Modelling and Profit Evaluation of a Repairable System Working with One Operative Unit and Three Cold Standby Units

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Abstract

The current paper deals with the modeling and profit evaluation of a repairable system working with one operative unit and three cold standby units. Initially, there is single unit in operation which is referred as main unit and three cold standby units. The system works properly as long as the main unit remain in operable mode and whenever the main unit comes at halt, all the cold standby units have to function together to keep the system operating. There is only one repairman available. The reliability and profit evaluation has been done for the current study. Various measures of system effectiveness such as MTSF and Profit are obtained using semi Markov process and Regenerative point technique.

Keywords - *Standby systems; semi Markov process; Regenerative point technique.*

I. INTRODUCTION

Failure is an undeniable phenomenon in the life of electric gadgets, instruments, etc. These failures bring the complete system to halt and in such a way hinder the productivity of an industry. The redundancy technique is a proposed way in order to decrease such losses. The application of redundancy or standby systems has greater applications in any industrial field. Numerous models with availability of standby systems have been developed by researchers [1-8] in the literature of reliability.

Our study deals with repairable system where there is one main unit in operative state along with three units kept as cold standby units. Initially, there is one main unit which is in operative state and three cold standby units which are kept as cold standby units. The system functions properly with single unit in operative mode. The problem arises when that single unit fails. In such situation, the three smaller units are kept in cold standby state such that the capacity of functioning of all smaller units is equal to that of single main unit. There is single repairman available for repair of the whole system. Various measures of system effectiveness such as MTSF and Profit are obtained using semi Markov process and Regenerative point technique. The graphical interpretation has also been done for the present study.

II. ASSUMPTIONS

1. There is a single repairman.
2. No inspection is carried out on occurrence of faults.
3. Repair preference is given to main unit.
4. There are three cold standby units in the system.
5. Functioning of every cold standby unit is necessary for system operation.
6. Only one repair is done at a time.

III. NOTATIONS

λ	Rate of occurrence of failure in main unit
$\lambda_1/ \lambda_2/ \lambda_3$	Rate of occurrence of failure in I^{st} / II^{nd} / III^{rd} cold standby unit
$g(t)/ G(t)$	pdf/ cdf of times to repair the main unit at failed state
$g_1(t)/ G_1(t)$	pdf/ cdf of times to repair the I^{st} cold standby unit at failed state

$g_2(t)/ G_2(t)$	pdf/ cdf of times to repair the IInd cold standby unit at failed state
$g_3(t)/ G_3(t)$	pdf/ cdf of times to repair the IIIrd cold standby unit at failed state
$O_I/ O_{II}/ O_{III}/ O_{IV}$	Ist/ IInd/ IIIrd/ IVth unit under operation
$S_{II}/ S_{III}/ S_{IV}$	IInd/ IIIrd/ IVth unit under cold standby state
F_{rI}/F_{wrI}	Ist unit under repair/ waiting for repair
F_{rII}/F_{wrII}	IInd unit under repair/ waiting for repair
F_{rIII}/F_{wrII}	IIIrd unit under repair/ waiting for repair
F_{rIV}/F_{wrIV}	IVth unit under repair/ waiting for repair
F_{RI}	Ist unit under repair continuing from the previous state
F_{RII}	IInd unit under repair continuing from the previous state
F_{RIII}	IIIrd unit under repair continuing from the previous state
F_{RIV}	IVth unit under repair continuing from the previous state

IV. TRANSITION PROBABILITIES AND MEAN SOJOURN TIMES

Table No. 1 shows various transitions of the system and the respective states are being given in Table No. 2. The epochs of entry into states 0, 1, 5, 6 and 7 are regenerative points and thus these are regenerative states. The states 2, 3 and 4 are failed states.

Table No. 1: Possible States with Status

State No.	Status
S_0	O_I, S_{II}, S_{III}, S_{IV}
S_1	F_{rI}, O_{II}, O_{III}, O_{IV}
S_2	F_{RI}, F_{wrII}, S_{III}, S_{IV}
S_3	F_{RI}, S_{II}, F_{wrIII}, S_{IV}
S_4	F_{RI}, S_{II}, S_{III}, F_{wrIV}
S_5	O_I, F_{rII}, S_{III}, S_{IV}
S_6	O_I, S_{II}, F_{rIII}, S_{IV}
S_7	O_I, S_{II}, S_{III}, F_{rIV}

