Study of Low Dimention Seperation Axioms and Weakly Generalized Pre - r_0 Spaces

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***Abstract**: The aim of this paper is to introduce the new class of low dimension sets called generalized pre difference sets and to introduce weakly generalized space called generalized R_0 spaces in topological spaces, and studied some of their properties and characterizations.*

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I. INTRODUCTION

In 1989 Nour [19] introduced the notions of pre - T_0, pre -T_1,and pre - T_2 spaces. In 2000 Saied Jafari [8] introduced the weaker separation axioms like pre – D_0, pre – D_1, pre – D_2 and pre-symmetric spaces. The first part deals with weak form of separation axioms like gp - D_0, gp - D_1,gp - D_2 space. The second part contains notions of generalized pre – symmetric spaces. The last part of this deals with Weakly generalized pre - R_0 spaces.

II. PRELIMINARIES

Throughout the thesis (X, τ) and (Y, σ) denote topological spaces on which no separation axioms are assumed unless explicitly stated and they simply written as X and Y respectively. All sets are considered to be subsets to topological spaces. The complement of A is denoted by X – A. The closure and interior of a set A are denoted by Cl(A) and int(A) respectively.

The following definitions are useful in the sequel :

DEFINITION 2.1 : A subset A of a space X is said to be

(i) semiopen [] if $A \subset Cl(Int(A))$.
(ii) semi-closed set[6] if $Int(cl(A)) \subseteq A$.
(iii) preopen [12] if$A \subset Int(Cl(A))$
(iv) preclosed [14] if$Cl(Int(A)) \subseteq A$
(v) α - open [15] if $A \subset Int(Cl(Int A)))$
(vi) α - closed [13] if$Cl(Int(Cl(A))) \subseteq A$
(vii) Semi - preopen [2] (= β - open [1]) if $A \subset Cl(Int(Cl(A)))$
(viii) a semi- pre closed set [1] if $Int(cl(Int(A))) \subset A$
(ix) generalized closed (briefly, g-closed) [11] if $Cl(A) \subset U$ whenever $A \subset U$ and Uis open in X.
(x) generalized preclosed (briefly, gp-closed) [18] if $pCl(A) \subset U$ whenever $A \subset U$ and U is open in X.

The family of all semi open sess (resp. semi-pre open sets) of X will be denoted by SO(X) SPO(X).

DEFINITION 2.2 : The semi-closure (resp. semi-pre closure) of a subset A of a space X is the intersection of all semi-closed (resp. semi preclosed) sets that contain A and is denoted by sCl(A) [4] (resp. spCl(A) [1])

DEFINITION 2.3 : The semi-interior (resp. semi-pre interior) of a subset A of a space X is the union of all semi-open (resp. semi preopen) sets that contained in A and is denoted by sInt(A) [4] (resp. spInt(A) [1])

DEFINITION 2.4 : A subset A of (X, τ) is called :
(i) generalized closed (in brief, g-closed) set [5] if $Cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open.
(ii) a semi-generalized-closed (briefly sg-closed) set [3] if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is semiopen in X
(iii) a generalized semi-closed (briefly gs-closed) set [2] if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X.

The complement of a g-closed (resp. sg-closed , gs-closed) set of a space X is called generalized open [5] (in brief, g-open) (resp. sg-open [3] , gs-open [2]) set of X .

DEFINITION 2.5: Pre T_i– spaces (i = 0, 1, 2) A topological space X is called

(i) Pre - T_0 [7] if for each pair of distinct points x, y of X there exists a preopen set U such that $x \in U$ and $y \notin U$.
(ii) Pre -T_1 [7] if to each pair of distinct points x, y of X there exists a preopen set U, V such that $x \in U$ and $y \in V$.
(iii) Pre -T_2 [9] if to each pair of distinct points x, y of X there exists disjoint preopen sets U, V such that $x \in U$ and $y \in V$.

DEFINITION 2.6: A topological space (X,τ) is said to be

(i) strongly regular [16] if for each preclosed set $A \subset X$ and each point $x \notin A$, there exist disjoint preopen sets $U, V \subset X$ such that $x \in U$ and $A \subset V$.
(ii) strongly normal [17] if for each pair of disjoint preclosed sets A and B of X, there exist disjoint preopen sets U and V containing them.

III. GENERALIZED PRE-DIFFERENCE SETS AND SPACES

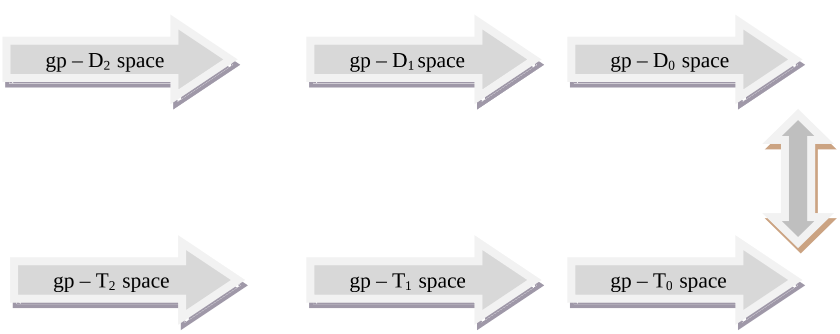
In this section we introduce some new separation axioms namely, gp-D_0 – spaces , gp-D_1 – spaces and gp-D_2 – spaces in topological spaces and investigate some of their properties.

