

# Translations of Q-Intuitionistic Fuzzy Subbigroup of a Bigroup

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## Abstract

In this paper, we made an attempt to study the algebraic nature of translations of Q-intuitionistic fuzzy subbigroup of a bigroup.

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## Key Words

*Bigroup, fuzzy subset, intuitionistic fuzzy subset, Q-fuzzy subbigroup, Q-intuitionistic fuzzy subbigroup, product, psudo Q-intuitionistic fuzzy coset, A<sup>o</sup>.*

## I. INTRODUCTION

In 1965, the fuzzy subset was introduced by L.A.Zadeh [10, 11], after that several researchers explored on the generalization of the concept of fuzzy sets. The concept of intuitionistic fuzzy subset was introduced by K.T.Aтанassов [2, 3], as a generalization of the notion of fuzzy set. The notion of fuzzy subgroups was introduced by Azriel Rosenfeld [4]. Palaniappan.N & K.Arjunan [7] defined the intuitionistic fuzzy subgroups of a group. T.Justin Prabu and K.Arjunan[5, 6] defined the Q-fuzzy subbigroups of a bigroup. In this paper, we introduce the some theorems in translations of Q-intuitionistic fuzzy subbigroup of a bigroup.

## II.PRELIMINARIES

**Definition 1.1[5]** A set  $(G, +, \bullet)$  with two binary operations  $+$  and  $\bullet$  is called a bigroup if there exist two proper subsets  $G_1$  and  $G_2$  of  $G$  such that (i)  $G = G_1 \cup G_2$  (ii)  $(G_1, +)$  is a group (iii)  $(G_2, \bullet)$  is a group.

**Definition 1.2[10]** Let  $X$  be a non-empty set. A **fuzzy subset**  $A$  of  $X$  is a function  $A: X \rightarrow [0, 1]$ .

**Definition 1.3[3]** Let  $X$  be a non-empty set. A **intuitionistic fuzzy subset**  $A$  in  $X$  is defined as an object of the form  $A = \{<x, \mu_A(x), v_A(x)> / x \in X\}$ , where  $\mu_A: X \rightarrow [0, 1]$  and  $v_A: X \rightarrow [0, 1]$  define the degree of membership and the degree of non-membership of the element  $x \in X$  respectively and for every  $x \in X$  satisfying  $\mu_A(x) + v_A(x) \leq 1$ .

**Definition 1.4[5]** Let  $(G, +)$  be a group and  $Q$  be a non-empty set. A fuzzy subset  $A$  of  $G \times Q$  is said to be a Q-fuzzy subgroup of  $G$  if  $\mu_A(x-y, q) \geq \min \{ \mu_A(x, q), \mu_A(y, q) \}$  for all  $x$  and  $y$  in  $G$  and  $q$  in  $Q$ .

**Definition 1.5.** Let  $(G, +)$  be a group and  $Q$  be a non-empty set. An intuitionistic fuzzy subset  $A$  of  $G \times Q$  is said to be a Q-intuitionistic fuzzy subgroup of  $G$  if it satisfies the following axioms: (i)  $\mu_A(x-y, q) \geq \min \{ \mu_A(x, q), \mu_A(y, q) \}$  (ii)  $v_A(x-y, q) \leq \max \{ v_A(x, q), v_A(y, q) \}$  for all  $x$  and  $y$  in  $G$  and  $q$  in  $Q$ .

**Definition 1.6.** Let  $G = (G_1 \cup G_2, +, \bullet)$  be a bigroup. Then a fuzzy set  $A$  of  $G$  is said to be a fuzzy subbigroup of  $G$  if there exist two fuzzy subsets  $A_1$  of  $G_1$  and  $A_2$  of  $G_2$  such that (i)  $A = A_1 \cup A_2$  (ii)  $A_1$  is a fuzzy subgroup of  $(G_1, +)$  (iii)  $A_2$  is a fuzzy subgroup of  $(G_2, \bullet)$ .

**Definition 1.7.** Let  $G = (G_1 \cup G_2, +, \bullet)$  be a bigroup. Then an Q-intuitionistic fuzzy set  $A$  of  $G$  is said to be an Q-intuitionistic fuzzy subbigroup of  $G$  if there exist two Q-intuitionistic fuzzy subsets  $A_1$  of  $G_1$  and  $A_2$  of  $G_2$  such that (i)  $A = A_1 \cup A_2$  (ii)  $A_1$  is an Q-intuitionistic fuzzy subgroup of  $(G_1, +)$  (iii)  $A_2$  is an Q-intuitionistic fuzzy subgroup of  $(G_2, \bullet)$ .

**Definition 1.8.** Let  $A$  be a Q-intuitionistic fuzzy subbigroup of a bigroup  $G$ . Then  $A^0 = M^0 \cup N^0$  is defined as  $\mu_M^0(x, q) = \mu_M(x, q) / \mu_M(0, q)$ ,  $v_M^0(x, q) = v_M(x, q) v_M(0, q)$  for all  $x \in E$ ,  $q \in Q$ , identity  $0 \in E$  and  $\mu_N^0(x, q) = \mu_N(x, q) / \mu_N(1, q)$ ,  $v_N^0(x, q) = v_N(x, q) v_N(1, q)$  for all  $x \in F$ ,  $q \in Q$  and identity  $1 \in F$ .

**Definition 1.9.** Let  $A$  be a Q-intuitionistic fuzzy subbigroup of a bigroup  $G$ ,  $a \in E$  and  $b \in F$ . Then the **pseudo Q-intuitionistic fuzzy coset**  $(abA)^p = \langle (a\mu_M)^p \cup (b\mu_N)^p, (av_M)^p \cup (bv_N)^p \rangle$  is defined by  $(a\mu_M)^p(x, q) = p(a)\mu_M(x, q)$ ,

$(av_M)^P(x, q) = p(a)v_M(x, q)$  for every  $x \in E$ ,  $p \in P$ ,  $q \in Q$  and  $(b\mu_N)^P(x, q) = p(b)\mu_N(x, q)$ ,  $(bv_N)^P(x, q) = p(b)v_N(x, q)$  for every  $x \in F$ ,  $p \in P$  and  $q \in Q$ .

