Reliability of a Multicomponent System Using Pareto Distributions

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Abstract- This paper deals with the stress-strength problem incorporating multi component standby system is considered. The reliability has been derived when strength and stress follow pareto distribution. The general expression for the reliability of a multi component standby system is obtained and the system reliability is computed numerically for different values of the stress and strength parameters.

Key words: Pareto distribution, Reliability, Stress-strength model, Standby system.

I. INTRODUCTION

If X denotes the strength of a component and Y is the stress imposed on it. The component operates as long Y is less than X and the Reliability of the component may therefore be defined as R=(X<Y) The Probability of the failure of a system depends upon the stress and strength of the system .Kapur and Lamberson [1]. The reliability of an n-cascade system with stress attenuation was proposed by Pandit and Sriwastav [2].The reliability for multi component systems when stress-strength follows exponential distributions. Sandhya and Uma mheshwaei [3].Sriwastav and Kokati [4] used cascade system for reliability estimation by considering that the components stress-strength is identically distributed. The purpose of this paper is to study the variations in system reliability for different parameter values in a multicomponent strength-stress based on X and Y being two independent random variables. And considered follow pareto distributions.

II.GENERAL MODEL

Consider a system of n-components, out of which only one is working under the impact of stresses and the remaining (n-1) are standbys. Whenever the working component fails, one of standby components takes the place of a failed component and is subjected to impact of stress then the system works. The system fails when all the components fail. Let $X_1, X_2, \ldots X_n$ be the strengths of the n components arranged in order of activation in the system. And let $Y_1, Y_2 \cdots Y_n$ are the stresses on the n components respectively then the system reliability R_n is given by. Where the marginal reliability R (n) is the reliability of the system by the n^{th} component then

$$R(n) = P\left[X_1 < Y_1, X_2 < Y_2, \cdots, X_{n-1} < Y_{n-1}, X_n > Y_n\right]$$

Let $f_i(x)$ and $g_i(y)$ are the probability density functions of X_i & Y_i $i = 1,2,\cdots n$ respectively and assumed X_i and Y_i are independent, then

$$R(n) = \int_{-\infty}^{\infty} F_1(y) g_1(y) dy \int_{-\infty}^{\infty} F_2(y) g_2 dy \cdots \int_{-\infty}^{\infty} \overline{F_n}(y) g_n(y) dy$$

$$F_i(y) = \int_0^y f_i(x) dx \cdot \overline{F_i}(y) = 1 - F_i(y).$$

Let X be the Strength and Y be the Stress of a system with p.d.f.

$$f_i(x) = \frac{\lambda_i k^{\lambda_i}}{x^{\lambda_i + 1}}, k \le x \le \infty; \lambda_{i,} k > 0, i = 1,2,3\cdots n$$

$$g_i(y) = \frac{\mu_i k^{\mu_i}}{y^{\mu_i + 1}}\ k \le y \le \infty; \mu_i, k > 0, i = 1,2,3 \ldots n$$

III. RELIABILITY COMPUTATIONS

A. Strength and stress follow Pareto distribution.

$$R(1) = \int_0^\infty \overline{F_1}(y) g_1(y) dy$$

$$= \int_k^\infty \left(\frac{k}{y}\right)^{\lambda_1} \frac{\mu_1 k^{\mu_1}}{y^{\mu_1+1}} dy$$

$$R(1) = \left[\frac{\mu_1}{\mu_1 + \lambda_1}\right]$$

$$R(2) = \left[\int_0^\infty F_1(y) g_1(y) dy\right]\left[\int_0^\infty \overline{F_2}(y) g_2(y) dy\right]$$

$$= \left[\int_k^\infty \left[1 - \left(\frac{k}{y}\right)^{\lambda_1}\right] \frac{\mu_1 k^{\mu_1}}{y^{\mu_1+1}} dy\right]\left[\int_k^\infty \left(\frac{k}{y}\right)^{\lambda_2} \frac{\mu_2 k^{\mu_2}}{y^{\mu_2+1}} dy\right]$$