We define the following
DEFINITION 3.1: A subset M of a topological space X is called generalized pre-Difference set (briefly gpD-set) if there are two $U, V \in GPO(X)$ such that $U \neq X$ and $M = U \setminus V$.
DEFINITION 3.2: A topological space (X, τ) is called gp - D_0 if for any distinct pair of points x and y of X there exists a gpD-set of X containing x but not y.

DEFINITION 3.3: A topological space (X, τ) is called gp – D_1 if for any distinct pair of points x and y of X there exists a gpD-set of X containing x but not y and a gpD-set of X containing y but not x.

DEFINITION 3.4: A topological space (X, τ) is called gp – D_2 if for any distinct pair of points x and y of X there exists disjoint gpD-sets M and N of X containing x and y respectively.

REMARK 3.5: By the definitions it follows the implication :



THEOREM 3.6: For a topological space (X,τ), the following properties hold :

(1) (X,τ) is gp - D_0 iff gp - T_0.

(2) (X,τ) is gp - D_1 iff gp - D_2.

PROOF : Necessary condition (1): Suppose that (X,τ) is gp - D_0 and for any distinct pair of distinct pair of points x and y of X at least one belongs to a gpD-set M. So we choose $x \in M$ and $y \notin M$. Let $M = U \setminus V$ for which $U \neq X$ and $U, V \in GPO(X)$. It follows that $x \in U$. For $y \notin M$ we have two cases : (a) $y \notin U$, (b) $y \in U$ and $y \in V$. For the case (a), X is gp-T_0 since $x \in U$ and $y \notin U$. In case (b) $y \in V$ and $x \notin V$.

Necessary condition (2) : Assume that X is gp-D_1. So by definition, for any distinct pair of points x and y of X there exist gp -D sets M and N such that $x \in M$ and $y \notin M$ and $y \in N$ and $x \notin N$. Suppose $M = U \setminus V$ and $N = R \setminus S$ where $U, V, R, S \in GPO(X)$. Since $x \notin N$ which implies $x \notin R$ or $x \in R$ and $x \in S$. If $x \notin R$, then from $y \notin M$ either (i) $y \notin U$ or (ii) $y \in U$ and $y \in V$. For the case (i) $x \in U \setminus V$ i.e. $x \in U \setminus (V \cup R)$ and $y \in R \setminus S$ follows that $y \in R \setminus (U \cup S)$. Now, we have $U \setminus (V \cup R)$ and $R \setminus (U \cup S)$ are disjoint sets. For the case (ii) since $y \in U$ and $y \in V$, it follows that $x \in U \setminus V$ and $y \in V$. So $U \setminus V$ and V are disjoint. Now if $x \in R$ and $x \in S$ follows that $y \in R \setminus S$ and $x \in S$. Thus $R \setminus S$ and S are disjoint. All steps shows that X is gp- D_2 .

The sufficient conditions for (1) and (2) follows from Remark 3.2.5.

REMARK 3.7 : From remark 3.5 and Theorem 3.6 it follows that gp-D_1 space gp-T_0 but converse is not true. Take $X = \{a, b\}$ and $\tau = \{ X, \phi, \{a\}\}$. Then $GPO(X) = \{ X, \phi, \{a\}\}$.The space (X,τ) is gp-T_0. But not gp-D_1. For $X = \{a,b,c,d\}$ with topology $\sigma = \{ X, \phi, \{a\}, \{a, d\}, \{a, b, d\}, \{a, c, d\}\}$. The space (X,σ) is gp-D_1but not gp-T_1.

IV. WEAKLY GENERALIZED PRE - R_0 SPACES

DEFINITION 4.1: A topological space (X, τ) is said to be weakly gp - R_0 if and only if $\bigcap_{x\in X} gpCl(\{x\}) = \phi$.

REMARK 4,2: Every weakly gp - R_0 space is weakly pre - R_0

PROOF: since $\bigcap_{x\in X} gpCl(\{x\}) \subseteq \bigcap_{x\in X} pCl(\{x\})$ hence the result follows.

EXAMPLE 4.3: Take $X = \{a, b, c \}$ and $\tau = \{ X, \phi, \{a\}, \{b, c\}\}$ then (X, τ) is weakly gp - R_0.

DEFINITION 4.4: Let (X,τ) be a topological space and $A \subset X$. Then the gp-kernel of the point x is the set denoted by $gpKer(\{x\}) = \{ y \mid \{x\}\bigcap_{x\in X} gpCl(\{y\})\neq\phi\}$ equivalently $gpKer(\{x\}) = \cap \{G \in GPO(X,\tau)/ x\in G\}$.

THEOREM 4.5 : A topological space (X, τ) is weakly gp - R_0 if and only if $gpKer(\{x\})\neq X$.

