Reluctant Intuitionistic Anti-Fuzzy Soft Prime Ideals of BCK-Algebras

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Abstract

The aim of this paper is to apply the notion of a reluctant intuitionistic anti -fuzzy soft set for dealing with several kinds of theories in BCK-algebras. The notions of reluctant anti-fuzzy soft commutative ideals and reluctant intuitionistic anti- fuzzy soft prime ideals in BCK-algebras are introduced and related properties are investigated. Relations between a reluctant intuitionistic anti-fuzzy soft ideals and reluctant intuitionistic anti-fuzzy soft commutative ideals are discussed. Conditions for a reluctant anti-intuitionistic fuzzy soft prime ideal to be a reluctant intuitionistic anti-fuzzy soft prime ideal are given and provided.

Keywords: *BCK-algebra, fuzzy set, soft set, Reluctant intuitionistic fuzzy soft set, andReluctant intuitionistic anti-fuzzy ideal (commutative ideal, Prime ideal).*

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I. INTRODUCTION

Fuzzy sets were introduced by L. A. Zadeh [21] in 1965. A fuzzy set is a class of objects with a continum of grades of membership. Such a set characterized by a membership function which assigns to each objects grade of membership ranging between zero and one. The notions of inclusion, union, intersection, complement, relation, convexity, etc., are extended to such sets, and various properties of these notions in the context of fuzzy sets are established. Intuitionistic fuzzy set (IFS) developed by Atanassov [4] is a powerful tool to deal with vagueness. A prominent characteristic of IFS is that it assigns to each element a membership degree and a non-membership degree, and thus, the IFS constitutes an extension of Zadeh's fuzzy set, which only assigns to each element a membership degree.Imai and Iseki [6, 7] introduced a new algebra induced by theBCI-system of propositional calculus.The soft set theory offers a general mathematical tool for dealing with uncertain, fuzzy, not clearly defined objects. Molodstov [16] introduced the basic notions of the theory of soft sets, to present the first results of the theory, and to discuss some problems of the future.Maji [12] introduced new operations on intuitionistic fuzzy soft sets. Some results relating to the properties of these operations have been established. An example has also been introduced as an application of these operations. The theory of soft sets was initiated by Maji et al. [14]. They defined equality of two soft sets, subset and superset of a soft set, the complement of a soft set, null soft set, and absolute soft set with examples. Soft binary operations like AND, OR and also the operations of union, intersection are defined. DeMorgan's laws and a number of results are verified in soft set theory.Maji et al. [13-15] introduced the notion of intuitionistic fuzzy soft set and proved that operations of the intuitionistic fuzzy soft set such as union, intersection, and subset etc. Jun [8] proposed the concept of soft BCK/BCI-algebras. They applied the notion of soft sets by Molodtsov to the theory of BCK/BCI-algebras. Also, the notion of soft BCK/BCI-algebras and soft subalgebras are introduced, and their basic properties are derived.

Jun et al. [10] introduced the notion of soft set theory applied to ideals in d-algebras and fuzzy soft set theory applied to BCI/BCK-algebras. Jun et al. [11] proposed fuzzy soft set theory applied to BCK/BCI-algebras. They applied a fuzzy soft set for dealing with several kinds of theories in BCK/BCI-algebras. The notions of fuzzy soft BCK/BCI-algebras, (closed) fuzzy soft ideals and fuzzy soft p-ideals are introduced, and related properties are investigated. Torra [18] introduced the concept of the hesitant fuzzy set as one of the extensions of Zadeh's fuzzy set allows the membership degree of an element to a set presented by several possible values, and it can express the hesitant information more comprehensively than other extensions of a fuzzy set and proved that the envelope of the hesitant fuzzy sets is an intuitionistic fuzzy set. Also,they proved that the operations we propose are consistent with the ones of intuitionistic fuzzy sets when applied to the envelope of the hesitant fuzzy sets. Babitha et al. [5] defined another important soft set, hesitant fuzzy soft set. They introduced basic operations such as union, intersection, complement, and De Morgan's law was proven. Jun et al. [9] suggest the concept of hesitant fuzzy soft sets to subalgebras and ideals in BCK/BCI-algebras. Abdullah [1] introduced the concept of intuitionistic fuzzy prime ideals of BCK-algebras.

