Some Aspects on 2-Fuzzy 2-Anti Metric Space

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Abstract

*In this paper the paper 2- fuzzy 2- anti equicontinuity, **α**-2- anti standard metric, **α**-2- anti standard bounded metric, **α**-2 anti uniform metric are introduced. It turns out that 2-fuzzy 2- anti metric space is compact if and only if it is fuzzy complete and fuzzy totally bounded. Some more theorems related to these concepts are proved*

Keywords —- *fuzzy 2- anti equicontinuity, α-2- anti standard metric, α-2- anti standard bounded metric, α-2 anti uniform metric*.

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I. INTRODUCTION

Fuzzy set theory is a useful tool to describe situations in which the data are imprecise or vague. Fuzzy set handle such situation by attributing a degree to which certain object belongs to a set. The idea of fuzzy set was initiated by Zadeh [15] in 1965and thereafter several authors diversified it in various branches of pure and applied mathematics. The concept of fuzzy norm was investigated by Katsaras [10]in 1984. After that in 1992 ,Felbin [5,6] inducted the theory of fuzzy normed linear space. Cheng and Mordeson [4] 1994 established an idea of a fuzzy norm on a linear space.

A satisfactory theory of 2- norm on a linear space has been brought out and developed by Gahler [7]. The concept of fuzzy 2- normed linear space introduced by A.K. Meenakshi and Cokilavani [11] in 2001. R.M SomasundaramamdThangarajBeaula[13] defined the concept of 2-fuzzy 2- normed linear space in 2009. Later, Jebril and Samanta [8] established the definition of fuzzy anti- normed linear space depending on the idea of fuzzy anti norm was introduced by Bag and Samanta [1,2,3] and investigated their important properties.Jialuzhang [9] have defined fuzzy norm in a different way. In 2011 B.Surender Reddy introduced the idea of fuzzy anti 2- normed linear space. In 2012 ParijitSinha, Divya Mishra and GhanshyamLal [12] developed some results on fuzzy anti 2- normed linear space.ThangarajBeaula and Beulah Mariya [14] introduced some apscets on 2- fuzzy 2- anti normed linear space in 2017.

In this paper the concept of 2- fuzzy 2- anti equicontinuity,α-2- anti standard metric, α-2- anti standard bounded metric, α-2 anti uniform metric are defined. Further 2-fuzzy 2-anti euclidean space is developed and some theorems related to above concepts are proved.

II. PRELIMINARIES

For the sake of completeness we reproduce the following definitions due to Gahler[7]RM.Somasundaram and ThangarajBeaula[13] and ThangarajBeaula and Beulah Mariya[14].

Definition2.1

A fuzzy set in X is a map from X to [0,1] it is an element of $[0,1]^{X}$

Definition 2.2[7]

Let X be a real linear space of dimension greater that one and let $\|.,.\|$be a real valued function on X xX satisfying the following conditions

(i) $\|x,y\| = 0$ if and only if x , y are linearly dependent.

(ii) $\|x,y\| = \|y,x\|$.

(iii) $\|\alpha x,y\| = |\alpha|\|x,y\|$ whereα is real.

(iv) $\|x,y+z\| \leq \|x,y\| + \|x,z\|$.

$\|.,.\|$is called a 2- norm on X and the pair (X, $\|.,.\|$) is called 2- norm linear space.

Definition 2.3[13]

Let F(X) =$[0,1]^{X}$ denote the set of all fuzzy sets in X. A 2- fuzzy set on X is a fuzzy set on F(X)

Definition 2.4[13]

Let F(X) be a linear space over the real field K a fuzzy subset N of F(X) x F(X) xR, (R is the set of all real numbers) is called a 2- fuzzy 2-norm on X if and only if

(N1) for all $t \in R$ with $t \leq 0$, $N(f_1, f_2, t) = 0$

(N2) for $t \in R$ with $t > 0$, $N(f_1, f_2, t) = 1$ if and only if f_1, f_2 are linearly dependent

(N3) $N(f_1, f_2, t)$ is invariant under any permutation of f_1, f_2

(N4) for all $t \in R$ with $t \geq 0$, $N(f_1, cf_2, t) = N(f_1, f_2, \frac{t}{|c|})$ if $c \neq 0$ $c \in K$ (field)

(N5) for all $s,t \in R$, $N(f_1, f_2+ f_3, s+t) \geq \min N\{(f_1, f_2, s) , N(f_1, f_3, t)\}$

(N6) $N(f_1, f_2, .) = (0,\infty) \longrightarrow [0,1]$ is continuous.