$$R(2) = \left[\frac{\lambda_1}{\mu_1 + \lambda_1}\right]\left[\frac{\mu_2}{\mu_2 + \lambda_2}\right]$$

$$R(3) = \left[\int_0^\infty F_1(y) g_1(y) dy\right]\left[\int_0^\infty F_2(y) g_2(y) dy\right]\left[\int_0^\infty \overline{F_3}(y) g_3(y) dy\right]$$

$$= \left[\int_k^\infty \left[1 - \left(\frac{k}{y}\right)^{\lambda_1}\right] \frac{\mu_1 k^{\mu_1}}{y^{\mu_1+1}} dy\right]\left[\int_k^\infty \left[1 - \left(\frac{k}{y}\right)^{\lambda_2}\right] \frac{\mu_2 k^{\mu_2}}{y^{\mu_2+1}} dy\right]\left[\int_k^\infty \left(\frac{k}{y}\right)^{\lambda_3} \frac{\mu_3 k^{\mu_3}}{y^{\mu_3+1}} dy\right]$$

$$R(3) = \left[\frac{\lambda_1}{\mu_1 + \lambda_1}\right]\left[\frac{\lambda_2}{\mu_2 + \lambda_2}\right]\left[\frac{\mu_3}{\mu_3 + \lambda_3}\right]$$

In general

$$R(n) = \left[\prod_{i=1}^{n-1}\left[\frac{\lambda_i}{\mu_i + \lambda_i}\right]\right]\left[\frac{\mu_n}{\mu_n + \lambda_n}\right]$$

B. Strength is Pareto distribution and stress follows mixture of two Pareto distributions.

$$R(1) = \int_0^\infty \overline{F_1}(y) g_1(y) dy$$

$$=\int_{k}^{\infty}\left(\frac{k}{y}\right)^{\lambda_1}\left(p_1\frac{\mu_{11}k^{\mu_{11}}}{y^{\mu_{11}+1}}+(1-p_1)\frac{\mu_{21}k^{\mu_{21}}}{y^{\mu_{21}+1}}\right)dy$$

$$=\left[p_1\left[\frac{\mu_{11}}{\mu_{11}+\lambda_1}\right]+(1-p_1)\left[\frac{\mu_{21}}{\mu_{21}+\lambda_1}\right]\right]$$

$$R(2)=\left[\int_0^{\infty}F_1(y)g_1(y)dy\right]\left[\int_0^{\infty}\overline{F_2}(y)g_2(y)dy\right]$$

$$=\left[\int_k^{\infty}\left[1-\left(\frac{k}{y}\right)^{\lambda_1}\right]\left[p_1\frac{\mu_{11}k^{\mu_{11}}}{y^{\mu_{11}+1}}+(1-p_1)\frac{\mu_{21}k^{\mu_{21}}}{y^{\mu_{21}+1}}\right]dy\right]\left[\int_k^{\infty}\left[\left(\frac{k}{y}\right)^{\lambda_2}\right]\left[p_2\frac{\mu_{12}k^{\mu_{12}}}{y^{\mu_{12}+1}}+(1-p_2)\frac{\mu_{22}k^{\mu_{22}}}{y^{\mu_{22}+1}}\right]dy\right]$$

$$R(2)=\left[1-p_1\left[\frac{\mu_{11}}{\mu_{11}+\lambda_1}\right]-(1-p_1)\left[\frac{\mu_{21}}{\mu_{21}+\lambda_1}\right]\right]\times\left[p_2\left[\frac{\mu_{12}}{\mu_{12}+\lambda_2}\right]+(1-p_2)\left[\frac{\mu_{22}}{\mu_{22}+\lambda_2}\right]\right]$$

$$R(3)=\left[\int_0^{\infty}F_1(y)g_1(y)dy\right]\left[\int_0^{\infty}F_2(y)g_2(y)dy\right]\left[\int_0^{\infty}\overline{F_3}(y)g_3(y)dy\right]$$