New types of hesitant fuzzy soft ideals in BCK-algebras were introduced by Alshehri et al. [2]. They proved that hesitant fuzzy soft implicative ideal, hesitant fuzzy soft positive implicative ideal and hesitant fuzzy soft commutative ideal in BCK-algebras are introduced and related properties are investigated. Relations between a hesitant fuzzy soft subalgebra (ideal) and hesitant fuzzy soft (implicative, positive implicative and commutative) ideals are discussed. Conditions for a hesitant fuzzy soft ideal to be a hesitant fuzzy soft implicative ideal (positive implicative and commutative) are given and provided. Application of hesitant fuzzy soft sets in decision making is investigated.Hesitant intuitionistic fuzzy soft sets were introduced by Nazra et al. [17]. They extended the hesitant fuzzy soft sets to hesitant intuitionistic fuzzy soft sets by merging the concept of hesitant intuitionistic fuzzy sets and soft sets. Also, defined some operations on hesitant intuitionistic fuzzy sets, such as complement, union, and intersection, and obtain related properties. The similar operations are defined on hesitant intuitionistic fuzzy soft sets, and also some properties such as associative and De Morgan's laws are obtained. Alshehri et al. [3] introduced the notions of the hesitant anti-fuzzy soft set (subalgebras and ideals) and provide the relation between them. However, they studied new types of hesitant anti-fuzzy soft ideals (implicative, positive implicative and commutative). Also, they proved some theorems which determine the relationship between these notions.

In this paper, we discussed reluctant intuitionistic anti-fuzzy soft commutative ideals with example and proved that every reluctant intuitionistic anti-fuzzy soft commutative ideal is a reluctant intuitionistic anti-fuzzy soft ideal with converse also. we defined reluctant intuitionistic anti-fuzzy soft prime ideals and proved that a reluctant intuitionistic anti-fuzzy soft prime ideal is a reluctant intuitionistic ani- fuzzy soft ideal of commutative BCK-algebra.

II. PRELIMINARIES

In this section, some basic aspects are used to present the paper.

2.1 Definition:An algebra $(X; *, 0)$ of type (2, 0) is called a BCK-algebra if for each x, y, z in X satisfies,
(BCK-1). $((x * y) * (x * z)) \leq (z * y)$;
(BCK-2) $(x * (x * y)) \leq y$;
(BCK-3). $x \leq x$;
(BCK-4) $0 \leq x$;
(BCK-5) $x \leq y$, and $y \leq x$ imply $x = y$.
For briefness, we call X as a BCK-algebra.

2.2 Definition:A nonempty subset I of X is an ideal of X if it satisfies (I_1) 0 in I, and (I_2) $x * y$ in I and y in I imply x in I.

2.3 Definition*:* A nonempty subset I of X is a commutative ideal of X if it satisfies (I_1) and (I_3) $(x * y) * z$ in I and z in I imply $x * (y * (y * x))$ in I.

2.4 Definition:An ideal I of commutative BCK-algebra X is said to be prime if $x \wedge y$ in I imply x in I or y in I.
2.5 Definition*:* Let A be any set. A fuzzy set (abbreviated as FS) A in X is defined by $r_A : X \rightarrow [0,1]$.

2.6 Definition:Let X be primary set under consideration and let E be the set of factors. Let $\mathcal{F}(X)$ denotes the set of all FSs in X. Then $(\tilde{R}, A)$ is called a fuzzy soft set (FSS) over X and A is a subset of E,where $\tilde{R} : A \rightarrow \mathcal{F}(X)$.

***2.7 Definition*:**Let E be a reference set. A reluctant fuzzy set (RFS) on E is defined by$R_E = \{(e, r_E(e)) : e \in E\}$, where $r_E : E \rightarrow P([0, 1])$.

***2.8Definition*:**Let$\mathcal{RF}(X)$ be the set of all reluctant FSs. A couple $(\tilde{R}, A)$ is called a reluctant Fuzzy soft set (RFSS) over X, where $\tilde{R} : A \rightarrow \mathcal{RF}(X)$.