(N7) $\lim_{t \to \infty} N(f_1, f_2, t) = 1$.

Then the pair (F(X), N) is a fuzzy 2- normed linear space.

Definition2.5[14]

Let X be the linear space over the real field K. A fuzzy subset N*of F(X) x F(X) x R. (R, the set of all real numbers) is called a 2- fuzzy 2 - anti norm on X if and only if

(A-N1) for all $t \in R$ with $t \leq 0$, $N^*(f_1,f_2,t) = 1$.

(A-N2) for all $t \in R$ with $t \geq 0$, $N^*(f_1,f_2,t) = 0$ if and only if f_1 and f_2 are linearly dependent.

(A-N3) $N^*(f_1,f_2,t)$ is invariant under any permutation of f_1,f_2.

(A-N4) for all $t \in R$ with $t \geq 0$, $N^*(f_1, cf_2, t) = N^*(f_1, f_2 ,\frac{t}{|c|})$ if $c \neq 0$, $c \in K$ (field).

(A-N5) for all $s, t \in R$, $N^*(f_1, f_2 + f_3, s + t) \leq \max \{N^*(f_1, f_2, s), N^*(f_1, f_3, t)\}$.

(A-N6) $N^*(f_1, f_2, \cdot) : (0, \infty) \longrightarrow [0,1]$ is continuous.

(A-N7) $\lim_{t \to \infty} N^*(f_1, f_2, t) = 0$.

Then (F(X), N*) is a fuzzy 2- anti normed linear space or (X, N*) is a 2- fuzzy 2- anti normed linear space.

III. MAIN RESULTS

First of all we define 2- fuzzy metric space and the induced α −metric. Later 2- fuzzy 2- anti metric on $\mathcal{C}(\mathcal{F}(X), \mathcal{F}(Y)$, the space of all continuous functions from $\mathcal{F}(X)$ to $\mathcal{F}(Y)$ is defined. Various metrics like Standard bounded metric, uniform metric and supremum metric are defined on it.

3. 2- Fuzzy 2- anti metric space

Definition3.1

A 3 - triple (X , M , *) is said to be a 2 fuzzy metric space if X is anarbitrary set, *is continuous t - norm and M is a fuzzy set on F(X) x F(X) x $(0,\infty)$ satisfying the following condition for all $f, g, h \in F(X)$ with $s , t > 0$.

(i) $M (f , g , t) > 0$.

(ii) $M (f , g , t) = 1$ if and only if $x = y$.

(iii) $M (f , g , t) = M (g , f , t)$.

(iv) $M (f , g , t) * M (g , h , s) \leq M (f , h , t + s)$.

(v) $M (f, g, t) : (0, \infty) \rightarrow (0,1]$ is continuous.

Then M is called a fuzzy metric on X.

Theorem 3.1

Let (X, M, *) be a fuzzy metric space assume that $M(f,g,t) > 0$ for all $t > 0$ and $M(f,g,0) = 0$ for all $f,g \in F(X)$.Define $d_\alpha (f, g) = \inf \{ t : M(f, g, t) \geq \alpha \}$ is an ascending family of α −metric on X. induced by the fuzzy metric M.

Proof

Let $M : F(X) \times F(X) \times (0,\infty) \longrightarrow [0,1]$ be the fuzzy metric on F(X)

(i) If $\inf \{t: M(f, g, t)) \geq \alpha / \alpha \in (0,1)\} = 0$

Then by the definition $d_\alpha (f, g) = 0$ if and only if $f = g$. also if $\inf \{t: M(f, g, t)) \geq \alpha / \alpha \in (0,1)\} > 0$ it is implies that $d_\alpha (f, g) > 0$ for all $f,g \in F(X)$

(ii) $d_\alpha (x, y) = \inf\{ t: M(f, g, t)) \geq \alpha / t \geq 0\}$ for all f, g $g \in F(X)$