**Definition 1.10.** Let A and B be any two Q-intuitionistic fuzzy subbigroups of bigroups G and S respectively. The product of A and B, denoted by  $A \times B = (M \times O) \cup (N \times O) \cup (M \times P) \cup (N \times P)$ , is defined as  $\mu_{M \times O}(x, y, q) = \min \{ \mu_M(x, q), \mu_O(y, q) \}$ ,  $v_{M \times O}(x, y, q) = \max \{ v_M(x, q), v_O(y, q) \}$  for all  $x \in E$ ,  $y \in H$ ,  $q \in Q$  and  $\mu_{N \times O}(x, y, q) = \min \{ \mu_N(x, q), \mu_O(y, q) \}$ ,  $v_{N \times O}(x, y, q) = \max \{ v_N(x, q), v_O(y, q) \}$  for all  $x \in F$ ,  $y \in H$ ,  $q \in Q$ ,  $\mu_{M \times P}(x, y, q) = \min \{ \mu_M(x, q), \mu_P(y, q) \}$ ,  $v_{M \times P}(x, y, q) = \max \{ v_M(x, q), v_P(y, q) \}$  for all  $x \in E$ ,  $y \in I$ ,  $q \in Q$ ,  $\mu_{N \times P}(x, y, q) = \min \{ \mu_N(x, q), \mu_P(y, q) \}$ ,  $v_{N \times P}(x, y, q) = \max \{ v_N(x, q), v_P(y, q) \}$  for all  $x \in F$ ,  $y \in I$  and  $q \in Q$ .

**Definition 1.11.** Let A be a Q-intuitionistic fuzzy subbigroup of a bigroup G. The strongest Q-intuitionistic fuzzy relation on G is a Q-intuitionistic fuzzy relation  $V = (M \times M) \cup (N \times N) \cup (M \times N) \cup (N \times M)$  with respect to A given by  $\mu_{M \times M}(x, y, q) = \min \{ \mu_M(x, q), \mu_M(y, q) \}$ ,  $v_{M \times M}(x, y, q) = \max \{ v_M(x, q), v_M(y, q) \}$  for all  $x, y \in E$ ,  $q \in Q$ ,  $\mu_{N \times N}(x, y, q) = \min \{ \mu_N(x, q), \mu_N(y, q) \}$ ,  $v_{N \times N}(x, y, q) = \max \{ v_N(x, q), v_N(y, q) \}$  for all  $x, y \in F$ ,  $q \in Q$ ,  $\mu_{M \times N}(x, y, q) = \min \{ \mu_M(x, q), \mu_N(y, q) \}$ ,  $v_{M \times N}(x, y, q) = \max \{ v_M(x, q), v_N(y, q) \}$  for all  $x \in E$ ,  $y \in F$ ,  $q \in Q$ ,  $\mu_{N \times M}(x, y, q) = \min \{ \mu_N(x, q), \mu_M(y, q) \}$ ,  $v_{N \times M}(x, y, q) = \max \{ v_N(x, q), v_M(y, q) \}$  for all  $x \in F$ ,  $y \in E$  and  $q \in Q$ .

**Definition 1.12[3]** Let A be an intuitionistic fuzzy subset of X. Then the following translations are defined as (i)  $\Lambda(A) = \{ \langle x, \min \{ \frac{1}{2}, \mu_A(x) \}, \max \{ \frac{1}{2}, v_A(x) \} \rangle / \text{for all } x \in X \}$ .

(ii)  $\Theta(A) = \{ \langle x, \max \{ \frac{1}{2}, \mu_A(x) \}, \min \{ \frac{1}{2}, v_A(x) \} \rangle / \text{for all } x \in X \}$ .

(iii)  $Q_{\alpha, \beta}(A) = \{ \langle x, \min \{ \alpha, \mu_A(x) \}, \max \{ \beta, v_A(x) \} \rangle / \text{for all } x \in X, \alpha, \beta \in [0, 1], \alpha + \beta \leq 1 \}$ .

(iv)  $P_{\alpha, \beta}(A) = \{ \langle x, \max \{ \alpha, \mu_A(x) \}, \min \{ \beta, v_A(x) \} \rangle / \text{for all } x \in X, \alpha, \beta \in [0, 1], \alpha + \beta \leq 1 \}$ .

(v)  $G_{\alpha, \beta}(A) = \{ \langle x, \alpha \mu_A(x), \beta v_A(x) \rangle / \text{for all } x \in X \text{ and } \alpha, \beta \text{ in } [0, 1] \}$ .

**Example 1.13.**  $A = \{ \langle a, 0.5, 0.4 \rangle, \langle b, 0.1, 0.2 \rangle, \langle c, 0.3, 0.2 \rangle \}$  is an intuitionistic fuzzy subset of  $X = \{ a, b, c \}$ . If  $\alpha = 0.3$  and  $\beta = 0.4$ ,

(i)  $\Lambda(A) = \{ \langle a, 0.5, 0.5 \rangle, \langle b, 0.1, 0.5 \rangle, \langle c, 0.3, 0.5 \rangle \}$ .

(ii)  $\Theta(A) = \{ \langle a, 0.5, 0.4 \rangle, \langle b, 0.5, 0.2 \rangle, \langle c, 0.5, 0.2 \rangle \}$ .

(iii)  $Q_{0.3, 0.4}(A) = \{ \langle a, 0.3, 0.4 \rangle, \langle b, 0.1, 0.4 \rangle, \langle c, 0.3, 0.4 \rangle \}$ .

(iv)  $P_{0.3, 0.4}(A) = \{ \langle a, 0.5, 0.4 \rangle, \langle b, 0.3, 0.2 \rangle, \langle c, 0.3, 0.2 \rangle \}$ .