2.9 Definition:Let $(\tilde{R}, A)$ be a reluctant FSSover X and A is a subset of E. Then $(\tilde{R}, A)$ is a reluctant AFSIDof X if the reluctant FS$\tilde{R}[\alpha] = \{(x, r_{\tilde{R}[\alpha]}(x)) \mid x$ in X and α in A$\}$ is a reluctant AFID of X satisfies for each x, y in X,

(RAFSI1)$r_{\tilde{R}[\alpha]}(0) \leq r_{1_{\tilde{R}[\alpha]}}(x)$.

(RAFSI2)$r_{\tilde{R}[\alpha]}(x) \leq r_{\tilde{R}[\alpha]}(x * y) \vee r_{\tilde{R}[\alpha]}(y)$.

2.10 Definition**:** Let X is a primary set under consideration and let E be the set of factors. Let $I\mathcal{F}(X)$ denote the set of all IFSs in X. Then $(\widetilde{R}, A)$ is called an IFSS over X and A is a subset of E, where $\widetilde{R} : A \to I\mathcal{F}(X)$.

III. RELUCTANT INTUITIONISTIC ANTI-FUZZY SOFT COMMUTATIVE IDEALS

In this section, reluctant intuitionistic anti-fuzzy soft commutative ideals are defined with few results investigated.

3.1 Definition**:**Let$\mathcal{R}I\mathcal{F}(X)$ be the set of all reluctant IFSs. A couple $(\widetilde{R}, A)$ is called a reluctant IFSS over X, where $\widetilde{R} : A \to \mathcal{R}I\mathcal{F}(X)$.

3.2 Definition**:**Let $(\widetilde{R}, A)$ be a reluctant FSSover X and $A \subset E$. Then $(\widetilde{R}, A)$ is a reluctant AFSCID of X if the reluctant FS$\widetilde{R}$ $[\alpha] = \{(x, r_{\widetilde{R}[\alpha]}(x)) \mid x \in X$ and, α in A} is a reluctant AFCID of X satisfies for each x, y, z in X,

(RAFSCI1)$r_{\widetilde{R}[\alpha]}(0) \le r_{1_{\widetilde{R}[\alpha]}}(x)$.

(RAFSCI2)$r_{\widetilde{R}[\alpha]}(x * (y * (y * x))) \le r_{\widetilde{R}[\alpha]}((x * y) * z) \vee r_{\widetilde{R}[\alpha]}(z)$.

3.3 Definition**:**Let $(\widetilde{R}, A)$ be a reluctant IFSSover X and A is a subset of E. Then$(\widetilde{R}, A)$ is a reluctant IAFSCID of X if the reluctant IFS$\widetilde{R}[\alpha] = \{\langle x, r_{1_{\widetilde{R}[\alpha]}}(x), r_{2_{\widetilde{R}[\alpha]}}(x)\rangle \mid$ x in X, and α in A} is a reluctant IAFCID of X satisfies for each x, y, z in X,

(RIAFSCI1)$r_{1_{\widetilde{R}[\alpha]}}(0) \le r_{1_{\widetilde{R}[\alpha]}}(x)$ and $r_{2_{\widetilde{R}[\alpha]}}(0) \ge r_{2_{\widetilde{R}[\alpha]}}(x)$.

(RIAFSCI2)$r_{1_{\widetilde{R}[\alpha]}}(x * (y * (y * x))) \le r_{1_{\widetilde{R}[\alpha]}}((x * y) * z) \vee r_{1_{\widetilde{R}[\alpha]}}(z)$.

(RIAFSCI3)$r_{2_{\widetilde{R}[\alpha]}}(x * (y * (y * x))) \ge r_{2_{\widetilde{R}[\alpha]}}((x * y) * z) \wedge r_{2_{\widetilde{R}[\alpha]}}(z)$.

3.4 Example**:**Let X = {0, 1, 2} be a BCK-algebra with Cayley table:

Table 1.

$*$	0	1	2
0	0	0	0
1	1	0	0
2	2	2	0

Let $\widetilde{R}[\alpha]$ be a reluctant IFSS is defined as

Table 2.