$= \inf\{ t: M(g, f, t)) \leq \alpha / t \leq 0\}$

which implies $d_\alpha (f, g) = d_\alpha (g, f)$

(iii) Further to prove that $d_\alpha (f, g) \leq d_\alpha (f, h) + d_\alpha (h, g)$

$d_\alpha(f, h) + d_\alpha (h, g) = \inf \{ t: M(f, h, t)) \geq \alpha / t \geq 0\} + \inf \{ t: M(h, g, s)) \geq \alpha / t \geq 0\}$

$= \inf \{ t+ s: M(f, h, t)) \geq \alpha \text{ and } M(h, g, s)) \geq \alpha\}$
$= \inf \{ r: M(f, g, t)) \geq \alpha \text{ where } t + s = r\}$
$= d_\alpha (f, g)$

Therefore $\{ d_\alpha (.,.) : \alpha \in (0,1)\}$ is a family of α metrics on corresponding to fuzzy metric on X.

Definition3.2

Let $\mathcal{C}(\mathcal{F}(X), \mathcal{F}(Y)$ be the space of all continuous functionsfrom$\mathcal{F}(X)$to$\mathcal{F}(Y)$. The 2- fuzzy 2- anti metric, M on $\mathcal{C}$ (X , Y) is a mapping from $\mathcal{C}$ (X , Y) x $\mathcal{C}$ (X , Y) x $(0, \infty)$ to [0, 1].
such that M(f, g , h , t) = N*(f-g, h, t) for every f, g, h ∈F(X), and t ,s ∈R where N* is a 2- fuzzy 2- anti norm satisfying the following conditions

(i) N* (f, g , h , t) = N*(f-g, h, t) = 1 with $t \leq 0$ for t ∈R

(ii) N*(f, g , h , t) = N*(f-g, h, t) = 0 with t > 0 for t∈R

(iii) N*(f, g , h , t) = N*(f-g, h, t)
= N*(-(g-f), h, t)
= N*(g-f, h, $\frac{t}{|-1|}$)
= N*(g-f, h, t)
= M (g, f, h, t)

(iv) N*(f, g, h, s + t) = N*(f-g, h, s+t)
$\leq$max { (N*(f-g, h, s), N*(f-g, h, t)}
= N* (f, g, h,t) ◊N*(f, g, h, s) where '◊' is the t- co norm.

(v) N*(f, g , h , t) = N*(f-g, h, t) is a continuous mapping from $(0, \infty)$ to [0, 1]

Definition 3.3

Let (F(X), M ,◊) be a 2- fuzzy 2- anti metric space. Let $\mathcal{F}$be the subset of the$\mathcal{C}(\mathcal{F}(X), \mathcal{F}(Y))$. If $f_0 \in \mathcal{F}$, the space $\mathcal{F}$ of functions is said to be 2- fuzzy 2- anti equicontinuous at f_0 if for given$\varepsilon \in (0,1)$

N*(F(f) – F(f_0), G(g), t) < ε for F,G∈ $\mathcal{C}(\mathcal{F}(X), \mathcal{F}(Y))$ and f ,g , f_0 ∈F(X).

Definition 3.4

The α - 2- standard metric d_α corresponding to the 2- fuzzy 2- anti metric N* on $\mathcal{C}(\mathcal{F}(X), \mathcal{F}(Y))$ is defined as $d_\alpha(F, G)$= inf{ t : N* (F(f) - G(g), H, t) ≤ αforall F,G, H ∈ $\mathcal{C}(\mathcal{F}(X), \mathcal{F}(Y))$}.

Definition 3.5

The α - 2- standard bounded metric $\overline{d}_\alpha$corresponding to the 2- fuzzy 2- anti metric N* on $\mathcal{C}(\mathcal{F}(X), \mathcal{F}(Y))$ is defined as $\overline{d}_\alpha$ (F ,G) = min { d_α (F, G) , 1}

Definition 3.6

The α - 2- uniform metric$\overline{\rho}_\alpha$ on $\mathcal{C}(\mathcal{F}(X), \mathcal{F}(Y))$ is defined as $\overline{\rho}_\alpha$ (F ,G) = sup{ $\overline{d}_\alpha$ (F ,G) / f, g ∈ $\mathcal{F}(X)$}

Definition 3.7

A fuzzy 2- anti metric space (F(X), M , ◊) is said to be fuzzy totally bounded if for everyr ∈(0,1), there is a finite covering for $\mathcal{F}$ by r- balls, B(f, g, r, t) with 0 < t <1.

