Some Special Type of CI-Algebras

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ABSTRACT

In this paper we introduce some special type of CI-algebras which are obtained from a given CI-algebra. They are Cartesian product of CI–algebra, function algebra of CI–algebra and CI-algebra of matrices.

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1. INTRODUCTION

In 1966,Y.Imai and K.Iseki [2] introduced the notion of a BCK-algebra. There exist several generalizations of BCK-algebras,such as BCI-algebras [3],BCH-algebras [1],BH-algebras [4],d-algebras [8],etc. In [5], H.S.Kim and Y.H.kim introduced the notion of a BE-algebra as a dualization of a generalization of a BCK-algebra.As a generalization of BE-algebras,B.L.Meng [7] introduced the notion of CI-algebras and discussed its important properties. In this paper we introduce some special type of CI-algebras which are obtained from a given CI-algebra.They are Cartesian product of CI–algebra, function algebra of CI–algebra and CI-algebra of matrices.

2. PRELIMINARIES

Definition 2.1. ([5]) **-** A system $(X; *, 1)$ of type (2, 0) consisting of a non-empty set X, a binary operation $*$ and a fixed element 1 is called a BE–algebra if the following conditions are satisfied:

1. (BE 1) $x * x = 1$
2. (BE 2) $x * 1 = 1$
3. (BE 3) $1 * x = 1$
4. (BE 4) $x * (y * z) = y * (x * z)$

for all $x, y, z \in X$.

Definition 2.2 ([7]) **-** A CI–algebra is an algebra (X; $*$, 1) of type (2, 0) consisting of a non-empty set X, a binary operation $*$ and a fixed element 1satisfying the following conditions:

(CI 1) $x * x = 1$

(CI 2) $1 * x = x$

(CI 3) $x * (y * z) = y * (x * z)$

for all x, y, z $\in$ X.

Example 2.3.[9] - Let $X = R^+ = \{x \in R : x > o\}$

For x , y $\in$ X, we define

$x * y = y \cdot \frac{1}{x}$

Then (X; $*$, 1) is a CI – algebra

Example 2.4.- The simplest example of a BE–algebra and a CI –algebra are the following.

Let X = {0, 1}. We consider binary operations $*$ and o given by the Cayley tables

$*$	0	1
0	1	1
1	0	1

Table 2.1

o	0	1
0	1	0
1	0	1

Table 2.2

Then (i) (X; $*$, 1) is a BE–algebra,

(ii) (X; o, 1) is a CI–algebra but not a BE–algebra.

Example 2.5 ([7])**:-** Let X = {1, a, b, c, d} and let the binary operation $*$ be given by the Cayley table

$*$	1	a	b	c	d
1	1	a	b	c	d
a	1	1	b	b	d
b	1	a	1	a	d
c	1	1	1	1	d
d	d	d	d	d	1

Then $(X; *, 1)$ is a CI-algebra.

Lemma 2.6. ([6]) **-** In a CI–algebra following results are true:

(1) $x * ((x * y) * y) = 1$
(2) $(x * y) * 1 = (x * 1) * (y * 1)$
(3) $1 \leq x$ imply $x = 1$

for all $x, y \in X$.

§. 3 SOME SPECIAL CI–ALGEBRAS

Here we establish the following results:

Theorem 3.1.- Let $(X; *, 1)$ be a system consisting of a non–empty set X, a binary operation $*$ and a distinct element 1. Let $Y = X \times X = \{(x_1, x_2) : x_1, x_2 \in X\}$. For $u, v \in Y$ with $u = (x_1, x_2)$, $v = (y_1, y_2)$, we define an operation $\otimes$ in Y as

$$u \otimes v = (x_1 * y_1, x_2 * y_2)$$

Then $(Y; \otimes, (1, 1))$ is a CI–algebra iff $(X; *, 1)$ is a CI–algebra.

Proof: Suppose that $(Y; \otimes, (1, 1))$ is a CI–algebra. Let $x \in X$ and we choose $u = (x, 1) \in Y$. Then

(1) $u \otimes u = (1, 1) \Rightarrow (x * x, 1 * 1) = (1, 1)$
$\Rightarrow x * x = 1$, since $1 * 1 = 1$.

(2) $(1, 1) \otimes u = u \Rightarrow (1 * x, 1 * 1) = (x, 1)$
$\Rightarrow 1 * x = x$.

(3) Let $x, y, z \in X$ and we choose $u = (x, 1)$, $v = (y, 1)$ and $w = (z, 1)$. Then
$u \otimes (v \otimes w) = v \otimes (u \otimes w)$
$\Rightarrow (x * (y * z), 1 * (1 * 1)) = (y * (x * z), 1 * (1 * 1))$

$\Rightarrow x * (y * z) = y * (x * z)$.

Thus we see that $(X; *, 1)$ is a CI–algebra .

Conversely, suppose that $(X; *, 1)$ is a CI–algebra. Let $U = (x_1, x_2) \in Y$. Then

(1) $u \otimes u = (x_1, x_2) \otimes (x_1, x_2)$
$= (x_1 * x_1, x_2 * x_2)$
$= (1, 1)$.

(2) $(1, 1) \otimes u = (1, 1) \otimes (x_1, x_2)$
$= (1 * x_1, 1 * x_2)$
$= (x_1, x_2)$
$= u$.

(3) Let $u = (x_1, x_2)$, $v = (y_1, y_2)$ and $w = (z_1, z_2)$ be any three elements of Y.