X	0	1	2
$\widetilde{R}[\alpha]$	[0.25,0.66]	[0.33,0.5]	[0.33,0.5]

Then $\widetilde{R}[\alpha]$ is a reluctant IAFCID of X. Therefore $(\widetilde{R}, A)$ is a reluctant IAFSCID of X.

3.5 Theorem*:* Every reluctant IAFSCID of X is order-preserving.

Proof :.Let $(\widetilde{R}, A)$ be a reluctant IAFSCID of X. Let α in A, and x, y, z in X so that $x \le y$ implies $x * y = 0$. Then $r_{1_{\widetilde{R}[\alpha]}}(x * (y * (y * x))) \le r_{1_{\widetilde{R}[\alpha]}}((x * y) * z) \vee r_{1_{\widetilde{R}[\alpha]}}(z) = r_{1_{\widetilde{R}[\alpha]}}(0 * z) \vee r_{1_{\widetilde{R}[\alpha]}}(z) = r_{1_{\widetilde{R}[\alpha]}}(0) \vee r_{1_{\widetilde{R}[\alpha]}}(z) = r_{1_{\widetilde{R}[\alpha]}}(z)$ $\Rightarrow r_{1_{\widetilde{R}[\alpha]}}(x * (y * (y * x))) \le r_{1_{\widetilde{R}[\alpha]}}(z)$ and $r_{2_{\widetilde{R}[\alpha]}}(x * (y * (y * x))) \ge r_{2_{\widetilde{R}[\alpha]}}((x * y) * z) \wedge r_{2_{\widetilde{R}[\alpha]}}(z) = r_{2_{\widetilde{R}[\alpha]}}(0 * z) \wedge r_{2_{\widetilde{R}[\alpha]}}(z) = r_{2_{\widetilde{R}[\alpha]}}(0) \wedge r_{2_{\widetilde{R}[\alpha]}}(z) = r_{2_{\widetilde{R}[\alpha]}}(z) \Rightarrow r_{2_{\widetilde{R}[\alpha]}}(x * (y * (y * x))) \ge r_{2_{\widetilde{R}[\alpha]}}(z)$. Putting y = 0 and z = y, we obtain $r_{1_{\widetilde{R}[\alpha]}}(x) \le r_{1_{\widetilde{R}[\alpha]}}(y)$ and $r_{2_{\widetilde{R}[\alpha]}}(x) \ge r_{2_{\widetilde{R}[\alpha]}}(y)$.

3.6 Theorem**:**Every reluctant IAFSCID of X is a reluctant IAFSID of X.

*Proof.*Let $(\widetilde{R}, A)$ is a reluctant IAFSCID of X. Then for every α in A and x, y, z in X. Then $r_{1_{\widetilde{R}[\alpha]}}(x) = r_{1_{\widetilde{R}[\alpha]}}(x * (0 * (0 * x))) \le r_{1_{\widetilde{R}[\alpha]}}((x * 0) * z) \vee r_{1_{\widetilde{R}[\alpha]}}(z) \Rightarrow r_{1_{\widetilde{R}[\alpha]}}(x) \le r_{1_{\widetilde{R}[\alpha]}}(x * z) \vee r_{1_{\widetilde{R}[\alpha]}}(z)$, and $r_{2_{\widetilde{R}[\alpha]}}(x) = r_{2_{\widetilde{R}[\alpha]}}(x * (0 * (0 * x))) \ge r_{2_{\widetilde{R}[\alpha]}}((x * 0) * z) \wedge r_{2_{\widetilde{R}[\alpha]}}(z) \Rightarrow r_{2_{\widetilde{R}[\alpha]}}(x) \ge r_{2_{\widetilde{R}[\alpha]}}(x * z) \wedge r_{2_{\widetilde{R}[\alpha]}}(z)$. Hence $(\widetilde{R}, A)$ is a reluctant IAFSID of X.

3.7 Theorem: Every reluctant IAFSID of X satisfies the following conditions for each α in A and x, y, z in X if $x * y \leq z$, then $r_{1_{\tilde{R}[\alpha]}}(x) \leq r_{1_{\tilde{R}[\alpha]}}(y) \vee r_{1_{\tilde{R}[\alpha]}}(z)$, and $r_{2_{\tilde{R}[\alpha]}}(x) \geq r_{2_{\tilde{R}[\alpha]}}(y) \wedge r_{2_{\tilde{R}[\alpha]}}(z)$.

