On GP-Compactness & GP-Continuous Function in a Topological Spaces

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Abstract - *The aim of this paper is to introduce the new class of sets called generalized pre compact set (briefly gp-compact) sets in topological spaces and generalized pre continuous function (briefly gp-continuous functions and obtained some of their properties and studied some of their characterizations.*

Keywords - *Topological spaces, gp-compactness , gp-continuous functions.*

AMS Subject Classifications: 54A05, 54C08

I. INTRODUCTION

In 1987, Di Maio and Noiri [8] introduced and studied a new class of compact spaces called s-closed spaces using semi-open sets. Balachandran et al [4] introduced the concept of GO-compactness using g-open sets. Devi [7] introduced the notions of αGO-compactness by using αg-open sets. Recently Gnanambal et al [10] and Sheik John [19] introduced and investigated generalized pre-regular compact spaces (briefly GPR-compact)

The aim of this paper is to introduce the concepts of gp-compact, gp-continuous function and by using gp-open sets in topological spaces and investigate some of their properties.

II. PRELIMINARIES

Throughout this paper, the space (X, τ) (or simply X), the space (Y, σ) (or simply Y), always means a topological spaces on which no separation axioms are assumed unless explicitly stated. For a subset A of a space (X, τ), A^C denote the closure of A.

DEFINITION 2.1 : A subset A of a space X is said to be

(i) semiopen [11] if $A \subset Cl(Int(A))$.
(ii) semi-closed set[6] if $Int(cl(A)) \subseteq A$.
(iii) preopen [14] If $A \subset Int(Cl(A))$
(iv) preclosed [16] if $Cl(Int(A)) \subseteq A$
(v) α - open [17] if $A \subset Int(Cl(Int A)))$
(vi) α - closed [15] if $Cl(Int(Cl(A))) \subseteq A$
(vii) Semi - preopen [2] (= β - open [1]) if $A \subset Cl(Int(Cl(A)))$
(viii) a semi- pre closed set [1] if $Int(cl(Int(A))) \subset A$

(ix) generalized closed (briefly, g-closed) [12] if $Cl(A) \subset U$ whenever $A \subset U$ and Uis open in X.
(x) generalized preclosed (briefly, gp-closed) [18] if $pCl(A) \subset U$ whenever $A \subset U$ and U is open in X.
(xi) α - generalized closed (briefly, αg-closed) [13] if $\alpha Cl(A) \subset U$ whenever $A \subset U$ and U is open in X.
(xii) generalized preregularclosed (briefly, gpr-closed) [9] if $pCl(A) \subset U$ whenever $A \subset U$ and U is regular open in X.
(xiii) a generalized semi-closed (briefly gs-closed) set [3] if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X.

(xiv) a semi-generalized-closed (briefly sg-closed) set [5] if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is semiopen in X

Clearly, every open set is semiopen (resp.preopen, α - open, semipreopen (or β - open)). The complement of a semiopen (resp. preopen, α - open, semipreopen (or β - open)) is called semiclosed [6] (resp. preclosed [16], α - closed [15], semipreclosed [2] (or β - closed [1]).

Also every closed set is preclosed (resp. semiclosed, α - closed, semipreclosed (or β - closed)).

Every closed is g-closed, every preclosed is gp-closed. Every α - closed is αg-closed, every gp-closed is gpr-closed. The complement of a g-closed set (resp. αg –closed, gp-closed, gpr–closed, sg-closed, gs-closed) set in X is g – open [12] (resp. αg –open [13], gp-open [18], gpr-open [9], sg-open [5], gs-open [3]) set of X.Let PO(X) (resp. SO(X), αO(X), SPO(X), GO(X), αGO(X), GPO(X), GPRO(X)) denotes the family of all preopen (resp. semiopen, α-open, semipreopen, g-open, αg –open, gp-open, gpr-open) sets of X.

The family of all preclosed (resp. semiclosed, α -, g-closed, αg - closed, gp-closed, gpr-closed) sets of X is denoted by PF(X) (resp. SF(X), αF(X), GF(X), αGF(X), GPRF(X)).Also, every open is g-open, every preopen is gp-open, every α - open is αg –open

III. GP-COMPACT IN A TOPOLOGICAL SPACE

DEFINITION 3.1: A collection $\{A_i: i \in I\}$ of gp-open sets in a topological space X is called gp-open cover of a subset A in X if $A \subseteq \bigcup_{i \in I} A_i$.

DEFINITION 3.2: A topological space X is called gp-compact if every gp-open cover of X has a finite subcover.

DEFINITION 3.3: A subset A of a topological space X is called gp-compact relative to X if for every collection $\{A_i: i \in I\}$ of gp-open subsets of X such that $A \subseteq \bigcup_{i \in I} A_i$ there exists a finite subset I_0 of I such that $A \subseteq \bigcup_{i \in I_0} A_i$.

DEFINITION 3.4: A subset A of a topological space X is called gp-compact if A is gp-compact of the subspace of X.