$$=\left[\int_k^{\infty}\left[1-\left(\frac{k}{y}\right)^{\lambda_1}\right]\left[p_1\frac{\mu_{11}k^{\mu_{11}}}{y^{\mu_{11}+1}}+(1-p_1)\frac{\mu_{21}k^{\mu_{21}}}{y^{\mu_{21}+1}}\right]dy\right]\left[\int_k^{\infty}\left[\left(\frac{k}{y}\right)^{\lambda_2}\right]\left[p_2\frac{\mu_{12}k^{\mu_{12}}}{y^{\mu_{12}+1}}+(1-p_2)\frac{\mu_{22}k^{\mu_{22}}}{y^{\mu_{22}+1}}\right]dy\right]$$

$$\left[\int_k^{\infty}\left[\left(\frac{k}{y}\right)^{\lambda_3}\right]\left[p_3\frac{\mu_{13}k^{\mu_{13}}}{y^{\mu_{13}+1}}+(1-p_3)\frac{\mu_{23}k^{\mu_{23}}}{y^{\mu_{23}+1}}\right]dy\right]$$

$$R(3)=\left[1-p_1\left[\frac{\mu_{11}}{\mu_{11}+\lambda_1}\right]-(1-p_1)\left[\frac{\mu_{21}}{\mu_{21}+\lambda_1}\right]\right]\left[1-p_2\left[\frac{\mu_{12}}{\mu_{12}+\lambda_2}\right]-(1-p_2)\left[\frac{\mu_{22}}{\mu_{22}+\lambda_2}\right]\right]\times$$
$$\left[p_3\left[\frac{\mu_{13}}{\mu_{13}+\lambda_3}\right]+(1-p_3)\left[\frac{\mu_{23}}{\mu_{23}+\lambda_3}\right]\right]$$

In general

$$R(n)=\left[\prod_{i=1}^{n-1}\left[1-p_i\left[\frac{\mu_{1i}}{\mu_{1i}+\lambda_i}\right]-(1-p_i)\left[\frac{\mu_{2i}}{\mu_{2i}+\lambda_i}\right]\right]\right]\left[p_n\left[\frac{\mu_{1n}}{\mu_{1n}+\lambda_n}\right]+(1-p_n)\left[\frac{\mu_{2n}}{\mu_{2n}+\lambda_n}\right]\right]$$

C. Strength follows Pareto distribution and stress follows mixture of three Pareto distributions.

$$R(1)=\int_0^{\infty}\overline{F_1}(y)g_1(y)dy$$

$$=\int_k^\infty \left(\frac{k}{y}\right)^{\lambda_1}\left(p_{11}\frac{\mu_{11}k^{\mu_{11}}}{y^{\mu_{11}+1}}+p_{12}\frac{\mu_{12}k^{\mu_{12}}}{y^{\mu_{12}+1}}+p_{13}\frac{\mu_{13}k^{\mu_{13}}}{y^{\mu_{13}+1}}\right)dy$$

$$R(1)=\left[p_{11}\left[\frac{\mu_{11}}{\mu_{11}+\lambda_1}\right]+p_{12}\left[\frac{\mu_{12}}{\mu_{12}+\lambda_1}\right]+p_{13}\left[\frac{\mu_{13}}{\mu_{13}+\lambda_1}\right]\right]$$

$$R(2)=\left[\int_0^\infty F_1(y)g_1(y)dy\right]\left[\int_0^\infty \overline{F_2}(y)g_2(y)dy\right]$$

$$=\left[\int_k^\infty\left[1-\left(\frac{k}{y}\right)^{\lambda_1}\right]\left[p_{11}\frac{\mu_{11}k^{\mu_{11}}}{y^{\mu_{11}+1}}+p_{12}\frac{\mu_{12}k^{\mu_{12}}}{y^{\mu_{12}+1}}+p_{13}\frac{\mu_{13}k^{\mu_{13}}}{y^{\mu_{13}+1}}\right]dy\right]\left[\int_k^\infty\left[\left(\frac{k}{y}\right)^{\lambda_2}\right]\left[p_{21}\frac{\mu_{21}k^{\mu_{21}}}{y^{\mu_{21}+1}}+p_{22}\frac{\mu_{22}k^{\mu_{22}}}{y^{\mu_{22}+1}}+p_{23}\frac{\mu_{23}k^{\mu_{23}}}{y^{\mu_{23}+1}}\right]dy\right]$$