PROOF: Suppose (X, τ) is weakly gp - R_0 and there exist a point $z \in X$ such that $gpker(\{z\}) = X$. It follows that $z \notin U$, where U is some proper gp-open subset of X. It means that $z \in\bigcap_{x\in X} gpCl(\{x\})$ which against to the assumption. Now if $gpker(\{z\}) \neq X$ for every x in X. If there exists a point z in X such that $z \in\bigcap_{x\in X} gpCl(\{x\})$, then every gp-open set containing z must contain every point of X. It follows that X is the unique preopen set containing z. Therefore, $gpker(\{z\}) = X$ which is a contradiction. It follows that X is weakly gp - R_0.

DEFINITION 4.6 : A function $f : (X, \tau) \to (Y, \sigma)$ is called strongly gp-closed if the image of every gp-closed subset of X is gp-closed in Y.

THEOREM 4.7: If the function $f : (X, \tau) \to (Y, \sigma)$ is an injective always gp-closed function and X is weakly , then Y is weakly gp - R_0.

PROOF: From the assumption follows that

$$\bigcap_{y\in X} gpCl(\{y\})\subset\bigcap_{x\in X} gpCl(\{f(x)\})\subset f(\bigcap_{x\in X} gpCl(\{x\})) = f(\emptyset) = \emptyset.$$

THEOREM 4.8: Let the topological space be weakly gp - R_0 and Y be any topological space. Then the product space X xY is weakly gp - R_0.

PROOF: It is sufficient to show that $\bigcap_{(x,y)\in XxY} gpCl(\{(x,y)\}) = \emptyset$.

We have

$\bigcap_{(x,y)\in XxY} gpCl(\{(x,y)\}) = \bigcap_{(x,y)\in XxY}\big(gpCl(\{x\}) \times gpCl(\{y\})\big) =$
$\bigcap_{x\in X} gpCl(\{x\}) \times \bigcap_{y\in Y} gpCl(\{y\}) = \emptyset \times Y = \emptyset$.

V. CONCLUSION

In this paper, we have introduced the new class of low dimension separation axioms and weakly generalized R_0 –spaces and their properties and characterizations.

REFERENCES

[1] Abd-El - Monsef, M.E., El-Deeb, S.N. and Mahmoud, R.A., β - open and β - continuous mappings, Bull. Fac. Sci. Assiut Univ. 12 (1983), 70 – 90.
[2] D.Andrijevic, Semi-preopen sets, Mat. Vesnik 38 (1986), No.1, 24 – 32.
[3] S.P.Arya and R. Gupta, On strongly continuous mappings, Kyungpook Math., 14(1974)131-143.
[4] S.Arya and T. Nour, Characterizations of s – normal spaces, Indian J. Pure Appl. Math., 21 (1990). 717 – 719.
[5] K.Balachandran, P. Sundram and A. Rani, On generalized preclosed sets, preprint.
[6] N.Biswas , Characterization of semicontinuous mappings, Atti. Accad. Naz. Lience. Rend. Cl. Sci. Fis. Mat. Nat (8), 48 (1970), 399 – 402.
[7] Chattopadhyay. A, Pre - T_0 and Pre – T_1 topological spaces, J. Indian Acad. Math. Vol. 17 No.2 (1995), 156 – 159.
[8] S.Jafari and T. Noiri, On contra-precontinuous functions, Bull.Malaysian Math.Sci.Soc. (Second series)(2002), 115-128
[9] A.Kar and P. Bhattacharya., Some weak separation axioms, Bull. Calcutta. Math Soc. 82(1990), 415 - 422.
[10] N.Levine, Semi-open sets and semi-continuity in topological spaces, Amer. Math. Monthly 70
[11] (1963), 36-41. Mashhour, A.S., Abd El-Monsef, M.E. and El-Deeb, S.N., On pre-continuous and weak pre-continuous mappings, Proc. Math. Phys. Soc. Egypt (1982), No. 53, 47-53.
[12] Mashhour, A.S., I.A. Hasanein and S.N. El-Deeb, On α -continuous and α - open mappings, Acta. Math Hunga. 41 (1983), 213 – 218.
[13] Mashhour, A.S., Abd El-Monsef, M.E., and I.A. Hasanein: On pretopological spaces, Bull. Math. Soc.Sci.Math. R.S.R. 28(76) (1984), 39-45.
[14] Govindappa Navalagi, and Girishsajjanshettar, (2012),Some properties of (s, p)-open and (s, p)-closed functions, Int. J. of Matematical sciences and applications,Vol. 2 No.1(2012),Pp. 13-16.
[15] G.B.Navalagi, Completely preirresolute functions and completely gp-irresolute functions, IJMMS.
[16] G.B.Navalagi, Quasi α - closed, strongly α - closed and weakly α - irresolute mappings, TAP # 421,
[17] G.B.Navalagi, Generalization of preirresolute functions,Topology Atlas Preprint 420.
[18] T.Noiri, H. Maki and J. Umehara, Generalized preclosed functions, Mem. Fac.Sci. Kochi Univ. (Math.)19(1998), 13-20.
[19] T.Noiri and V. Popa, On g-regular spaces and some functions, Mem.Fac.Sci.Kochi Univ.Math 20 (1999), 67-74.