THEOREM 3.5: A gp -closed subset of gp-compact space is gp-compact relative to X.

PROOF: Let A be a gp-closed subset of a topological space X. Then A^c is gp-open in X. Let $S = \{A_i: i \in I\}$ be a gp-open cover of A by gp-open subsets in X. Then $S^* = S \cup A^c$ is a gp-open cover of X. That is $X = [\cup \{A_i: i \in I\}] \cup A^c$. By hypothesis X is gp-compact and hence S^* is reducible to a finite subcover of X say $X = A_{i1} \cup A_{i2} \cup \ldots \cup A_{in} \cup A^c$, $A_{ik} \in S^*$. But A and A^c are disjoint. Hence $A \subseteq A_{i1} \cup A_{i2} \cup \ldots \cup A_{in} \in S$. Thus a gp-open cover S of A contains a finite subcover. That is A is gp-compact relative to X.

THEOREM 3.6: Let $f: X \rightarrow Y$ be surjective, gp-continuous function. If X is gp-compact, then Y is compact.

PROOF: Let $\{A_i: i \in I\}$ be an open cover of Y. Since f is gp-continuous function, then $\{f^{-1}(A_i): i \in I\}$ is gp-open cover of X has a finite subcover say $\{f^{-1}(A_i): i=1\ldots n\}$. Therefore $X = \bigcup_{i=1}^{n} f^{-1}(A_i)$ which implies $f(X) = \bigcup_{i=1}^{n} A_i$. Since f is surjective, that is $Y = \bigcup_{i=1}^{n} A_i$. Thus $\{A_1, A_2, \ldots A_n\}$ is a finite subcover of $\{A_i: i \in I\}$ for Y. Hence Y is compact.

THEOREM 3.7: If a function $f: X \rightarrow Y$ is gp-irresolute and a subset B of X is gp-compact relative to X, then the image f(B) is gp-compact relative to Y.

PROOF: Let $\{A_i: i\in I\}$ be any collection of gp-open sets in Y such that $f(B) = \bigcup_{i\in I} A_i$. Then $B \subseteq \bigcup_{i\in I} f^{-1}(A_i)$, where $\{f^{-1}(A_i: i \in I\}$ is gp-open set in X. Since B is gp-compact relative to X, there exists finite subcollection $\{A_1, A_2, ..., A_n\}$ such that $B \subseteq \bigcup_{i\in I_0} f^{-1}(A_i)$ Therefore $f(B) \subseteq \bigcup_{i\in I_0} A_i$. Hence f(B) is gp-compact relative to Y.

THEOREM 3.8: If a function f: X → Y is strongly gp-continuous from a compact space X onto a topological space Y, then Y is gp-compact.

PROOF: Let $\{A_i: i\in I\}$ be a gp-open cover of Y. Since f is strongly gp-continuous, $\{f^{-1}(A_i) : i\in I\}$ is an open cover of X. Again since X is compact space, the open cover $\{f^{-1}(A_i): i\in I\}$ of X has a finite subcover say $\{f^{-1}(A_i): i = 1 ... n\}$. Therefore $X = \bigcup_{i=1}^{n} f^{-1}(A_i)$ which implies $f(X) = \bigcup_{i=1}^{n} A_i$ so that $Y = \bigcup_{i=1}^{n} A_i$. That is $\{A_1, A_2 ...... A_n\}$ is a finite subcover of $\{A_i : i\in I\}$ for Y. Hence Y is gp-compact.

THEOREM 3.9: A function f: X → Y is perfectly gp-continuous from a compact space X onto a topological space Y, then Y is gp-compact.

PROOF: Since every perfectly gp-continuous function is a strongly gp-continuous and by Theorem 5.2.8, Y is gp-compact.

THEOREM 3.10: Let f: X → Y be a gp-continuous function from a gp-compact space X onto a topological space Y. If Y is ${}_{gp}T^*_{1/2}$-space, then Y is gp-compact.

PROOF: Let $\{A_i: i\in I\}$ be a gp-open cover of Y, by gp-open sets in Y. As Y is ${}_{gp}T^*_{1/2}$-space, $\{A_i, i\in I\}$ is an open cover of Y. Since f is gp-continuous, $\{f^{-1}(A_i): i\in I\}$ is a gp-open cover of X. Again since X is gp-compact, the gp-open cover $\{f^{-1}(A_i):i\in I\}$ of X has a finite subcover say $\{f^{-1}(A_i): i = 1, ..., n\}$. Therefore $X = \bigcup_{i=1}^{n} f^{-1}(A_i)$ which implies $f(X) = \bigcup_{i=1}^{n} A_i$. So that $Y = \bigcup_{i=1}^{n} A_i$. That is $\{A_1, A_2 ..., A_n\}$ is a finite subcover of $\{A_i: i\in I\}$ for Y. Hence Y is gp-compact.

THEOREM 3.11: Every gp-compact space is compact.