$$R(2)=\left[1-p_{11}\left[\frac{\mu_{11}}{\mu_{11}+\lambda_1}\right]-p_{12}\left[\frac{\mu_{12}}{\mu_{12}+\lambda_1}\right]-p_{13}\left[\frac{\mu_{13}}{\mu_{13}+\lambda_1}\right]\right]\times\left[p_{21}\left[\frac{\mu_{21}}{\mu_{21}+\lambda_2}\right]+p_{22}\left[\frac{\mu_{22}}{\mu_{22}+\lambda_2}\right]+p_{23}\left[\frac{\mu_{23}}{\mu_{23}+\lambda_2}\right]\right]$$

$$R(3)=\left[\int_0^\infty F_1(y)g_1(y)dy\right]\left[\int_0^\infty F_2(y)g_2(y)dy\right]\left[\int_0^\infty \overline{F_3}(y)g_3(y)dy\right]$$

$$=\left[\int_k^\infty\left[1-\left(\frac{k}{y}\right)^{\lambda_1}\right]\left[p_{11}\frac{\mu_{11}k^{\mu_{11}}}{y^{\mu_{11}+1}}+p_{12}\frac{\mu_{12}k^{\mu_{12}}}{y^{\mu_{12}+1}}+p_{13}\frac{\mu_{13}k^{\mu_{13}}}{y^{\mu_{13}+1}}\right]dy\right]\left[\int_k^\infty\left[1-\left(\frac{k}{y}\right)^{\lambda_2}\right]\left[p_{21}\frac{\mu_{21}k^{\mu_{21}}}{y^{\mu_{21}+1}}+p_{22}\frac{\mu_{22}k^{\mu_{22}}}{y^{\mu_{22}+1}}+p_{23}\frac{\mu_{23}k^{\mu_{23}}}{y^{\mu_{23}+1}}\right]dy\right]\times$$
$$\left[\int_k^\infty\left[\left(\frac{k}{y}\right)^{\lambda_3}\right]\left[p_{31}\frac{\mu_{31}k^{\mu_{31}}}{y^{\mu_{31}+1}}+p_{32}\frac{\mu_{32}k^{\mu_{32}}}{y^{\mu_{32}+1}}+p_{33}\frac{\mu_{33}k^{\mu_{33}}}{y^{\mu_{33}+1}}\right]dy\right]$$

$$R(3)=\left[1-p_{11}\left[\frac{\mu_{11}}{\mu_{11}+\lambda_1}\right]-p_{12}\left[\frac{\mu_{12}}{\mu_{12}+\lambda_1}\right]-p_{13}\left[\frac{\mu_{13}}{\mu_{13}+\lambda_1}\right]\right]\times\left[1-p_{21}\left[\frac{\mu_{21}}{\mu_{21}+\lambda_2}\right]-p_{22}\left[\frac{\mu_{22}}{\mu_{22}+\lambda_2}\right]+p_{23}\left[\frac{\mu_{23}}{\mu_{23}+\lambda_2}\right]\right.$$
$$\left[p_{31}\left[\frac{\mu_{31}}{\mu_{31}+\lambda_3}\right]+p_{32}\left[\frac{\mu_{32}}{\mu_{32}+\lambda_3}\right]+p_{33}\left[\frac{\mu_{33}}{\mu_{33}+\lambda_3}\right]\right]$$