Table No. 2: Transition Rate

S.No.	From State	To State	Rate
1	S_0	S_1	λ
2	S_1	S_0	$g(t)$
3	S_1	S_2	λ_1
4	S_1	S_3	λ_2
5	S_1	S_4	λ_3
6	S_2	S_5	$g(t)$
7	S_3	S_6	$g(t)$
8	S_4	S_7	$g(t)$
9	S_5	S_0	$g_1(t)$
10	S_6	S_0	$g_2(t)$
11	S_7	S_0	$g_3(t)$

Transition Probabilities:

The transition probabilities are given by:

$$dQ_{01}(t) = \lambda e^{-\lambda t} dt \qquad dQ_{10}(t) = g(t) e^{-(\lambda_1+\lambda_2)t} dt$$

$$dQ_{12}(t) = \lambda_1 e^{-(\lambda_1+\lambda_2)t} \overline{G}(t) \qquad dQ_{13}(t) = \lambda_2 e^{-(\lambda_1+\lambda_2)t} \overline{G}(t)$$

$$dQ_{24}(t) = g(t) dt \qquad dQ_{35}(t) = g(t) dt$$

$$dQ_{40}(t) = g_1(t) dt \qquad dQ_{50}(t) = g_2(t) dt$$

$$dQ_{14}^{(2)}(t) = (\lambda_1 e^{-(\lambda_1+\lambda_2)t} \copyright 1) g(t) dt \qquad dQ_{15}^{(3)}(t) = (\lambda_2 e^{-(\lambda_1+\lambda_2)t} \copyright 1) g(t) dt$$

The non-zero elements p_{ij}, are obtained as under:

$$p_{01} = 1 \qquad p_{10} = g^*(\lambda_1 + \lambda_2)$$

$$p_{12} = \frac{\lambda_1[1 - g^*(\lambda_1 + \lambda_2)]}{\lambda_1 + \lambda_2} = p_{14}^{(2)} \qquad p_{40} = g_1^*(0)$$

$$p_{13} = \frac{\lambda_2[1 - g^*(\lambda_1 + \lambda_2)]}{\lambda_1 + \lambda_2} = p_{15}^{(3)} \qquad p_{50} = g_2^*(0)$$

$$p_{24} = g^*(0) = p_{35}$$

By these transition probabilities, it can be verified that

$$p_{01} = 1 \qquad p_{10} + p_{14}^{(2)} + p_{15}^{(3)} = 1 \qquad p_{10} + p_{12} + p_{13} = 1 \qquad p_{24} = 1$$

$$p_{35} = 1 \qquad p_{40} = 1 \qquad p_{50} = 1$$

The unconditional mean time taken by the system to transit for any regenerative state j, when it is counted from epoch of entrance into that state i, is mathematically stated as –

$$m_{ij} = \int_0^\infty t\, dQ_{ij}(t) = -q_{ij}^{*'}(0), \text{Thus} \ -$$

$$m_{01} = \frac{1}{\lambda} \qquad m_{10} + m_{12} + m_{13} = \mu_1$$

$$m_{24} = k_1 = m_{35} \qquad m_{40} = k_2 \qquad m_{50} = k_3$$

where ,

$$k_1 = \int_0^\infty \overline{G}(t) dt \qquad k_2 = \int_0^\infty \overline{G}_1(t) dt \qquad k_3 = \int_0^\infty \overline{G}_2(t) dt$$

The mean sojourn time in the regenerative state i (μ_i) is defined as the time of stay in that state before transition to any other state, then we have –

$$\mu_0 = \frac{1}{\lambda} \qquad \mu_1 = \frac{1 - g^*(\lambda_1 + \lambda_2)}{\lambda_1 + \lambda_2}$$

$$\mu_2 = -g^*(0) = \mu_3 \qquad \mu_4 = -g_1^*(0)$$

$$\mu_5 = -g_2^*(0)$$

V. MEAN TIME TO SYSTEM FAILURE

The expressions for the mean time to system failure (MTSF) are obtained on taking the failed states of the system as absorbing states. By probabilistic arguments, we obtain the following recursive relations for $\phi_i(t)$, c.d.f. of the first passage time from regenerative state i to failed state :

$$\phi_0(t) = Q_{01}(t)(s)\phi_1(t)$$
$$\phi_1(t) = Q_{10}(t)(s)\phi_0(t) + Q_{12}(t) + Q_{13}(t)$$
$$\phi_4(t) = Q_{40}(t)(s)\phi_0(t)$$
$$\phi_5(t) = Q_{50}(t)(s)\phi_0(t)$$