(v)  $G_{0.3, 0.4}(A) = \{ \langle a, 0.15, 0.16 \rangle, \langle b, 0.03, 0.08 \rangle, \langle c, 0.09, 0.08 \rangle \}$ .

## II. PROPERTIES

**Theorem 2.1.** If A is a Q-intuitionistic fuzzy subbigroup of a bigroup G, then  $\Lambda A = \Lambda M \cup \Lambda N$  is a Q-intuitionistic fuzzy subbigroup of G.

**Proof.** For every  $x, y \in E$  and  $q \in Q$ , then  $\mu_{\Lambda M}(x-y, q) = \min \{ \frac{1}{2}, \mu_M(x-y, q) \} \geq \min \{ \frac{1}{2}, \min \{ \mu_M(x, q), \mu_M(y, q) \} \} = \min \{ \min \{ \frac{1}{2}, \mu_M(x, q) \}, \min \{ \frac{1}{2}, \mu_M(y, q) \} \} = \min \{ \mu_{\Lambda M}(x, q), \mu_{\Lambda M}(y, q) \}$  for all  $x, y \in E$ ,  $q \in Q$ . For every  $x, y \in E$  and  $q \in Q$ , then  $v_{\Lambda M}(x-y, q) = \max \{ \frac{1}{2}, v_M(x-y, q) \} \leq \max \{ \frac{1}{2}, \max \{ v_M(x, q), v_M(y, q) \} \} = \max \{ \max \{ \frac{1}{2}, v_M(x, q) \}, \max \{ \frac{1}{2}, v_M(y, q) \} \} = \max \{ v_{\Lambda M}(x, q), v_{\Lambda M}(y, q) \}$  for all  $x, y \in E$ ,  $q \in Q$ . For every  $x, y \in F$  and  $q \in Q$ , then  $\mu_{\Lambda N}(xy^{-1}, q) = \min \{ \frac{1}{2}, \mu_N(xy^{-1}, q) \} \geq \min \{ \frac{1}{2}, \min \{ \mu_N(x, q), \mu_N(y, q) \} \} = \min \{ \min \{ \frac{1}{2}, \mu_N(x, q) \}, \min \{ \frac{1}{2}, \mu_N(y, q) \} \} = \min \{ \mu_{\Lambda N}(x, q), \mu_{\Lambda N}(y, q) \}$  for all  $x, y \in F$ ,  $q \in Q$ . For every  $x, y \in F$ ,  $q \in Q$ , then  $v_{\Lambda N}(xy^{-1}, q) = \max \{ \frac{1}{2}, v_N(xy^{-1}, q) \} \leq \max \{ \frac{1}{2}, \max \{ v_N(x, q), v_N(y, q) \} \} = \max \{ \max \{ \frac{1}{2}, v_N(x, q) \}, \max \{ \frac{1}{2}, v_N(y, q) \} \} = \max \{ v_{\Lambda N}(x, q), v_{\Lambda N}(y, q) \}$  for all  $x, y \in F$ ,  $q \in Q$ . Hence  $\Lambda A = \Lambda M \cup \Lambda N$  is a Q-intuitionistic fuzzy subbigroup of G.

**Theorem 2.2.** If A is a Q-intuitionistic fuzzy subbigroup of a bigroup G, then  $\Theta A = \Theta M \cup \Theta N$  is a Q-intuitionistic fuzzy subbigroup of G.

**Proof.** For every  $x, y \in E$  and  $q \in Q$ , then  $\mu_{\Theta M}(x-y, q) = \max \{ \frac{1}{2}, \mu_M(x-y, q) \} \geq \max \{ \frac{1}{2}, \min \{ \mu_M(x, q), \mu_M(y, q) \} \} = \min \{ \max \{ \frac{1}{2}, \mu_M(x, q) \}, \max \{ \frac{1}{2}, \mu_M(y, q) \} \} = \min \{ \mu_{\Theta M}(x, q), \mu_{\Theta M}(y, q) \}$  for all  $x, y \in E$ ,  $q \in Q$ . For every  $x, y \in E$  and  $q \in Q$ , then  $v_{\Theta M}(x-y, q) = \min \{ \frac{1}{2}, v_M(x-y, q) \} \leq \min \{ \frac{1}{2}, \max \{ v_M(x, q), v_M(y, q) \} \} = \max \{ \min \{ \frac{1}{2}, v_M(x, q) \}, \min \{ \frac{1}{2}, v_M(y, q) \} \} = \max \{ v_{\Theta M}(x, q), v_{\Theta M}(y, q) \}$  for all  $x, y \in E$ ,  $q \in Q$ . For every  $x, y \in F$  and  $q \in Q$ , then  $\mu_{\Theta N}(xy^{-1}, q) = \max \{ \frac{1}{2}, \mu_N(xy^{-1}, q) \} \geq \max \{ \frac{1}{2}, \min \{ \mu_N(x, q), \mu_N(y, q) \} \} = \min \{ \max \{ \frac{1}{2}, \mu_N(x, q) \}, \max \{ \frac{1}{2}, \mu_N(y, q) \} \} = \min \{ \mu_{\Theta N}(x, q), \mu_{\Theta N}(y, q) \}$  for all  $x, y \in F$ ,  $q \in Q$ . For every  $x, y \in F$ ,  $q \in Q$ , then  $v_{\Theta N}(xy^{-1}, q) = \min \{ \frac{1}{2}, v_N(xy^{-1}, q) \} \leq \min \{ \frac{1}{2}, \max \{ v_N(x, q), v_N(y, q) \} \} = \max \{ \min \{ \frac{1}{2}, v_N(x, q) \}, \min \{ \frac{1}{2}, v_N(y, q) \} \} = \max \{ v_{\Theta N}(x, q), v_{\Theta N}(y, q) \}$  for all  $x, y \in F$ ,  $q \in Q$ . Hence  $\Theta A = \Theta M \cup \Theta N$  is a Q-intuitionistic fuzzy subbigroup of G.

