On Intuitionistic Fuzzy Normal Groups

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 Abstract

In this present paper the Author discussed the concepts of Intutionistic fuzzy normal subgroups. The concepts of ordinary groups and ordinary normal groups are already in existence. Zadeh [1] Introduced the notion of fuzzy set theory , over an extra edge on ordinary set theory. A remarkable and beneficial research work is done in the field of fuzzy set theory, then the researchers started to think about the applications of fuzzy groups and fuzzy subgroups and fuzzy normal subgroups. The Intutionistic fuzzy sets and the Intutionistic fuzzy groups are same as Vague sets and vague groups, as justified by Bustince and Burillo in [2]. Consequently, in this paper the two terminologies 'vague set' and 'Intutionistic fuzzy set' have been used with same meaning and objectives
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Keywords **—** *Fuzzy Sets, Fuzzy Groups, Fuzzy Relations , Intutionistic Fuzzy Sets, Intutionistic Fuzzy Relations, Intutionistic Fuzzy Groups, Intutionistic Fuzzy Normal Groups*

I. INTRODUCTION

As we are aware with the concepts of ordinary set theory, that 'A Set is a collection of well defined objects, and the different operations of set theory like union, intersection, etc. Zadeh[1] in 1965 introduced the notion of fuzzy set theory. Which played an important role in many field of daily lives. The basic concepts of fuzzy set theory is the extended version of ordinary set theory. To understand the basic concepts of fuzzy set theory one could see [[3], [4], [61], [5], [6], [8- 12], Precision assumes that the parameters of a model represent exactly either our perception of the phenomenon modelled or the features of the real system that has been modelled. Generally precision indicates that the model is unequivocal, that is, it contains no ambiguities. By crisp we mean yes-or-no type rather than more-or-less type. In conventional dual logic, for instance, a statement can be true or false-and definitely nothing in between. Vagueness, imprecision and uncertainty have so far been modelled by classical set-theoretic approach. According to this approach, borderline elements can be either put into the set or should be kept outside it. Hence it becomes inadequate for applying to humanistic type of problems. The Zadeh[1] fuzzy set theory had attracted to researchers on time to time. Later on the fuzzy groups and fuzzy relations were introduced. If we look at the developmental history of mathematical systems or structures, we see that a mathematical system is, in general, suggested by situations which, while they are different, have some basic features in common so that the emergence of a mathematical system is essentially the result of a process of unification and abstraction. A mathematical system, thus, lays bare the structurally essential relations between otherwise distinct entities. So, it may be accepted that the results of the study of a mathematical system will be valid for each of those otherwise different situations which provided motivation and inspiration for the same. Such a study also provided an economy of effort and leads to a better and fuller understanding of the motivation situations.

Even without considering the motivation situations inherent in cybernetics and general systems prevailing in the emerging man- machine civilization, if we just consider everyday language, we see that we are concerned with statements which are often distinguished as interrogative, imperative, exclamatory or declarative. In classical mathematical systems, we deal with only those statements which are declarative in nature and which may be either true or false. Fuzzy mathematical systems, whose foundation was laid by Zadeh [125] are able to deal with interrogative and imperative statements also.

Definition 1.1

 Let A be an IFG of a group G. Then A is called an Intutionistic fuzzy normal group (IFNG)   if $\forall$ x,y$\in$G, $V_A(xy) = V_A(yx)$.

Alternatively, we can say that an IFG A is said to be an IFNG of G   if

$$V_A(x) = V_A(yxy^{-1}) \quad \forall \ x,y \in G.$$

We now prove the following propositions for Intutionistic fuzzy normal groups

Proposition 1.1

Let A be an IFNG of a group G.   Then $\forall$ x,y$\in$G, $V_A([x,y]) = V_A(e)$

(The notation [x,y] stands for the expression $x^{-1}y^{-1}xy$)

Proof :   Since A is an IFNG of G,

we have $V_A(x) = V_A(yxy^{-1})$ $\forall$ x,y $\in$ G.