PROOF: Let X be a gp-compact space. Let $\{A_i : i\in I\}$ be an open cover of X. Then $\{A_i: i\in I\}$ is a gp-open cover of X as every open set is gp-open set. Since X is gp-compact, the gp-open cover $\{A_i: i\in I\}$ of X has a finite subcover say $\{A_i: i=1...n\}$ for X. Hence X is compact.

THEOREM 3.12: If X is compact and ${}_{gp}T^*_{1/2}$-space, then X is gp-compact.

PROOF: Let $\{A_i: i\in I\}$ be a gp-open cover of X. As X is ${}_{gp}T^*_{1/2}$-space, $\{A_i: i\in I\}$ is an open cover of X. Since X is compact, the open cover $\{A_i: i\in I\}$ of X has a finite subcover say $\{A_i: i=1,..., n\}$. Hence X is gp-compact.

THEOREM 3.13: Every αGO-compact space is gp-compact.

PROOF: Let X be a αGO-compact space. Let $\{A_i: i\in I\}$ be a gp-open cover of X by gp-open set in X. By Theorem 1.2.44 (v), $\{A_i: i \in I\}$ is αg-open cover of X. Since X is αGO-compact, the αg-open cover $\{A_i: i\in I\}$ of X has a finite subcover say $\{A_i: i=1...n\}$ of X. Hence X is gp-compact.

THEOREM 3.14: A topological space X is gp-compact if and only if every family of gp-closed sets of X having finite intersection property has a non-empty intersection.

PROOF: Suppose X is gp-compact. Let $\{A_i: i\in I\}$ be a family of gp-closed sets with finite intersection property. To prove that $\bigcap_{i\in I} A_i \neq \phi$ .Suppose $\bigcap_{i\in I} A_i = \phi$. Then $X - \bigcup_{i\in I} A_i = X$. Which implies $\bigcup_{i\in I}(X - A_i) = X$. Thus the cover $\{X - A_i; i\in I\}$ is a gp-open cover of X. Since X is gp-compact, the gp-open cover $\{X - A_i; i\in I\}$ has a finite subcover say $\{X - A_i; i=1\ldots\ldots n\}$. This implies $X = \bigcup_{i=1}^{n}(X - A_i)$ which implies that $X = X - \bigcap_{i=1}^{n} A_i$ which implies $X - X = X - \left[X - \bigcap_{i=1}^{n} A_i\right]$ implies that $\phi = \bigcap_{i=1}^{n} A_i$. This contradicts the assumption. Hence $\bigcap_{i=1}^{n} A_i \neq \phi$.

Conversely, suppose every family of gp-closed sets of X with finite intersection property has a non-empty intersection. To prove that X is gp-compact. Suppose X is not a gp-compact. Then there exists a gp-open cover of X say $\{G_i: i\in I\}$ having no finite subcover. This implies for any finite sub family $\{G_i: i=1\ldots n\}$ of $\{G_i: i\in I\}$ we have $\bigcup_{i=1}^{n} G_i \neq X$ which implies that $X - \bigcup_{i=1}^{n} G_i \neq X - X$, which implies $\bigcap_{i=1}^{n}(X - G_i) \neq \phi$. Then the family $\{X - G_i: i\in I\}$ of gp-closed sets has a finite intersection property. Also by assumption $\bigcap_{i=1}^{n}(X - G_i) \neq \phi$ which implies $X - \bigcup_{i=1}^{n} G_i \neq \phi$. So that $\bigcup_{i=1}^{n} G_i \neq X$. This implies $\{G_i: i \in I\}$ is not a cover of X. This contradicts the fact that $\{G_i: i\in I\}$ is a cover for X. Thus a gp-open cove $\{G_i: i\in I\}$ has a finite subcover $\{G_i: i=1\ldots\ldots n\}$. Hence X is gp-compact.

THEOREM 3.15: The image of a gp-compact space under a strongly gp-continuous function is gp-compact.

PROOF: Let $f: X \to Y$ be a strongly gp-continuous function from a gp-compact space X onto a topological space Y. Let $\{A_i : i\in I\}$ be a gp-open cover of Y. Then $\{f^{-1}(A_i) : i\in I\}$ is an open cover of X as f is strongly gp-continuous and so $\{f^{-1}(A_i) : i\in I\}$ is gp-open cover of X. Since X is gp-compact, the gp-open cover $\{f^{-1}(A_i): i\in I\}$ of X has a finite subcover say $\{f^{-1}(A_i): i=1\ldots\ldots n\}$. Therefore $X = \bigcup_{i=1}^{n} f^{-1}(A_i)$ which implies $f(X) = \bigcup_{i=1}^{n} A_i$. Thus $Y = \bigcup_{i=1}^{n} A_i$. That is $\{A_1, A_2, \ldots\ldots, A_n\}$ is a finite subcover of $\{A_i: i\in I\}$ for Y. Hence Y is gp-compact.

IV. CONCLUSION

In this paper, we have introduced the new class of generalized form of sets namely *gp- compactness* established their relationships with some generalized sets in topological space.

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