Definition 3.8

If F(X) is a 2- fuzzy 2- anti normed linear space and (F(Y), M , ◊) is a 2- fuzzy 2- anti metric space. Define asupremum metric on $\mathcal{B}(\mathcal{F}(X), \mathcal{F}(Y))$ the set of all bounded functions from $\mathcal{F}(X)$ to $\mathcal{F}(Y)$ as,

ρ_α (F ,G) = Sup{d_α (F(f) ,G(g)) / f, g ∈ $\mathcal{F}(X)$where d_α is the α-metric induced byM.

Definition 3.9

F(X) is said to be fuzzy compact space if every open covering of F(X) contains finite sub collection that also covers F(X).

Theorem 3.2

Let ($\mathcal{C}(\mathcal{F}(X), \mathcal{F}(Y)$, M, ◊) be a 2- fuzzy 2- anti metric space Assume that M (F, G , h , t) < 0 for all t > 0 and M (F, G, h, θ) = 1 for all F, G∈ $\mathcal{C}(\mathcal{F}(X), \mathcal{F}(Y)$ define the α metric
d_α (F, G) = inf { t : N*(F(f) – G(g), H, t) ≤ α / α ∈(0,1)} where H ∈ $\mathcal{C}(\mathcal{F}(X), \mathcal{F}(Y)$ is a descending family of α metric on $\mathcal{C}(\mathcal{F}(X), \mathcal{F}(Y)$.

(i) If inf {t : N*(F(f) –G(g), H, t) ≤ α / α ∈(0,1)}= 0 then d_α (F, G, H) = 0 if and only if F = G
By definition, inf{N*(F(f) –G(g), H, t) ≤ α / α ∈(0,1)} ≥ 0, so d_α (F, G, H) ≥ 0 for all f, g, h ∈ $\mathcal{F}(X)$

(ii) d_α (F, G, H) = inf {t : N*(F(f) – G(g), H, t) ≤ α}
= inf {t : N*(– G(g)– F(f), H, t) ≤ α}

$$= \inf\{t : N^*(G(g)- F(f), H, \tfrac{t}{|-1|}) \le \alpha\}$$
$$= d_\alpha (G, F, H)$$

(iii) $d_\alpha (F, G, H) = d_\alpha (F, P, H) + d_\alpha (P, G, H)\}$

$$d_\alpha (F, P, H) + d_\alpha (P, G, H) = \inf \{t : N^*(F(f) - P(p), H, t) \le \alpha + \inf \{s : N^*(P(p) - G(g), H, t) \le \alpha \}$$
$$= \inf \{t + s : N^*(F(f) - G(g), H, t) \le \alpha \}$$
$$= \inf \{ r : N^*(F(f) - G(g), H, t) \le \alpha ; r = s + t\}$$
$$= d_\alpha (F, G, H)$$

Therefore { $d_\alpha(. , .) : \alpha \in (0,1)\}$ is a descending family of α metric on $\mathcal{F}(X)$ corresponding to the 2-fuzzy 2- anti metric on $\mathcal{F}(X)$.

Let $0 < \alpha_1 < \alpha_2$ then $d_{\alpha_1} (F,G, H) = \inf \{ t : N^*(F(f) - G(g), H, t) \le \alpha_1\}$

$d_{\alpha_2} (F,G, H) = \inf \{ t : N^*(F(f) - G(g), H, t) \le \alpha_2\}$

As $\alpha_1 < \alpha_2$

$$\{t : N^*(F(f) - G(g), H, t) \ge \alpha_2\} \supseteq \{ t : N^*(F(f) - G(g), H, t) \ge \alpha_1\}$$

implies $\inf \{ t : N^*(F(f) - G(g), H, t) \ge \alpha_2\} \supseteq \inf \{ t : N^*(F(f) - G(g), H, t) \ge \alpha_1\}$

Therefore, $d_{\alpha_2} (F,G, H) \le d_{\alpha_1} (F,G, H)$

Hence, $0 < \alpha_1 < \alpha_2$ implies $d_{\alpha_1} (F,G, H) \ge d_{\alpha_2} (F,G, H)$ therefore it is a descending family of α – metric.