Proof. Let $\alpha \in$ A and x, y, z in X. If $x * y \leq z$, then $(x * y) * z = 0$. Since $r_{1_{\tilde{R}[\alpha]}}(x) \leq r_{1_{\tilde{R}[\alpha]}}(0) \vee r_{1_{\tilde{R}[\alpha]}}(z) = r_{1_{\tilde{R}[\alpha]}}((x * y) * z) \wedge r_{1_{\tilde{R}[\alpha]}}(z) \geq r_{1_{\tilde{R}[\alpha]}}(x * y)$ and $r_{2_{\tilde{R}[\alpha]}}(x) \geq r_{1_{\tilde{R}[\alpha]}}(0) \wedge r_{2_{\tilde{R}[\alpha]}}(z) = r_{2_{\tilde{R}[\alpha]}}((x * y) * z) \wedge r_{2_{\tilde{R}[\alpha]}}(z) \leq r_{2_{\tilde{R}[\alpha]}}(x * y)$, it follows that $r_{1_{\tilde{R}[\alpha]}}(x) \leq r_{1_{\tilde{R}[\alpha]}}(x * y) \vee r_{1_{\tilde{R}[\alpha]}}(y) = r_{1_{\tilde{R}[\alpha]}}(y) \vee r_{1_{\tilde{R}[\alpha]}}(z)$ and $r_{2_{\tilde{R}[\alpha]}}(x) \geq r_{2_{\tilde{R}[\alpha]}}(x * y) \wedge r_{2_{\tilde{R}[\alpha]}}(y) = r_{2_{\tilde{R}[\alpha]}}(y) \wedge r_{2_{\tilde{R}[\alpha]}}(z)$.

3.8 Theorem: If X is a commutative BCK-algebra, then every reluctant IAFSID of X is a reluctant IAFSCID of X.

Proof. Suppose $(\tilde{R}, A)$ is a reluctant IAFSID of X. It is suffices to show that $(\tilde{R}, A)$ is a reluctant AIFSCID of X. Let x, y, z in X, and α in A. Then $((x * (y * (y * x))) * ((x * y) * z)) * z = ((x * (y * (y * x))) * z) * ((x * y) * z) \leq (x * (y * (y * x))) * (x * y) = (x * (x * y)) * (y * (y * x)) = 0 \Rightarrow (x * (y * (y * x))) * ((x * y) * z) \leq z$. It follows fromTheorem 3.7 that, $r_{1_{\tilde{R}[\alpha]}}(x * (y * (y * x))) \leq r_{1_{\tilde{R}[\alpha]}}((x * y) * z) \vee r_{1_{\tilde{R}[\alpha]}}(z)$, and $r_{2_{\tilde{R}[\alpha]}}(x * (y * (y * x))) \geq r_{2_{\tilde{R}[\alpha]}}((x * y) * z) \wedge r_{2_{\tilde{R}[\alpha]}}(z)$ for every x, y, z in X, and α in A. Thus $(\tilde{R}, A)$ is a reluctant IAFSCID of X.

***3.9 Theorem**:* Let $(\tilde{R}, A)$ be a reluctant IAFSID of X. Then $(\tilde{R}, A)$ is a reluctant IAFSCID of Xif and only if for every x, y in X and α in A satisfies $r_{1_{\tilde{R}[\alpha]}}(x * (y * (y * x))) \leq r_{1_{\tilde{R}[\alpha]}}(x * y)$, and $r_{2_{\tilde{R}[\alpha]}}(x * (y * (y * x))) \geq r_{2_{\tilde{R}[\alpha]}}(x * y)$.