Then $u \otimes (v \otimes w) = (x_1, x_2) \otimes ((y_1, y_2) \otimes (z_1, z_2))$

$= (x_1, x_2) \otimes (y_1 * z_1, y_2 * z_2)$
$= (x_1 * (y_1 * z_1), x_2 * (y_2 * z_2))$

$= (y_1 * (x_1 * z_1), y_2 * (x_2 * z_2))$

$= (y_1, y_2) \otimes (x_1 * z_1, x_2 * z_2)$

$= (y_1, y_2) \otimes ((x_1, x_2) \otimes (z_1, z_2))$

$= v \otimes (u \otimes w)$.

Hence $(Y; \otimes, (1, 1))$ is a CI–algebra.

Corollary 3.2. - If $(X; *, 1)$ and $(Y; o, e)$ are two CI–algebras, then $Z = X \times Y$ is also a CI–algebra under the binary operation defined as follows:

For $u = (x_1, y_1)$ and $v = (x_2, y_2)$ in Z,

$$u \otimes v = (x_1 * x_2, y_1 \, o \, y_2)$$

Here the distinct element of Z is $(1, e)$.

Note 3.3. - The above result can be extended for finite number of CI-algebras.

Theorem 3.4. - Let $(X; *, 1)$ be a CI–algebra and let $F(X)$ be the class of all functions $f : X \to X$. Let a binary operation o be defined in $F(X)$ as follows:

For $f, g \in F(X)$ and $x \in X$,

$$(f \, o \, g)(x) = f(x) * g(x).$$

Then (F(X); o, $1^{\sim}$) is a CI–algebra where $1^{\sim}$ is defined as $1^{\sim}(x) = 1$ for all $x \in X$.

Here two functions $f, g \in F(X)$ are equal iff $f(x) = g(x)$ for all $x \in X$.

Proof: Let $f, g, h \in F(X)$. Then for $x \in X$, we have

(i) $(f \text{ o } f)(x) = f(x) * f(x) = 1 = 1^{\sim}(x) \Rightarrow f \text{ o } f = 1^{\sim}$;
(ii) $(1^{\sim} \text{ o } f)(x) = 1^{\sim}(x) * f(x) = f(x) \Rightarrow 1^{\sim} \text{ o } f = f$;
(iii) $(f \text{ o } (g \text{ o } h))(x) = f(x) * (g \text{ o } h)(x)$

$$= f(x) * (g(x) * h(x))$$

$$= g(x) * (f(x) * h(x))$$

$$= g(x) * (f \text{ o } h)(x)$$

$$= (g \text{ o } (f \text{ o } h))(x).$$

$$\Rightarrow f \text{ o } (g \text{ o } h) = g \text{ o } (f \text{ o } h).$$

This proves that (F(X); o, $1^{\sim}$) is a CI–algebra.

Theorem 3.5.- Let (X; ∗, 1) be a CI–algebra and let M(X) be the class of all m x n matrices $(a_{ij})_{m \times n}$ with entries $a_{ij} \in X$. For $A = (a_{ij})_{m \times n}$, $B = (b_{ij})_{m \times n}$ we define

(a) $A = B$ iff $a_{ij} = b_{ij}$; $1 \le i \le m$, $1 \le j \le n$,
(b) a binary operation o in M(X) as
$A \text{ o } B = C = (c_{ij})_{m \times n}$

Where $c_{ij} = a_{ij} * b_{ij}$; $1 \le i \le m$, $1 \le j \le n$.

Then (M(X); o, I) is a CI-algebra with distinct element

$I = (e_{ij})_{m \times n}$ where $e_{ij} = 1$ for $1 \le i \le m$, $1 \le j \le n$.

Proof :- Let $A = (a_{ij})_{m \times n} \in M(X)$. Then

(i) $A \text{ o } A = (l_{ij})_{m \times n}$ where $l_{ij} = a_{ij} * a_{ij} = 1 = e_{ij}$;
$1 \le i \le m$, $1 \le j \le n$, which means that $A \text{ o } A = I$;
(ii) $I \text{ o } A = (k_{ij})_{m \times n}$ where $k_{ij} = e_{ij} * a_{ij} = 1 * a_{ij} = a_{ij}$;

$1 \le i \le m$, $1 \le j \le n$, which means that $I \text{ o } A = A$;

(iii) Let $A = (a_{ij})_{m \times n}$, $B = (b_{ij})_{m \times n}$ and $C = (c_{ij})_{m \times n}$ be elements of M(X). Then
$A \text{ o } (B \text{ o } C) = (x_{ij})_{m \times n}$

Where $x_{ij} = (a_{ij} * (b_{ij} * c_{ij}))$; $1 \le i \le m$, $1 \le j \le n$.

Also $B \text{ o } (A \text{ o } C) = (y_{ij})_{m \times n}$

Where $y_{ij} = (b_{ij} * (a_{ij} * c_{ij}))$; $1 \leq i \leq m$, $1 \leq j \leq n$.

Since $(a_{ij} * (b_{ij} * c_{ij})) = (b_{ij} * (a_{ij} * c_{ij}))$; $1 \leq i \leq m$, $1 \leq j \leq n$,

we see that A o (B o C) = B o (A o C).

Hence (M (X); o, I) is a CI-algebra.

Conclusion: Here we discussed some special type of CI-algebras such as Cartesian product of CI–algebras, function algebra of CI–algebras and CI-algebra of matrices from a given CI-algebra. There is a scope of further study in different structures of CI-algebra relating to Cartesian product of CI–algebras, function algebra of CI–algebras and CI-algebra of matrices etc.

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