A Key Note on Dual Simplex Method

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Abstract

The main purpose of this paper is to overcome the failure in solving an LPP by the dual simplex method. For the easy understanding the illustrations have been incorporated along with the proposed algorithm.

Keywords - *LPP, simplex method, dual simplex method, pivotal element*

I. INTRODUCTION

The standard simplex method used to solve an LPP can be tabulated and implemented for the calculations to arrive an optimal solution. It takes large calculations and time as the variables and constraints increase (See [1] & [6]). The dual simplex method is the easiest way of solving the large scale problems and finding the optimum solution when the right hand of the constraints is negative in sign. It was developed in 1954 and modified in the 90's. (See [1]). The original method in the dual simplex method fails when the coefficients in the objective function are positive in the maximization problems, which causes the net evaluation, is negative. A constructive algorithm is provided below for the dual simplex method when the net evaluations are negative.

II. PROPOSED ALGORITHM

1. The objective function is in the maximization, otherwise convert by Min Z = -Max(-Z)
2. Constraints are either $\leq$ or $\geq$ or = type.
3. Convert the constraints into $\leq$ type.
4. Introduce slack variables in $\leq$, to get into the standard form of LPP.
5. Introduce the new constraint not more than M, whose variable coefficient in the objective function is positive and its cost is taken as M in the objective function, where M is the largest positive quantity.
6. Apply the general dual simplex method process to get the optimum solution.

III. NUMERICAL ILLUSTRATION

Example 3.1: Solve

$$\text{Max } Z = -x_1 + 3x_2 - 2x_3$$
$$\text{Subject to } 3x_1 - x_2 + 2x_3 \leq 7$$
$$4x_1 - 3x_2 - 8x_3 \geq 10$$
$$\text{And } \quad x_1, x_2, x_3 \geq 0$$

Solution:

Since the coefficient of x_2 is positive in the objective function, generally the dual simplex method is not applicable. To avoid this, introducing a new constraint as $x_2 \leq M$

The given LPP becomes

$$\text{Max } Z = -x_1 + 3x_2 - 2x_3$$
$$\text{Subject to } \quad 3x_1 - x_2 + 2x_3 \leq 7$$
$$-4x_1 + 3x_2 + 8x_3 \leq -10$$
$$x_2 \leq M$$
$$\text{And } \quad x_1, x_2, x_3 \geq 0$$

Introducing the slack variables s_1, s_2, s_3 in the constraints, the IBFS is given by

$$s_1 = 7,\ s_2 = -10,\ s_3 = M$$

Constructing the dual simplex table-1,

		C_j	**-1**	3	-2	0	0	M
c_B	X_B	Y_B	$\mathbf{x_1}$	x_2	x_3	s_1	s_2	s_3
0	s_1	7	**3**	-1	2	1	0	0
0	$\mathbf{s_2}$	**-10**	**-4**	**3**	**8**	**0**	**1**	**0**
M	s_3	M	**0**	1	0	0	0	1
z_j			**0**	M	0	0	0	M
$z_j - c_j$			**1**	M-3	2	0	0	0
$\theta_j = (z_j - c_j)/a_{kj},\ a_{kj}<0$			**-0.25**	-	-	-	-	-

Since all $z_j - c_j$ are positive and one of Y_B is negative. Selecting it as most negative the corresponding variable s_2 leaves the basic matrix. To find the entering variable, selecting the maximum of θ_j as -0.25, the corresponding variable x_1 enters into the basic matrix. The new revised simplex table-2 is given by

	C_j		-1	3	-2	0	0	M
c_B	X_B	Y_B	x_1	x_2	x_3	s_1	s_2	s_3
0	**s_1**	**-0.5**	**0**	**5/4**	**8**	**1**	**3/4**	**0**
-1	x_1	2.5	1	-3/4	-2	0	-1/4	0
M	s_3	M	0	1	0	0	0	1
z_j			-1	¾+M	2	0	1/4	M
$z_j - c_j$			0	M-(9/4)	0	0	1/4	0
$\theta_j = (z_j-c_j)/a_{kj}$, $a_{kj}<0$			-	-	-	-	-	-

Since all $z_j - c_j$ are positive and one of Y_B is negative. Selecting it as most negative the corresponding variable s_1 leaves the basic matrix. To find the entering variable, the maximum of θ_j cannot be calculated as all a_{kj} are positive.

Hence the given LPP has unbounded solution.

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Remark 3.1:

One can easily verify the optimal solution obtained in the example 3.1 with the proposed algorithm, is same to that of one obtained by general dual simplex method. The same is also applicable for getting the optimum solution. However, the proposed algorithm is simple and easy to implement. It is not requiring the huge matrix calculations. (See [2],[3],[4],[5]).

IV. CONCLUSIONS

The final conclusions of this paper is that the proposed algorithm is systematic than the existing method. It is applicable whenever the general dual simplex is failed for the smooth functioning of the algorithm.

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