Proof. Suppose that $(\tilde{R}, A)$ is a reluctant IAFSCID of X, $r_{1_{\tilde{R}[\alpha]}}(x * (y * (y * x))) \leq r_{1_{\tilde{R}[\alpha]}}((x * y) * z) \vee r_{1_{\tilde{R}[\alpha]}}(z)$, and $r_{2_{\tilde{R}[\alpha]}}(x * (y * (y * x))) \geq r_{2_{\tilde{R}[\alpha]}}((x * y) * z) \wedge r_{2_{\tilde{R}[\alpha]}}(z)$ for every x, y, z in X and α in A. Taking z = 0, we have $r_{1_{\tilde{R}[\alpha]}}(x * (y * (y * x))) \leq r_{1_{\tilde{R}[\alpha]}}((x * y) * 0) \vee r_{1_{\tilde{R}[\alpha]}}(0)$ and $r_{2_{\tilde{R}[\alpha]}}(x * (y * (y * x))) \geq r_{2_{\tilde{R}[\alpha]}}((x * y) * 0) \wedge r_{2_{\tilde{R}[\alpha]}}(0) \Rightarrow r_{1_{\tilde{R}[\alpha]}}(x * (y * (y * x))) \leq r_{1_{\tilde{R}[\alpha]}}(x * y)$ and $r_{2_{\tilde{R}[\alpha]}}(x * (y * (y * x))) \geq r_{2_{\tilde{R}[\alpha]}}(x * y)$ for every x, y in X and α in A.

Conversely, assume that $(\tilde{R}, A)$ satisfies $r_{1_{\tilde{R}[\alpha]}}(x * (y * (y * x))) \leq r_{1_{\tilde{R}[\alpha]}}(x * y)$, and $r_{2_{\tilde{R}[\alpha]}}(x * (y * (y * x))) \geq r_{2_{\tilde{R}[\alpha]}}(x * y)$ for every x, y in X and α in A as $(\tilde{R}, A)$ be a reluctant IAFSID of X. Hence $r_{1_{\tilde{R}[\alpha]}}(x * y) \leq r_{1_{\tilde{R}[\alpha]}}((x * y) * z) \vee r_{1_{\tilde{R}[\alpha]}}(z)$, and $r_{2_{\tilde{R}[\alpha]}}(x * y) \geq r_{2_{\tilde{R}[\alpha]}}((x * y) * z) \wedge r_{2_{\tilde{R}[\alpha]}}(z)$ for every x, y, z in X and α in A. Therefore, $r_{1_{\tilde{R}[\alpha]}}(x * (y * (y * x))) \leq r_{1_{\tilde{R}[\alpha]}}(x * y) \leq r_{1_{\tilde{R}[\alpha]}}((x * y) * z) \vee r_{1_{\tilde{R}[\alpha]}}(z)$, and $r_{2_{\tilde{R}[\alpha]}}(x * (y * (y * x))) \geq r_{2_{\tilde{R}[\alpha]}}(x * y) \geq r_{2_{\tilde{R}[\alpha]}}((x * y) * z) \wedge r_{2_{\tilde{R}[\alpha]}}(z)$ for every x, y, z in X and α in A. Hence$(\tilde{R}, A)$ is a reluctant IAFSCID of X.

3.10 Theorem: A reluctant IAFSID $(\tilde{R}, A)$ of X is a reluctant IAFSCID of X if and only if for every x, y in X and α in A satisfies the identities: $r_{1_{\tilde{R}[\alpha]}}(x * (y * (y * x))) = r_{1_{\tilde{R}[\alpha]}}(x * y)$, and $r_{2_{\tilde{R}[\alpha]}}(x * (y * (y * x))) = r_{2_{\tilde{R}[\alpha]}}(x * y)$.

Proof. By using Theorem 3.9, we have $r_{1_{\tilde{R}[\alpha]}}(x * (y * (y * x))) \leq r_{1_{\tilde{R}[\alpha]}}(x * y)$ and $r_{2_{\tilde{R}[\alpha]}}(x * (y * (y * x))) \geq r_{2_{\tilde{R}[\alpha]}}(x * y)$. If $(x * y) \leq (x * (y * (y * x)))$, we have $r_{1_{\tilde{R}[\alpha]}}(x * y) \leq r_{1_{\tilde{R}[\alpha]}}(x * (y * (y * x)))$ and $r_{2_{\tilde{R}[\alpha]}}(x * y) \geq r_{2_{\tilde{R}[\alpha]}}(x * (y * (y * x)))$ for every x, y in X and α in A. Therefore, $r_{1_{\tilde{R}[\alpha]}}(x * (y * (y * x))) = r_{1_{\tilde{R}[\alpha]}}(x * y)$ and $r_{2_{\tilde{R}[\alpha]}}(x * (y * (y * x))) = r_{2_{\tilde{R}[\alpha]}}(x * y)$.