Replacing x by y^{-1} and   y by x^{-1},   we get

$\quad V_A(y^{-1}) = V_A(x^{-1}y^{-1}xy)$

or, $V_A(x^{-1}y^{-1}xyy^{-1}) = V_A(y^{-1})$

or, $V_A([x,y]y^{-1}) = V_A(y^{-1})$

or, $V_A([x,y]) = V_A(e)$.   Hence Proved.

Now we will define Intutionistic Fuzzy Characteristic Groups (IFCG) and then we will study their properties.

For this, first of all we define the notations t_A^{θ}, f_A^{θ} and V_A^{θ} which will be

useful in our next discussion.

Definition 2.1

Let A be an IFS of a group G.   Let θ :   G $\rightarrow$ G   be a map.   Define

the maps t_A^{θ} :   G $\rightarrow$ [0,1]   and f_A^{θ} :   G $\rightarrow$ [0,1]   given by,

respectively.

$\quad\quad$ (i) $t_A^{\theta}(g)$ = $t_A(\theta(g))$ $\forall$ g $\in$ G

and   (ii) $f_A^{\theta}(g)$ = $f_A(\theta(g))$ $\forall$g $\in$ G.

In such case we write $V_A^{\theta}(g)$ = $V_A(\theta(g))$ $\forall$g $\in$ G.

Definition 2.2

An IFG A of a group G is called an IFCG of   G   if

$\quad\quad$ (i) t_A^{θ} = t_A and

$\quad\quad$ (ii) f_A^{θ} =   fA,

for every automorphism θ of G.

We now prove the following propositions.

Proposition 2.3

If A is an IFG of a group G and θ is a homomorphism of G, then the IFS A^{θ}

of G given by $A^{\theta} = \{ < g, t_A{}^{\theta}, f_A{}^{\theta} > : g \in G \}$ is also an IFG of G.

Proof : Let x, y $\in$ G

Then $t_A{}^{\theta}(xy) = t_A (\theta(xy))$

$= t_A (\theta(x) \theta(y))$

$\geq \min \{ t_A (\theta(x)), t_A (\theta(y)) \}$

$= \min \{ t_A{}^{\theta} (x), t_A{}^{\theta} (y) \}$

Also, $f_A{}^{\theta}(xy) = f_A (\theta (xy))$

$= f_A (\theta (x) \theta (y))$

$\leq \max \{ f_A (\theta (x)), f_A (\theta (y)) \} = \max \{ f_A{}^{\theta} (x), f_A{}^{\theta} (y) \}$

Again, $t_A \theta_{(x-1)}$

$= t_A (\theta (x^{-1}))$

$= t_A ((\theta (x))^{-1})$

$= t_A (\theta (x))$

$= t_A{}^{\theta} (x) \qquad \forall \ x \in G$

Similarly we see that

$f_A{}^{\theta} (x^{-1}) = f_A{}^{\theta}(x)) \quad \forall x \in G$

Thus, A^{θ} is an IFG of the group G. Proved.

Proposition 2.4

If A is an IFCG of a group G,   then A   is   an IFNG of   G.

Proof: Let   x, y ∈ G.   Consider the map θ : G → G   given

by

$$\theta(g) \quad = \quad x^{-1}gx \qquad\qquad \forall\ g \in G.$$

Clearly, θ is an automorphism   of G.

Now, $t_A(xy) \quad = \quad t_A{}^{\theta}(xy),$

$$= \quad t_A(\theta(xy))$$

$$= \quad t_A(x^{-1}xyx),$$

$$= \quad t_A(yx)$$

Similarly,   we find that $f_A(xy) \quad = \quad f_A(yx) \qquad \forall\ x, y \in G.$

Therefore,   A is an IFNG of the group   G.   Proved.

The following proposition generates a new type of   classical subgroup of the group G.