Example3.1

Let $\mathcal{F}(X)$ be the non empty set and $N^* : \mathcal{C}(X , Y) \times \mathcal{C} (X , Y) \times (0, \infty) \to [0, 1]$ be defined by

$$N^*(F(f) - G(g), H, t) = \begin{cases} 0 & \text{for } t > d(f,g,t) \\ 1 & \text{for } t \le d(f,g,t) \end{cases}$$

Then fore $\alpha \in [0,1)$, $d_\alpha (f, g) = d (f, g)$ is the 2- fuzzy 2- anti crisp metric.

Theorem3.3

The space ($\mathcal{F}(X)$, d_α) is complete if every cauchy sequence in $\mathcal{F}(X)$ has a convergent subsequence.

Proof:

Let (f_n) be a Cauchy sequence in ($\mathcal{F}(X)$, d_α) to show that if (f_n) has a subsequence { f_{n_i} } that converges to a point f then the sequence (f_n) itself converges to f.

Given ε ($0<\varepsilon < 1$) choose N large enough that $d_\alpha(f_n , f_m) < \frac{\varepsilon}{2}$ for n,m ≥ N. [using the fact (f_n) is a Cauchy sequence]

Then choose an integer i large enough that $n_i \ge N$ and $d_\alpha(f_{n_i} , f ,g) < \frac{\varepsilon}{2}$.

Using the fact $n_1 < n_2 < \ldots\ldots$ is an increasing sequence of integer and suppose f_{n_i} converges to f for all n≥ N,

$$d_\alpha(f_n , f) \le d_\alpha (f_n ,f_{n_i}) + d_\alpha (f_{n_i} , f) \; d_\alpha (f_n ,f_{n_i}) + d_\alpha (f_{n_i}, f)$$
$$= \inf \{ t : N^*(f_n - f_{n_i}, g , t) \le \alpha \} + \inf \{s : N^*(f_{n_i} - f, g , t) \le \alpha \}$$
$$= \inf\{ t+s : N^*(f_n - f, g , t) \le \alpha \}$$
$$= \inf\{ r : N^*(f_n - f, g , t) \le \alpha \}$$
$$= d_\alpha(f_n , f) < \varepsilon.$$

Theorem 3.4

A fuzzy 2- anti metric space $(F(X),d_\alpha)$ is compact if and only if it is fuzzy complete and fuzzy totally bounded.

Proof

Suppose F(X) is a fuzzy compact metric space, every open covering of F(X) contains a finite sub collection covering F(X) and so F(X) is fuzzy totally bounded.

Conversely, let F(X) be a fuzzy 2 complete and fuzzy2- totally bounded to prove that F(X) is sequentially fuzzy compact.

Let (f_n) be a sequence of functions in F(X). Construct a subsequence (f_{n_i}) which is a cauchy sequence, so that it is necessarily converges. First cover F(X) by finitely many balls B (f, g, r , t_1) with radius r, where 0< r < 1 so that atleast one of these balls contains infinitely many f_n ‘s

Let J_1 be the subset of $\mathbb{Z}_+$ consisting of those indices n for which $f_n \in B_1$ (f, g, r ,t_1). Next cover F(X) by finitely many balls with radius $\frac{r}{2}$. Because J_1 is infinite at least one of these balls say B_2 (f, g ,$\frac{r}{2}$, t_2) must contain f_n for infinitely many values of n in J_1.