**Theorem 2.3.** If A is a Q-intuitionistic fuzzy subbigroup of a bigroup G, then  $Q_{\alpha, \beta}(A) = Q_{\alpha, \beta}(M) \cup Q_{\alpha, \beta}(N)$  is a Q-intuitionistic fuzzy subbigroup of G.

**Proof.** For every  $x, y \in E$  and  $q \in Q$  and  $\alpha, \beta \in [0, 1]$  and  $\alpha + \beta \leq 1$ , then  $\mu_{Q_{(\alpha, \beta)}(M)}(x-y, q) = \min \{ \alpha, \mu_M(x-y, q) \} \geq \min \{ \alpha, \min \{ \mu_M(x, q), \mu_M(y, q) \} \} = \min \{ \min \{ \alpha, \mu_M(x, q) \}, \min \{ \alpha, \mu_M(y, q) \} \} = \min \{ \mu_{Q_{(\alpha, \beta)}(M)}(x, q), \mu_{Q_{(\alpha, \beta)}(M)}(y, q) \}$  for all  $x, y \in E, q \in Q$ . And  $\nu_{Q_{(\alpha, \beta)}(M)}(x-y, q) = \max \{ \beta, v_M(x-y, q) \} \leq \max \{ \beta, \max \{ v_M(x, q), v_M(y, q) \} \} = \max \{ \max \{ \beta, v_M(x, q) \}, \max \{ \beta, v_M(y, q) \} \} = \max \{ \nu_{Q_{(\alpha, \beta)}(M)}(x, q), \nu_{Q_{(\alpha, \beta)}(M)}(y, q) \}$  for all  $x, y \in E, q \in Q$ . For every  $x, y \in F$  and  $q \in Q$  and  $\alpha, \beta \in [0, 1]$  and  $\alpha + \beta \leq 1$ , then  $\mu_{Q_{(\alpha, \beta)}(N)}(xy^{-1}, q) = \min \{ \alpha, \mu_N(xy^{-1}, q) \} \geq \min \{ \alpha, \min \{ \mu_N(x, q), \mu_N(y, q) \} \} = \min \{ \min \{ \alpha, \mu_N(x, q) \}, \min \{ \alpha, \mu_N(y, q) \} \} = \min \{ \mu_{Q_{(\alpha, \beta)}(N)}(x, q), \mu_{Q_{(\alpha, \beta)}(N)}(y, q) \}$  for all  $x, y \in F, q \in Q$ . And  $\nu_{Q_{(\alpha, \beta)}(N)}(xy^{-1}, q) = \max \{ \beta, v_N(xy^{-1}, q) \} \leq \max \{ \beta, \max \{ v_N(x, q), v_N(y, q) \} \} = \max \{ \max \{ \beta, v_N(x, q) \}, \max \{ \beta, v_N(y, q) \} \} = \max \{ \nu_{Q_{(\alpha, \beta)}(N)}(x, q), \nu_{Q_{(\alpha, \beta)}(N)}(y, q) \}$  for all  $x, y \in F, q \in Q$ . Hence  $Q_{\alpha, \beta}(A)$  is a Q-intuitionistic fuzzy subbigroup of G.

**Theorem 2.4.** If A is a Q-intuitionistic fuzzy subbigroup of a bigroup G, then  $P_{\alpha, \beta}(A) = P_{\alpha, \beta}(M) \cup P_{\alpha, \beta}(N)$  is a Q-intuitionistic fuzzy subbigroup of G.

**Proof.** For every  $x, y \in E$ ,  $q \in Q$  and  $\alpha, \beta \in [0, 1]$  and  $\alpha + \beta \leq 1$ , then  $\mu_{P_{(\alpha, \beta)}(M)}(x-y, q) = \max \{ \alpha, \mu_M(x-y, q) \} \geq \max \{ \alpha, \min \{ \mu_M(x, q), \mu_M(y, q) \} \} = \min \{ \max \{ \alpha, \mu_M(x, q) \}, \max \{ \alpha, \mu_M(y, q) \} \} = \min \{ \mu_{P_{(\alpha, \beta)}(M)}(x, q), \mu_{P_{(\alpha, \beta)}(M)}(y, q) \}$  for all  $x, y \in E, q \in Q$ . And  $\nu_{P_{(\alpha, \beta)}(M)}(x-y, q) = \min \{ \beta, v_M(x-y, q) \} \leq \min \{ \beta, \max \{ v_M(x, q), v_M(y, q) \} \} = \max \{ \min \{ \beta, v_M(x, q) \}, \min \{ \beta, v_M(y, q) \} \} = \max \{ \nu_{P_{(\alpha, \beta)}(M)}(x, q), \nu_{P_{(\alpha, \beta)}(M)}(y, q) \}$  for all  $x, y \in E, q \in Q$ . For every  $x, y \in F$  and  $q \in Q$ , then  $\mu_{P_{(\alpha, \beta)}(N)}(xy^{-1}, q) = \max \{ \alpha, \mu_N(xy^{-1}, q) \} \geq \max \{ \alpha, \min \{ \mu_N(x, q), \mu_N(y, q) \} \} = \min \{ \max \{ \alpha, \mu_N(x, q) \}, \max \{ \alpha, \mu_N(y, q) \} \} = \min \{ \mu_{P_{(\alpha, \beta)}(N)}(x, q), \mu_{P_{(\alpha, \beta)}(N)}(y, q) \}$  for all  $x, y \in F, q \in Q$ . For every  $x, y \in F$  and  $q \in Q$ , then  $\nu_{P_{(\alpha, \beta)}(N)}(xy^{-1}, q) = \min \{ \beta, v_N(xy^{-1}, q) \} \leq \min \{ \beta, \max \{ v_N(x, q), v_N(y, q) \} \} = \max \{ \min \{ \beta, v_N(x, q) \}, \min \{ \beta, v_N(y, q) \} \} = \max \{ \nu_{P_{(\alpha, \beta)}(N)}(x, q), \nu_{P_{(\alpha, \beta)}(N)}(y, q) \}$  for all  $x, y \in F, q \in Q$ . Hence  $P_{\alpha, \beta}(A)$  is a Q-intuitionistic fuzzy subbigroup of G.