Taking Laplace Stieltjes Transformation of these equations and solving for $\phi_0^{**}(s)$, we obtain

$$\phi_0^{**}(s) = \frac{N(s)}{D(s)}$$

The mean time to system failure when the system starts from the state 0, is

$$T_0 = \lim_{s\to 0} R^*(s) = \lim_{s\to 0} \frac{1-\phi_0^{**}(s)}{s} = \frac{N}{D}$$

Where $R^*(s)$ is the Laplace Transformation of the Reliability R(t). The Reliability R(t) of the system at time 't' can be obtained taking inverse Laplace transform of $R^*(s)$ Using L 'Hospital rule and putting the value of ϕ_0^{**} (s) we have

$$N = (\mu_0 + \mu_1)$$
$$D = 1 - p_{10}$$

VI. EXPECTED UP-TIME OF THE SYSTEM

Using the arguments of the theory of regenerative processes, the availability $AF_i(t)$, the probability that the system is up at instant 't' with full capacity given that it entered regenerative state 'i' at time t = 0, is seen to satisfy the following recursive relations.

$$AF_0(t) = M_0(t) + q_{01}(t) © AF_1(t)$$
$$AF_1(t) = M_1(t) + q_{10}(t) © AF_0(t) + q_{14}^{(2)}(t) © AF_4(t) + q_{15}^{(3)}(t) © AF_5(t)$$
$$AF_4(t) = M_4(t) + q_{40}(t) © AF_0(t)$$
$$AF_5(t) = M_5(t) + q_{50}(t) © AF_0(t)$$

Taking Laplace transform of the above equations and solving for $AF_0^{**}(s)$, we have

$$AF_0^{**}(s) = \frac{N_1(s)}{D_1(s)}$$

The steady state availability of the system is given by

$$AF_0 = \lim_{s\to 0} (sAF_0^*(s)) = \frac{N_1}{D_1}$$

Where

$M_0(t) = e^{-\lambda t} dt$ $\quad M_1(t) = e^{-(\lambda_1+\lambda_2)}\overline{G}(t)$

$M_4(t) = \overline{G_1}(t)$ $\quad M_5(t) = \overline{G_2}(t)$

$N_1 = \mu_0 + \mu_1 + k_2 p_{14}^{(2)} + k_3 p_{15}^{(3)}$

$D_1 = \mu_0 + \mu_1 + k_2 p_{14}^{(2)} + k_3 p_{15}^{(3)}$

VII. BUSY PERIOD OF A REPAIRMAN

Using the probabilistic arguments for regenerative process, the following recursive relation for $B_i(t)$ are obtained.

$$B_0(t) = q_{01}(t) © B_1(t)$$
$$B_1(t) = q_{10}(t) © B_0(t) + q_{14}^{(2)}(t) © B_4(t) + q_{15}^{(3)}(t) © B_5(t)$$
$$B_4(t) = W_4(t) + q_{40}(t) © B_0(t)$$
$$B_5(t) = W_5(t) + q_{50}(t) © B_0(t)$$

The steady state busy period of the system is given by:

$$B_R = \frac{N_2}{D_1}$$
$$N_2 = k_2 p_{14}^{(2)} + k_3 p_{15}^{(3)}$$

And D_1 is already specified above.

VIII. EXPECTED NO. OF VISITS OF REPAIRMAN

Using the probabilistic arguments for regenerative process, the following recursive relation for $V_i(t)$ are obtained.

$$V_0(t) = Q_{01}(t)(s)[1 + V_1(t)]$$
$$V_1(t) = Q_{10}(t)(s)V_0(t) + Q_{14}^{(2)}(t)(s)V_4(t) + Q_{15}^{(3)}(t)(s)V_5(t)$$
$$V_4(t) = Q_{40}(t)(s)V_0(t)$$
$$V_5(t) = Q_{50}(t)(s)V_0(t)$$

The steady state expected no. of visits of the repairman is given by:

$$V_R = \frac{N_3}{D_1}$$
$$N_3 = N_3(0) = 1$$

And D_1 is already specified above.