Choose J_2 to be the set of those indices n for $n \in J_1$ and $f_n \in B_2$ (f, g ,$\frac{r}{2}$, t_2). In general given J_k set of positive integer. Choose J_{k+1} to be the subset of J_k such that there is a ball B_{k+1} (f, g , $\frac{r}{k+1}$, t_{k+1}) of radius $\frac{r}{k+1}$ that contains f_n for all $n \in J_{k+1}$

Choose $n_1 \in J_1$ given n_k, choose $n_{k+1} \in J_{K+1}$ such that $n_{k+1} > n_k$. Now i , j ≥ k the indices both n_i and n_j belongs to J_k (because $J_1 \supset J_2$…… nested sequence) Therefore the function f_{n_i} and f_{n_j} are contained in a ball B_k (f, g , $\frac{r}{k}$, t_k). It follows that the sequence (f_{n_i}) is a Cauchy sequence as desired.

Theorem3.5

Let F(X) be a fuzzy 2- anti normed linear space. Let (F(Y), M ,◊) be a 2- fuzzy 2- anti metric space. If the subspace $\mathcal{F}$ of $\mathcal{C}(\mathcal{F}(X), \mathcal{F}(Y))$ is totally bounded under the uniform metric $\overline{\rho}_\alpha$ corresponding to the 2- fuzzy 2- anti metric d_α then $\mathcal{F}$ is equicontinuous under the metric d_α.

Proof

Assume $\mathcal{F}$ is totally bounded, $\mathcal{F}$ is covered by finite number of ∈- balls, B(F_1, G,ε , t), B(F_2, G,ε , t), ……. B(F_n, G,∈ , t). where $\delta = \frac{\varepsilon}{3}$ $0 < \delta < 1$ and $0 < \varepsilon < 1$ in $\mathcal{C}(\mathcal{F}(X), \mathcal{F}(Y))$, $F_i \in \mathcal{C}(\mathcal{F}(X), \mathcal{F}(Y))$. As each F_i is continuous at f_0 ,d_α (F_i (f), F_0 (f)) < δ for each f ∈ B(f, f_0, ε, t)

Let F be the arbitrary function of $\mathcal{F}$ then F belongs to at least one of the δ- balls say B(F_i, G, ε , t). Then for any f ∈ B(f, f_0, ε , t)

$$\overline{d}_\alpha (F(f) , F_i (f)) < \delta$$
$$d_\alpha (F_i (f), F_i(f_0)) < \delta$$
$$\overline{d}_\alpha (F_i (f_0), F(f_0)) < \delta$$

The first and third inequality holds because $\overline{\rho}_\alpha$ (F, F_i , G, H) < δ and the second inequality holds because f ∈ B(f, f_0,ε , t). Since δ< 1 the first and third inequality also hold if $\overline{d}_\alpha$ is replaced by d_α then the triangle inequality implies that for all f ∈ B(f, f_0,ε , t) and F, G ∈ $\mathcal{C}(\mathcal{F}(X), \mathcal{F}(Y))$.

$$d_\alpha(F(f), F(f_0)) < \varepsilon \text{ as desired.}$$

Theorem3.6

Let $\mathcal{F}$ be a subset of $\mathcal{C}(\mathcal{F}(X), \mathcal{F}(Y))$. $\mathcal{F}$ is totally bounded under supremum metric ρ_α is equivalent to $\mathcal{F}$ is totally bounded under uniform metric $\overline{\rho}_\alpha$.

Proof

To show that every ε – ball under ρ_α is equivalent to the ε – ball under $\overline{\rho}_\alpha$ where ρ_α is supremum metric and $\overline{\rho}_\alpha$ is the uniform metric corresponding to the 2- fuzzy 2- anti normed linear space.

By definition, ρ_α (F, H) = sup { d_α (F(f), H(h))/ f, g ∈F(X) } and

$$\overline{\rho}_\alpha (F, H) = \sup \{\overline{d}_\alpha (F(f) , H(h))/ f, g \in F(X) \}$$

$$\text{where } \overline{d}_\alpha (F, H) = \min \{ d_\alpha (F, H) , 1\} \text{ implies } \overline{d}_\alpha (F, G) \leq d_\alpha(F, G)$$

Therefore,

$$\begin{aligned}\overline{\rho}_\alpha (F, H) &= \sup \{\overline{d}_\alpha (F(f) , H(h) / f, g \in F(X) \}\\ &\leq \sup \{d_\alpha (F(f), H(h)/ f, g \in F(X) \}\\ &= \rho_\alpha(F, G)\end{aligned}$$

Which implies

$$\overline{\rho}_\alpha (F, H) \leq \rho_\alpha(F, H)$$

If H is an element in B_{ρ_α} (F, H, ε , t) then ρ_α (F, H) < ε (By using the above inequality)

Implies $\overline{\rho}_\alpha$ (F, H) < ε .Therefore, H ∈ B_{ρ_α} (F, G, ε , t).