**Theorem 2.5.** If A is a Q-intuitionistic fuzzy subbigroup of a bigroup G, then  $G_{\alpha, \beta}(A) = G_{\alpha, \beta}(M) \cup G_{\alpha, \beta}(N)$  is a Q-intuitionistic fuzzy subbigroup of G.

**Proof.** For every  $x, y \in E$  and  $q \in Q$  and  $\alpha, \beta \in [0, 1]$  and  $\alpha + \beta \leq 1$ , then  $\mu_{G_{(\alpha, \beta)}(M)}(x-y, q) = \alpha \mu_M(x-y, q) \geq \alpha (\min \{ \mu_M(x, q), \mu_M(y, q) \}) = \min \{ \alpha \mu_M(x, q), \alpha \mu_M(y, q) \} = \min \{ \mu_{G_{(\alpha, \beta)}(M)}(x, q), \mu_{G_{(\alpha, \beta)}(M)}(y, q) \}$  for all  $x, y \in E, q \in Q$ . And  $\nu_{G_{(\alpha, \beta)}(M)}(x-y, q) = \beta v_M(x-y, q) \leq \beta (\max \{ v_M(x, q), v_M(y, q) \}) = \max \{ \beta v_M(x, q), \beta v_M(y, q) \} = \max \{ \nu_{G_{(\alpha, \beta)}(M)}(x, q), \nu_{G_{(\alpha, \beta)}(M)}(y, q) \}$  for all  $x, y \in E, q \in Q$ . For every  $x, y \in F$  and  $q \in Q$ , then  $\mu_{G_{(\alpha, \beta)}(N)}(xy^{-1}, q) = \alpha \mu_N(xy^{-1}, q) \geq \alpha (\min \{ \mu_N(x, q), \mu_N(y, q) \}) = \min \{ \alpha \mu_N(x, q), \alpha \mu_N(y, q) \} = \min \{ \mu_{G_{(\alpha, \beta)}(N)}(x, q), \mu_{G_{(\alpha, \beta)}(N)}(y, q) \}$  for all  $x, y \in F, q \in Q$ . And  $\nu_{G_{(\alpha, \beta)}(N)}(xy^{-1}, q) = \beta v_N(xy^{-1}, q) \leq \beta (\max \{ v_N(x, q), v_N(y, q) \}) = \max \{ \beta v_N(x, q), \beta v_N(y, q) \} = \max \{ \nu_{G_{(\alpha, \beta)}(N)}(x, q), \nu_{G_{(\alpha, \beta)}(N)}(y, q) \}$  for all  $x, y \in F, q \in Q$ . Hence  $G_{\alpha, \beta}(A)$  is a Q-intuitionistic fuzzy subbigroup of G.

**Theorem 2.6.** Let A be a Q-intuitionistic fuzzy subbigroup of a bigroup G. Then  $A^0 = M^0 \cup N^0$  is a Q-intuitionistic fuzzy subbigroup of the bigroup G.

**Proof.** For any  $x, y \in E$  and  $q \in Q$ , then  $\mu_M^0(x-y, q) = \mu_M(x-y, q) / \mu_M(0, q) \geq (1/\mu_M(0, q)) (\min \{ \mu_M(x, q), \mu_M(y, q) \}) = \min \{ \mu_M(x, q) / \mu_M(0, q), \mu_M(y, q) / \mu_M(0, q) \} = \min \{ \mu_M^0(x, q), \mu_M^0(y, q) \}$  for all  $x, y \in E$  and  $q \in Q$ . And  $v_M^0(x-y, q) = v_M(x-y, q) v_M(0, q) \leq v_M(0, q) (\max \{ v_M(x, q), v_M(y, q) \}) = \max \{ v_M(x, q) v_M(0, q), v_M(y, q) v_M(0, q) \} = \max \{ v_M^0(x, q), v_M^0(y, q) \}$  for all  $x, y \in E$  and  $q \in Q$ . For any  $x, y \in F$  and  $q \in Q$ , then  $\mu_N^0(xy^{-1}, q) = \mu_N(xy^{-1}, q) / \mu_N(1, q) \geq (1/\mu_N(1, q)) (\min \{ \mu_N(x, q), \mu_N(y, q) \}) = \min \{ \mu_N(x, q) / \mu_N(1, q), \mu_N(y, q) / \mu_N(1, q) \} = \min \{ \mu_N^0(x, q), \mu_N^0(y, q) \}$  for all  $x, y \in F$  and  $q \in Q$ . And  $v_N^0(xy^{-1}, q) = v_N(xy^{-1}, q) v_N(1, q) \leq v_N(1, q) (\max \{ v_N(x, q), v_N(y, q) \}) = \max \{ v_N(x, q) v_N(1, q), v_N(y, q) v_N(1, q) \} = \max \{ v_N^0(x, q), v_N^0(y, q) \}$  for all  $x, y \in F$  and  $q \in Q$ . Hence  $A^0$  is a  $Q$ -intuitionistic fuzzy subbigroup of the bigroup  $G$ .

**Theorem 2.7.** Let  $A$  be a  $Q$ -intuitionistic fuzzy subbigroup of a bigroup  $G$ . Then the pseudo  $Q$ -intuitionistic fuzzy coset  $(abA)^P = \langle (a\mu_M)^P \cup (b\mu_N)^P, (av_M)^P \cup (bv_N)^P \rangle$  is a  $Q$ -intuitionistic fuzzy subbigroup of the bigroup  $G$  for every  $a \in E$  and every  $b \in F$ .

