_{j}')_{p_{1}},\cdots, (c_{j}^{(r)})_{p_{r}} \\ \cdot \\ (b_{j})_{q'} : (d_{j}')_{q_{1}},\cdots, (d_{j}^{(r)})_{q_{r}} \end{pmatrix} \right\} = \frac{t^{\lambda-1} \Gamma_{q}(\lambda+\alpha)}{\Gamma_{q}(\lambda+\alpha+\mu)}$$

$$G_{p+1,q'+1:p_{1},q_{1};\cdots;p_{r},q_{r}}^{0,n+1:m_{1},n_{1};\cdots;m_{r},n_{r}} \left( \begin{array}{c} z_{1}t \\ \cdot \\ \vdots \\ z_{r}t \end{pmatrix} \left| \begin{pmatrix} (1-\lambda-\eta), (a_{j})_{p} : (c_{j}')_{p_{1}},\cdots, (c_{j}^{(r)})_{p_{r}} \\ \cdot \\ \vdots \\ z_{r}t \end{pmatrix} \right|$$

$$(3.6)$$

where  $\operatorname{Re}(\lambda + \alpha) > 0$ ,  $\operatorname{Re}(\operatorname{slog}(z_i) - \log \sin \pi s_i) < 0, i = 1, \cdots, r$ 

## 4. Riemann-Liouville fractional q-integral.

It is intereting to observe that several result similar to be above results can be deduced from the other result (3.3). Further, the results proved in this paper may find certain applications to the solution of the q- differintegral equations associated with the aforementioned function. If  $\alpha = 0$ , we obtain

$$I_q^{\mu}(f(t) = t^{\mu} I_q^{0,\mu} f(t)$$
(4.1)

where  $I_q^{\mu}(f(t))$  denotes the Riemann-Liouville fractional q-integral operator defined as :

$$I_q^{\mu}\{f(x)\} = \frac{1}{\Gamma_q(\mu)} \int_0^x (x - tq)_{\mu - 1} f(t) d_q t \quad (Re(\mu) > 0, |q| < 1).$$
(4.2)

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We have the following corollary

### Corollary.

$$I_{q}^{\alpha} \left\{ t^{\lambda-1} H_{p,q':V}^{0,n:U} \begin{pmatrix} \mathbf{x}_{1} t^{k_{1}} \\ \cdot \\ \cdot \\ \mathbf{x}_{r} t^{k_{r}} \end{pmatrix} \right\} = \frac{t^{\lambda+\mu-1} \Gamma_{q}(\lambda)}{\Gamma_{q}(\lambda+\alpha)}$$

$$H_{p+1,q'+1:V}^{0,n+1:U} \begin{pmatrix} \mathbf{x}_{1} t^{k_{1}} \\ \cdot \\ \cdot \\ \mathbf{x}_{r} t^{k_{r}} \end{pmatrix} \left| \begin{array}{c} (1-\lambda; k_{1}, \cdots, k_{r}), A:B \\ \cdot \\ \cdot \\ \mathbf{x}_{r} t^{k_{r}} \end{pmatrix} \right| C, (1-\lambda-\alpha; k_{1}, \cdots, k_{r}):D \end{pmatrix}$$

$$(4.3)$$

where  $(\lambda + \alpha) > 0$  and  $Re(slog(z_i) - \log \sin \pi s_i) < 0, i = 1, \cdots, r$ 

#### **Corollary 1.**

$$I_{q}^{\alpha} \left\{ t^{\lambda-1} G_{p,q':p_{1},q_{1};\cdots;p_{r},q_{r}}^{0,n:m_{1},n_{1};\cdots;m_{r},n_{r}} \begin{pmatrix} z_{1}t \\ \vdots \\ z_{r}t \end{pmatrix} \left\{ \begin{array}{c} (a_{j})_{p} : (c_{j}')_{p_{1}},\cdots, (c_{j}^{(r)})_{p_{r}} \\ \vdots \\ (b_{j})_{q'} : (d_{j}')_{q_{1}},\cdots, (d_{j}^{(r)})_{q_{r}} \end{array} \right\} = \frac{t^{\lambda+\mu-1}\Gamma_{q}(\lambda)}{\Gamma_{q}(\lambda+\alpha)}$$

$$G_{p+1,q'+1:p_{1},q_{1};\cdots;p_{r},q_{r}}^{0,n+1:m_{1},n_{1};\cdots;m_{r},n_{r}} \left\{ \begin{array}{c} z_{1}t \\ \vdots \\ z_{r}t \end{array} \right| \left( (1-\lambda), (a_{j})_{p} : (c_{j}')_{p_{1}},\cdots, (c_{j}^{(r)})_{p_{r}} \\ \vdots \\ z_{r}t \end{array} \right)$$

$$(4.4)$$

where  $\operatorname{Re}(\lambda + \alpha) > 0$ ,  $\operatorname{Re}(\operatorname{slog}(z_i) - \log \sin \pi s_i) < 0, i = 1, \cdots, r$ 

If the basic analogue of multivariable H-function reduces to basic of Srivastava-Daoust function, see the work of Yadav et al. [13].

We obtain the same relations with basic analogue of H-function of two variables and one variable and basic analogue of Meijer-function of two variables and one variable. The basic analogue of H-function of two variables and the basic analogue of H-function of one variables are defined by Saxena et al. [7] and [6] respectively.

### 5. Conclusion.

The importance of our all the results lies in their manifold generality. By specialising the various parameters as well as variables in the basic analogue of multivariable H-function and the basic analogue of multivariable Meijer-function, we obtain a large number of results involving remarkably wide variety of useful basic functions ( or product of such basic functions) which are expressible in terms of basic H-function, Basic Meijer's G-function, Basic E-function, basic hypergeometric function of one and two variables and simpler special basic functions of one and two variables. Hence the formulae derived in this paper are most general in character and may prove to be useful in several interesting cases appearing in literature of Pure and Applied Mathematics and Mathematical Physics.

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