IX. PROFIT ANALYSIS

The expected profit incurred of the system is –

$$P = C_0 AF_0 - C_1 B_R - C_2 V_R$$

C_0 = Revenue per unit up time of the system

C_1 = Cost per unit up time for which the repairman is busy in repair

C_2 = Cost per visit of the repairman

X. GRAPHICAL INTERPRETATION AND CONCLUSION

For graphical analysis following particular cases are considered:

$g(t) = \beta e^{-\beta t}$ $\quad g_1(t) = \beta_1 e^{-\beta_1 t}$ $\quad g_2(t) = \beta_2 e^{-\beta_2 t}$

$p_{01} = 1$ $\quad p_{10} = \dfrac{\beta}{\lambda_1 + \lambda_2 + \beta}$

$p_{12} = \dfrac{\lambda_1}{\lambda_1 + \lambda_2 + \beta} = p_{14}^{(2)}$ $\quad p_{13} = \dfrac{\lambda_2}{\lambda_1 + \lambda_2 + \beta} = p_{15}^{(3)}$

$p_{40} = 1$ $\quad p_{50} = 1$ $\quad p_{24} = 1 = p_{35}$

$\mu_0 = \dfrac{1}{\lambda}$ $\quad \mu_1 = \dfrac{1}{\lambda_1 + \lambda_2 + \beta}$

$\mu_2 = \dfrac{1}{\beta} = \mu_3$ $\quad \mu_4 = \dfrac{1}{\beta_1}$

$\mu_5 = \dfrac{1}{\beta_2}$

Table No. 3

	T_0 (in Hrs.)		
λ	λ_1 = 0.000015	λ_1 = 0.000020	λ_1 = 0.000025
0.000024	6824537.5	6119448.5	5551600
0.000026	6285040	5634906	5111430.5
0.000028	5823196	5220129.5	4734652.5
0.00003	5423442.5	4861136.5	4408563.5
0.000032	5074113	4547444.5	4123633.5
0.000034	4766282.5	4271034	3872581.25
0.000036	4493017.5	4025675.5	3649741.25
0.000038	4248840	3806447	3450643
0.00004	4029371.75	3609416.5	3271713.25
0.000042	3831067.5	3431397	3110055.25

The behaviour of MTSF w.r.t. V/s rate of failure of main unit (λ) for different values of rate of failure of Ist standby unit (λ_1) is shown in Table No. 3. From the table, it can be easily seen that MTSF gets decreased with the increase in the values of the failure of main unit (λ). It also depicts that with the increase in rate of failure of Ist standby unit (λ_1), the MTSF decreases.

Table No. 4

	PROFIT (in INR)		
C_0	λ = 0.000032	λ = 0.0032	λ = 0.32
200	-261.236023	-20415.89453	-77937.66406
15200	14738.38672	-5449.623535	-63084.17969
30200	29738.01172	9516.646484	-48230.69531
45200	44737.63672	24482.91797	-33377.20703

60200	59737.25781	39449.19141	-18523.72461
75200	74736.875	54415.46094	-3670.243408
90200	89736.5	69381.74219	11183.24902
105200	104736.125	84348.00781	26036.73242
120200	119735.75	99314.27344	40890.21875
135200	134735.3906	114280.5391	55743.69531
150200	149735.0156	129246.8203	70597.19531
165200	164734.6406	144213.0781	85450.67969
180200	179734.2656	159179.3594	100304.1641
195200	194733.8906	174145.625	115157.6484
210200	209733.5	189111.8906	130011.1328
225200	224733.125	204078.1719	144864.6094
240200	239732.75	219044.4375	159718.0938
255200	254732.375	234010.7031	174571.5781

In Table No. 4, the behaviour of the profit w.r.t. revenue per unit uptime of the system (C_0) for different values of rate failure of main unit (λ) is discussed. As seen from the trend above, the profit increases with increase in the values of C_0.The following conclusions are also interpreted from the above table:

1. For $\lambda = 0.000032$, profit is > or = or < according as C_0 > or = or < INR 461.20, i.e. the revenue per unit uptime of the system (C_0) should not be less than INR 461 in order to get positive profit.

2. For $\lambda = 0.0032$, profit is > or = or < according as C_0 > or = or < INR 20615, i.e. the revenue per unit uptime of the system (C_0) should not be less than INR 20615 in order to get positive profit.

3. For $\lambda = 0.32$, profit is > or = or < according as C_0 > or = or < INR 78136, i.e. the revenue per unit uptime of system (C_0) should not be less than INR 78136 in order to get positive profit.

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