**Proof.** For every  $x, y \in E$  and  $q \in Q$ , then  $(a\mu_M)^P(x-y, q) = p(a) \mu_M(x-y, q) \geq p(a) \min \{ \mu_M(x, q), \mu_M(y, q) \} = \min \{ p(a) \mu_M(x, q), p(a) \mu_M(y, q) \} = \min \{ (a\mu_M)^P(x, q), (a\mu_M)^P(y, q) \}$  for all  $x, y \in E, q \in Q$ . And  $(av_M)^P(x-y, q) = p(a) v_M(x-y, q) \leq p(a) \max \{ v_M(x, q), v_M(y, q) \} = \max \{ p(a) v_M(x, q), p(a) v_M(y, q) \} = \max \{ (av_M)^P(x, q), (av_M)^P(y, q) \}$  for all  $x, y \in E, q \in Q$ . For every  $x, y \in F$ ,  $q \in Q$ , then  $(b\mu_N)^P(xy^{-1}, q) = p(b) \mu_N(xy^{-1}, q) \geq p(b) \min \{ \mu_N(x, q), \mu_N(y, q) \} = \min \{ p(b) \mu_N(x, q), p(b) \mu_N(y, q) \} = \min \{ (b\mu_N)^P(x, q), (b\mu_N)^P(y, q) \}$  for all  $x, y \in F$  and  $q \in Q$ . And  $(bv_N)^P(xy^{-1}, q) = p(b) v_N(xy^{-1}, q) v_N(1, q) \leq v_N(1, q) (\max \{ v_N(x, q), v_N(y, q) \}) = \max \{ v_N(x, q) v_N(1, q), v_N(y, q) v_N(1, q) \} = \max \{ v_N^0(x, q), v_N^0(y, q) \}$  for all  $x, y \in F$  and  $q \in Q$ . Hence  $(abA)^P$  is a  $Q$ -intuitionistic fuzzy subbigroup of the bigroup  $G$ .

**Theorem 2.8.** If  $A$  and  $B$  are  $Q$ -intuitionistic fuzzy subbigroups of the bigroups  $G$  and  $S$  respectively, then  $A \times B = (M \times O) \cup (N \times O) \cup (M \times P) \cup (N \times P)$  is a  $Q$ -intuitionistic fuzzy sub four-group of  $G \times S$ .

**Proof.** Let  $A$  and  $B$  be  $Q$ -intuitionistic fuzzy subbigroups of the bigroups  $G$  and  $S$  respectively. Let  $x_1, x_2 \in E, y_1, y_2 \in H$  and  $q \in Q$ . Thus  $(x_1, y_1), (x_2, y_2) \in E \times H$ . Then  $\mu_{M \times O}[(x_1, y_1) - (x_2, y_2), q] = \mu_{M \times O}((x_1-x_2, y_1-y_2), q) \geq \min \{ \min \{ \mu_M(x_1, q), \mu_O(y_1, q) \}, \min \{ \mu_M(x_2, q), \mu_O(y_2, q) \} \} = \min \{ \mu_{M \times O}((x_1, y_1), q), \mu_{M \times O}((x_2, y_2), q) \}$  for all  $(x_1, y_1), (x_2, y_2) \in E \times H$ . And  $v_{M \times O}[(x_1, y_1) - (x_2, y_2), q] = v_{M \times O}((x_1-x_2, y_1-y_2), q) \leq \max \{ \max \{ v_M(x_1, q), v_M(x_2, q) \}, \max \{ v_O(y_1, q), v_O(y_2, q) \} \} = \max \{ \max \{ v_M(x_1, q), v_O(y_1, q) \}, \max \{ v_M(x_2, q), v_O(y_2, q) \} \} = \max \{ v_{M \times O}((x_1, y_1), q), v_{M \times O}((x_2, y_2), q) \}$  for all  $(x_1, y_1), (x_2, y_2) \in E \times H$ . Let  $x_1, x_2 \in F, y_1, y_2 \in H$  and  $q \in Q$ . Thus  $(x_1, y_1), (x_2, y_2) \in F \times H$ . Then  $\mu_{N \times O}[(x_1, y_1) - (x_2, y_2), q] = \mu_{N \times O}((x_1x_2^{-1}, y_1-y_2), q) \geq \min \{ \min \{ \mu_N(x_1, q), \mu_O(y_1, q) \}, \min \{ \mu_N(x_2, q), \mu_O(y_2, q) \} \} = \min \{ \min \{ \mu_{N \times O}((x_1, y_1), q), \mu_{N \times O}((x_2, y_2), q) \} \}$  for all  $(x_1, y_1), (x_2, y_2) \in F \times H$ . Also  $v_{N \times O}[(x_1, y_1) - (x_2, y_2), q] = v_{N \times O}((x_1x_2^{-1}, y_1-y_2), q) \leq \max \{ \max \{ v_N(x_1, q), v_N(x_2, q) \}, \max \{ v_O(y_1, q), v_O(y_2, q) \} \} = \max \{ \max \{ v_N(x_1, q), v_O(y_1, q) \}, \max \{ v_N(x_2, q), v_O(y_2, q) \} \} = \max \{ v_{N \times O}((x_1, y_1), q), v_{N \times O}((x_2, y_2), q) \}$  for all  $(x_1, y_1), (x_2, y_2) \in F \times H$ . Let  $x_1, x_2 \in E, y_1, y_2 \in I, q \in Q$ . Thus  $(x_1, y_1), (x_2, y_2) \in E \times I$ . Then  $\mu_{M \times P}[(x_1, y_1) - (x_2, y_2), q] = \mu_{M \times P}((x_1-x_2, y_1y_2^{-1}), q) \geq \min \{ \min \{ \mu_M(x_1, q), \mu_P(y_1, q) \}, \min \{ \mu_M(x_2, q), \mu_P(y_2, q) \} \} = \min \{ \min \{ \mu_M(x_1, q), \mu_P(y_1, q) \}, \min \{ \mu_M(x_2, q), \mu_P(y_2, q) \} \} = \min \{ \mu_{M \times P}((x_1, y_1), q), \mu_{M \times P}((x_2, y_2), q) \}$  for all  $(x_1, y_1), (x_2, y_2) \in E \times I$ . And  $v_{M \times P}[(x_1, y_1) - (x_2, y_2), q] = v_{M \times P}((x_1-x_2, y_1y_2^{-1}), q) \leq \max \{ \max \{ v_M(x_1, q), v_M(x_2, q) \}, \max \{ v_P(y_1, q), v_P(y_2, q) \} \} = \max \{ \max \{ v_M(x_1, q), v_P(y_1, q) \}, \max \{ v_M(x_2, q), v_P(y_2, q) \} \} = \max \{ v_{M \times P}((x_1, y_1), q), v_{M \times P}((x_2, y_2), q) \}$  for all  $(x_1, y_1), (x_2, y_2) \in E \times I$ . Let  $x_1, x_2 \in F, y_1, y_2 \in I$  and  $q \in Q$ . Thus  $(x_1, y_1), (x_2, y_2) \in F \times I$ . Then  $\mu_{N \times P}[(x_1, y_1)(x_2, y_2)^{-1}, q] = \mu_{N \times P}((x_1x_2^{-1}, y_1y_2^{-1}), q) \geq \min \{ \min \{ \mu_N(x_1, q), \mu_P(y_1, q) \}, \min \{ \mu_N(x_2, q), \mu_P(y_2, q) \} \} = \min \{ \min \{ \mu_{N \times P}((x_1, y_1), q), \mu_{N \times P}((x_2, y_2), q) \} \}$  for all  $(x_1, y_1), (x_2, y_2) \in F \times I$ . And  $v_{N \times P}[(x_1, y_1)(x_2, y_2)^{-1}, q] = v_{N \times P}((x_1x_2^{-1}, y_1y_2^{-1}), q) \leq \max \{ \max \{ v_N(x_1, q), v_N(x_2, q) \}, \max \{ v_P(y_1, q), v_P(y_2, q) \} \} = \max \{ \max \{ v_N(x_1, q), v_P(y_1, q) \}, \max \{ v_N(x_2, q), v_P(y_2, q) \} \} = \max \{ v_{N \times P}((x_1, y_1), q), v_{N \times P}((x_2, y_2), q) \}$  for all  $(x_1, y_1), (x_2, y_2) \in F \times I$ . Hence  $A \times B$  is a  $Q$ -intuitionistic fuzzy sub four-group of  $G \times S$ .