Proposition 2.5

Let G be a finite group and   A be an IFG of G.   Consider the subset H of   G

given by

$$H \quad = \quad \{\ g\ :\ g \in G,\quad t_A(g)\ = t_A(e),\quad f_A(g)\ =\ f_A(e)\ \}.$$

Then H   is a crisp subgroup   of G.

Proof : $\forall$ g, h ∈ H,   we have

$$t_A(gh) \quad \geq \quad \min\{\ t_A(g),\ t_A(h)\ \}$$

$$= \quad \min \{\, t_A\ (e), t_A\ (e)\,\}$$

$$= \quad t_A\ (e)$$

$$\geq \quad t_A\ (gh)$$

$$\Rightarrow \quad t_A\ (gh) \quad = \quad t_A\ (e)$$

$$\text{Also,} \quad f_A\ (gh) \quad \leq \quad \max \{\, f_A\ (g),\ f_A\ (h)\,\}$$

$$= \quad \max \{\, f_A\ (e),\ f_A\ (e)\,\}$$

$$= \quad f_A\ (e)$$

$$\leq \quad f_A\ (gh)$$

$$\text{Therefore,} \quad f_A\ (gh) \quad = \quad f_A\ (e)$$

Thus, $gh \in H$.   Since G is finite, it follows that H is a subgroup of G.

Proved

II. CONCLUSION

Therefore as in the case of ordinary set theory we are able to prove the groups. Subgroups, normal groups, and normal subgroups. On the same pattern Zadeh fuzzy set theory plays an important role to prove fuzzy groups, fuzzy subgroups, fuzzy normal groups, and fuzzy normal subgroups,With the help of Zadeh fuzzy set theory Bustince and Burillio introduced intuitionistic fuzzy set theory or vague set theory. Which was extended to the vague groups, vague relations, and vague narmal groups , and vague normal subgroups. These vague normal subgroups playing an important role in decision sciences and management sciences and medical sciences.

REFERENCES

[1] Zadeh,L.A., Fuzzy sets ,Infor. and Control.8 (1965),338-353.
[2] Bustince,H .and Burillo,P., Vague sets are intuitionistic fuzzy sets, Fuzzy Sets and Systems 79 (1996)403-405
[3] Dubois,D. and Prade,H., Fuzzy Sets and System :Theory   and Applications, Academic Press,(London)1980.
[4] Klir, G. J. and Yuan, B. Fuzzy Sets and Fuzzy Logic :Theory and Applications. Englewood Clifis,NJ :Prentice-Hall,1995.
[5] Lin,T.Y., Fuzzy reasoning and rough sets, in Rough Sets, Fuzzy Sets and Knowledge Discovery (W.P.Ziarko, Ed.), Springer-Verlag,London,343 - 348,1994.
[6] Lin,T.Y., and Liu,Q., Rough approximate operators : Axiomatic rough set   theory, in Rough Sets, Fuzzy Sets and Knowledge Discovery (W. P. Ziarko, Ed.), Springer- Verlag,London,256-260,1994.
[7] Yazici,A., George,R., Buckles,B.P. and   Petry,F.E., A survey of conceptual and logical data models for uncertainty management in : Fuzzy Logic for the Management of Uncertainty, eds. L.Zadeh and J.Kacprzyk (John Wiely, New York,1992)607 -643.
[8] Zimmermann, H.-J., Fuzzy Set Theory and Its Applications (Kluwer Academic Publisher,Dordrecht,1985)
[9] Zimmermann,H.-J., Results of empirical studies in fuzzy set theory, in : Klir,G.J.(Eds).,Applied   General   System   Research, Plenum,   New york,1978.
[10] Zimmermann,H.-J., Fuzzy Set Theory and Its Applications, Kluwer Ac.   Press., Dordrecht,1991.
[11] Zimmermann,H.-J., Zadeh,L.A. and Gaines,B.R., Fuzzy Sets and Decision Analysis, North Holland,1984.
[12] Zimmermann,H.-J., Fuzzy Sets, Decision Making and Expert Systems. Boston, MA: Kluwer Academic Publishers,1987.