Let $B_{\overline{\rho}_\alpha}$ (F, G, ε , t) ⊂ B_{ρ_α} (F, G, ε , t). Let H be an element in $B_{\overline{\rho}_\alpha}$ (F, G, ε , t) Then $\overline{\rho}_\alpha$ (F, H) < ε

$$\text{Sup } \{ \overline{d}_\alpha (F(f) , H(h)/ f, g \in F(X) \} < \varepsilon$$
$$\overline{d}_\alpha (F(f) , H(h)/ f, g \in F(X) \} < \varepsilon$$
$$\min \{ d_\alpha (F(f) , H(h) , 1\} < \varepsilon$$
$$\{\overline{d}_\alpha (F(f) , H(h)\} < \varepsilon$$
$$\text{Sup } \{ d_\alpha (F(f) , H(h) / f, g \in F(X) \} < \varepsilon$$
$$\Rightarrow \rho_\alpha(F, G, H) < \varepsilon$$

Assume that $\mathcal{F}$ is equicontinuous given ε > 0 cover $\mathcal{F}$- by finitely many ε- balls B_{ρ_α} (f, f_0, ε , t). with respect to metric such that

N*(F(f)- F(f_0), G(g), t) <δ for all f, f_0∈F(X) and F,G ∈ $\mathcal{C}(\mathcal{F}(X), \mathcal{F}(Y))$.

We cover F(X) by finitely many ε- balls, B_{d_α}(f_1, u, ε, t), B_{d_α}(f_2,u ,ε, t)……. B_{d_α}(f_k,u, ε, t)Then again cover F(Y) by finitely many ε- balls B_{d_α}(g_1, h, ε, t) , B_{d_α}(g_2, h, ε, t)…….. B_{d_α}(g_m, h, ε, t)Let J be the collection of functions exist a function α : { 1,……k} → {1,……m} given α ∈ J if there exist a function F of $\mathcal{F}$. Such that F(f_i) ∈ B_{d_α}(g_{α_i}, h, ε, t)for each i = 1,…..k, denote the function F_{f_α} the collection of {f_α} is indexed by a J' of the index J

To assert that the ε- balls B_{ρ_α} (F,f_α, ε , t) for α ∈J' cover $\mathcal{F}$. Let G be an element of $\mathcal{F}$ for each i=1,……k cover $\mathcal{F}$

Choose an integer α (i) such that $F(f_i) \in B_{d_\alpha}(G_{\alpha(i)}, H, \varepsilon, t)$ then the function α is in J' we assert that F belongs to the ball $B_{\rho_\alpha}(F, F_\alpha, \varepsilon, t)$.

Let g∈F(X) choose i so that $g \in B_{\rho_\alpha}(f, f_i, \varepsilon, t)$. Let g∈F(X), choose i so that $f \in (f, f_i, \varepsilon, t)$

Then we get $N^*(F(g) - F(f_i), H(h), t) < \delta$

$$N^*(F(f_i) - F_\alpha(f_i), H(h), t) < \delta$$
$$N^*(F_\alpha(f_i) - F_\alpha(g), H(h), t) < \delta$$

It follows that $N^*(F(g) - F_\alpha(g), H(h), t) \leq \max\{N^*(F(g) - F(f_i), H(h), t), N^*(F(f_i) - F_\alpha(f_i), H(h), t), N^*(F_\alpha(f_i) - F_\alpha(g), H(h), t)\} < \delta$

Since this inequality holds, $\rho_\alpha(F, F_\alpha, H) = \max\{d_\alpha(F(g), F_\alpha(g))\}$ and

$d_\alpha(F(g), F_\alpha(g), H\} = \inf\{t : N^*(F(g) - F_\alpha(g), H(h), t) \leq \alpha\}$. Therefore $F \in B_{\rho_\alpha}(F, F_\alpha, \varepsilon, t)$.

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