**Theorem 2.9.** Let  $G$  be a bigroup and  $Q$  be a non-empty set. If  $A$  is a  $Q$ -intuitionistic fuzzy subbigroup of  $G$ , then strongest  $Q$ -intuitionistic fuzzy relation  $V = (M \times M) \cup (N \times N) \cup (M \times N) \cup (N \times M)$  is a  $Q$ -intuitionistic fuzzy sub four-group of  $G \times G$ .

**Proof.** Assume that  $A$  is a  $Q$ -intuitionistic fuzzy subbigroup of  $G$ . For any  $x = (x_1, x_2), y = (y_1, y_2) \in E \times E$  and  $q \in Q$ . Then  $\mu_{M \times M}(x-y, q) = \mu_{M \times M}((x_1, x_2)-(y_1, y_2), q) = \min \{ \mu_M(x_1-y_1, q), \mu_M(x_2-y_2, q) \} \geq \min \{ \min \{ \mu_M(x_1, q), \mu_M(y_1, q) \}, \min \{ \mu_M(x_2, q), \mu_M(y_2, q) \} \} = \min \{ \mu_{M \times M}((x_1, x_2), q), \mu_{M \times M}((y_1, y_2), q) \} = \min \{ \mu_{M \times M}(x, q), \mu_{M \times M}(y, q) \}$  for all  $x, y \in E \times E$  and  $q \in Q$ . And  $v_{M \times M}(x-y, q) = v_{M \times M}((x_1, x_2)-(y_1, y_2), q) = \max \{ v_M(x_1-y_1, q), v_M(x_2-y_2, q) \} \leq \max \{ \max \{ v_M(x_1, q), v_M(y_1, q) \}, \max \{ v_M(x_2, q), v_M(y_2, q) \} \}$