In general

$$R(n)=\left[\prod_{i=1}^{n-1}\left[1-p_{i1}\left[\frac{\mu_{i1}}{\mu_{i1}+\lambda_i}\right]-p_{i2}\left[\frac{\mu_{i2}}{\mu_{i2}+\lambda_i}\right]-p_{i3}\left[\frac{\mu_{i3}}{\mu_{i3}+\lambda_i}\right]\right]\right]$$
$$\left[p_{n1}\left[\frac{\mu_{n1}}{\mu_{n1}+\lambda_n}\right]+p_{n2}\left[\frac{\mu_{n2}}{\mu_{n2}+\lambda_n}\right]+p_{n3}\left[\frac{\mu_{n3}}{\mu_{n3}+\lambda_n}\right]\right]$$

IV. Numerical Calculations

Table-1: Marginal reliabilities and system reliabilities R_2, R_3 when strength and stress follow Pareto distribution.

μ_1	R(1)	R(2)	R(3)	R_2	R_3
0.1	0.5	0.25	0.125	0.75	0.875
0.2	0.666	0.222	0.074	0.888	0.962
0.3	0.75	0.187	0.046	0.937	0.984
0.4	0.8	0.16	0.032	0.96	0.992
0.5	0.833	0.138	0.023	0.972	0.995
0.6	0.857	0.122	0.017	0.979	0.997
0.7	0.875	0.109	0.013	0.984	0.998
0.8	0.888	0.098	0.010	0.987	0.998
0.9	0.9	0.09	0.009	0.99	0.999
1	0.9090	0.082	0.007	0.991	0.999

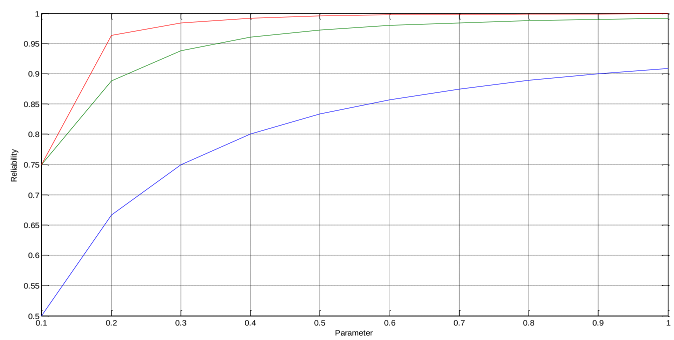

Figure-1

Table-2: Marginal reliabilities and system reliabilities R_2, R_3 when strength and stress follow Pareto distribution

λ_1	R(1)	R(2)	R(3)	R_2	R_3
0.1	0.666	0.222	0.04747	0.8888	0.962
0.2	0.5	0.25	0.125	0.75	0.875
0.3	0.4	0.24	0.144	0.64	0.784
0.4	0.3333	0.2222	0.14814	0.5555	0.703
0.5	0.2857	0.2040	0.14577	0.4897	0.635
0.6	0.25	0.1875	0.14062	0.4375	0.571
0.7	0.2222	0.1728	0.1344	0.3950	0.523
0.8	0.2	0.16	0.128	0.36	0.488
0.9	0.1818	0.1487	0.12171	0.3305	0.452
1	0.1666	0.1388	0.1157	0.3055	0.421

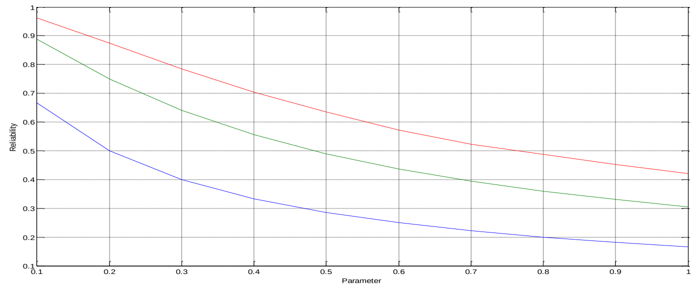

Figure-2

V. CONCLUSION

The reliability of a multi component standby system of stress - strength model is considered. Reliability has been derived for cases.Numerical calculations for R (r), r=1, 2, 3 have been obtained for the above particular cases. It has been observed from the graphs, the value of reliability increases when strength parameter decreases & the stress parameter increases.

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