$q)\}) = \max\{\max\{v_M(x_1, q), v_M(x_2, q)\}, \max\{v_M(y_1, q), v_M(y_2, q)\}\} = \max\{v_{M \times M}((x_1, x_2), q), v_{M \times M}((y_1, y_2), q)\} = \max\{v_{M \times M}(x, q), v_{M \times M}(y, q)\}$  for all  $x, y \in E \times E$  and  $q \in Q$ . For any  $x = (x_1, x_2), y = (y_1, y_2) \in E \times F$  and  $q \in Q$ , then  $\mu_{M \times N}(x-y, q) = \mu_{M \times N}[(x_1, x_2)-(y_1, y_2), q] = \min\{\mu_M(x_1-y_1, q), \mu_N(x_2y_2^{-1}, q)\} \geq \min\{\min\{\mu_M(x_1, q), \mu_M(y_1, q)\}, \min\{\mu_N(x_2, q), \mu_N(y_2, q)\}\} = \min\{\min\{\mu_M(x_1, q), \mu_N(x_2, q)\}, \min\{\mu_M(y_1, q), \mu_N(y_2, q)\}\} = \min\{\mu_{M \times N}((x_1, x_2), q), \mu_{M \times N}((y_1, y_2), q)\} = \min\{\mu_{M \times N}(x, q), \mu_{M \times N}(y, q)\}$  for all  $x, y \in E \times F$  and  $q \in Q$ . And  $v_{M \times N}(x-y, q) = v_{M \times N}[(x_1, x_2)-(y_1, y_2), q] = \max\{v_M(x_1-y_1, q), v_N(x_2y_2^{-1}, q)\} \leq \max\{\max\{v_M(x_1, q), v_M(y_1, q)\}, \max\{v_N(x_2, q), v_N(y_2, q)\}\} = \max\{\max\{v_M(x_1, q), v_N(x_2, q)\}, \max\{v_M(y_1, q), v_N(y_2, q)\}\} = \max\{v_{M \times N}((x_1, x_2), q), v_{M \times N}((y_1, y_2), q)\} = \max\{v_{M \times N}(x, q), v_{M \times N}(y, q)\}$  for all  $x, y \in E \times F$  and  $q \in Q$ . For any  $x = (x_1, x_2), y = (y_1, y_2) \in F \times E$  and  $q \in Q$ . Also  $\mu_{N \times M}(x-y, q) = \mu_{N \times M}[(x_1, x_2)-(y_1, y_2), q] = \min\{\mu_N(x_1y_1^{-1}, q), \mu_M(x_2-y_2, q)\} \geq \min\{\min\{\mu_N(x_1, q), \mu_N(y_1, q)\}, \min\{\mu_M(x_2, q), \mu_M(y_2, q)\}\} = \min\{\min\{\mu_N(x_1, q), \mu_M(x_2, q)\}, \min\{\mu_N(y_1, q), \mu_M(y_2, q)\}\} = \min\{\mu_{N \times M}((x_1, x_2), q), \mu_{N \times M}((y_1, y_2), q)\} = \min\{\mu_{N \times M}(x, q), \mu_{N \times M}(y, q)\}$  for all  $x, y \in F \times E$  and  $q \in Q$ . And  $v_{N \times M}(x-y, q) = v_{N \times M}[(x_1, x_2)-(y_1, y_2), q] = \max\{v_N(x_1y_1^{-1}, q), v_M(x_2-y_2, q)\} \leq \max\{\max\{v_N(x_1, q), v_N(y_1, q)\}, \max\{v_M(x_2, q), v_M(y_2, q)\}\} = \max\{\max\{v_N(x_1, q), v_M(x_2, q)\}, \max\{v_N(y_1, q), v_M(y_2, q)\}\} = \max\{v_{N \times M}((x_1, x_2), q), v_{N \times M}((y_1, y_2), q)\} = \max\{v_{N \times M}(x, q), v_{N \times M}(y, q)\}$  for all  $x, y \in F \times E$ ,  $q \in Q$ . For any  $x = (x_1, x_2), y = (y_1, y_2) \in F \times F$  and  $q \in Q$ . And  $\mu_{N \times N}(xy^{-1}, q) = \mu_{N \times N}[(x_1, x_2)(y_1, y_2)^{-1}, q] = \min\{\mu_N(x_1y_1^{-1}, q), \mu_N(x_2y_2^{-1}, q)\} \geq \min\{\min\{\mu_N(x_1, q), \mu_N(y_1, q)\}, \min\{\mu_N(x_2, q), \mu_N(y_2, q)\}\} = \min\{\min\{\mu_N(x_1, q), \mu_N(x_2, q)\}, \min\{\mu_N(y_1, q), \mu_N(y_2, q)\}\} = \min\{\mu_{N \times N}((x_1, x_2), q), \mu_{N \times N}((y_1, y_2), q)\} = \min\{\mu_{N \times N}(x, q), \mu_{N \times N}(y, q)\}$  for all  $x, y \in F \times F$  and  $q \in Q$ . Also  $v_{N \times N}(xy^{-1}, q) = v_{N \times N}[(x_1, x_2)(y_1, y_2)^{-1}, q] = \max\{v_N(x_1y_1^{-1}, q), v_N(x_2y_2^{-1}, q)\} \leq \max\{\max\{v_N(x_1, q), v_N(y_1, q)\}, \max\{v_N(x_2, q), v_N(y_2, q)\}\} = \max\{v_{N \times N}((x_1, x_2), q), v_{N \times N}((y_1, y_2), q)\} = \max\{v_{N \times N}(x, q), v_{N \times N}(y, q)\}$  for all  $x, y \in F \times F$  and  $q \in Q$ . Hence  $V$  is a  $Q$ -intuitionistic fuzzy sub four-group of  $G \times G$ .

### III. CONCLUSION

In this paper, definitions of  $Q$ -intuitionistic fuzzy subbigroup,  $A^0$ , pseudo  $Q$ -intuitionistic fuzzy coset, product, strongest  $Q$ -intuitionistic fuzzy relation have been given. Using these definitions, different types of translation Theorems and some more Theorems have been discussed. Using this concept, we can develop some new theorems and properties. And the concepts will be extended into multi fuzzy and bipolar valued fuzzy, bipolar valued multi fuzzy and so on.

### REFERENCE

- [1] Anthony.J.M. and Sherwood.H, Fuzzy groups Redefined, Journal of Mathematical analysis and applications, 69(1979), 124–130.
- [2] Atanassov.K.,  $Q$ -intuitionistic fuzzy sets, Fuzzy sets and systems, 20(1) (1986), 87–96.
- [3] Atanassov.K.T.,  $Q$ -intuitionistic fuzzy sets theory and applications, Physica-Verlag, A Springer-Verlag company, April 1999, Bulgaria.
- [4] Azriel Rosenfeld, Fuzzy Groups, Journal of Mathematical analysis and applications, 35 (1971), 512 – 517.
- [5] Justin Prabu, T. and K.Arjunan, “ $Q$ -Fuzzy subbigroup of a bigroup.” Bulletin of Mathematics and Statistics Research, Vol. 3, Iss. 2 (2015), 12 – 15.
- [6] Justin Prabu, T. and K.Arjunan, “Anti  $Q$ -fuzzy subbigroup of a bigroup.” International Journal of Mathematical Archive, 6(7) (2015), 102 –105.
- [7] Palaniappan. N & K.Arjunan, Some properties of  $Q$ -intuitionistic fuzzy subgroups, Acta Ciencia Indica, Vol.XXXIII (2) (2007), 321 – 328.
- [8] Rajesh Kumar, Fuzzy Algebra, University of Delhi Publication Division, Vol.1, July-1993.
- [9] Vasantha kandasamy.W.B, Smarandache fuzzy algebra, American research press, Rehoboth-2003.
- [10] Zadeh.L.A, Fuzzy sets, Information and control, Vol.8 (1965), 338 –353.
- [11] Zadeh.L.A, The concept of a linguistic variable and its application to approximation reasoning-1, Inform. Sci. 8 (1975), 199 –249.