IV. RELUCTANT INTUITIONISTIC ANTI-FUZZY SOFT PRIME IDEALS

In this section, reluctant intuitionistic anti- fuzzy soft prime ideals are defined with few results discussed. For short, we call X as a commutative BCK-algebra.

4.1 Definition: Let $(\tilde{R}, A)$ be a reluctant FSS over X and A is a subset of E. Then $(\tilde{R}, A)$ is a reluctant AFSPID of X if the reluctant FS$\tilde{R}[\alpha] = \{(x, r_{\tilde{R}[\alpha]}(x)) \mid x$ in X, and α in A$\}$ is a reluctant AFPID of X satisfies $r_{\tilde{R}[\alpha]}(x \wedge y) \geq r_{\tilde{R}[\alpha]}(x) \wedge r_{\tilde{R}[\alpha]}(y)$.

***4.2 Definition*:**Let $(\widetilde{R}, A)$ be a reluctant IFSS over X and A is a subset of E. Then $(\widetilde{R}, A)$ is a reluctant IAFSPID of X if the reluctant IFS$\widetilde{R}[\alpha] = \{\langle x, r1_{\widetilde{R}[\alpha]}(x), r2_{\widetilde{R}[\alpha]}(x)\rangle \mid x \text{ in } X, \text{ and } \alpha \text{ in } A\}$ is a reluctant AIFSPID of X satisfies for every x, y in X,

(RIAFSPI1)$r1_{\widetilde{R}[\alpha]}(x \wedge y) \geq r1_{\widetilde{R}[\alpha]}(x) \wedge r1_{\widetilde{R}[\alpha]}(y)$.

(RIAFSPI2)$r2_{\widetilde{R}[\alpha]}(x \wedge y) \leq r2_{\widetilde{R}[\alpha]}(x) \vee r2_{\widetilde{R}[\alpha]}(y)$.

4.3 Example:Let X = {0, 1, 2, 3} be a commutative BCK-algebra with Cayley table:

Table 3.

∗	0	1	2	3
0	0	0	0	0
1	1	0	0	0
2	2	1	0	0
3	3	2	1	0

Let $(\widetilde{R}, A)$ be a reluctant IFSS of X is defined as

Table 4.

X	0	1	2	3
$\widetilde{R}[\alpha_1]$	[0,1]	[0.1,0.9]	[0.5,0.5]	[1,0]
$\widetilde{R}[\alpha_2]$	[0.1, 0.9]	[0.2,0.8]	[0.4,0.4]	[0.9,0.1]

Then $\widetilde{R}[\alpha]$ is a reluctant IAFPID of X based on factors α_1, $\alpha_2$in A. Therefore $(\widetilde{R}, A)$ is a reluctant intuitionistic anti- fuzzy soft prime ideal over X.

4.4 Theorem*:* Every reluctant IAFSPID of X is a reluctant IAFSID of X.

Proof. Trivial.

4.5 Theorem*:*Let $(\widetilde{R}, A)$ be a reluctant IAFSS of X. Then $(\widetilde{R}, A)$ is areluctantIAFSPID of X if and only if $r1_{\widetilde{R}[\alpha]}(x)$ and $r^C2_{\widetilde{R}[\alpha]}(x)$ are ant-fuzzy prime ideals of X, where $r^C2_{\widetilde{R}[\alpha]}(x) = 1- r2_{\widetilde{R}[\alpha]}(x)$.

Proof. Let $(\widetilde{R}, A)$be a reluctant IAFSPID of X. Since by Theorem 4.4, $(\widetilde{R}, A)$is a reluctant IAFSID and so $r1_{\widetilde{R}[\alpha]}(x)$ and $r^C2_{\widetilde{R}[\alpha]}(x)$ are anti-fuzzy ideals of X. Let for every x, y in X and α in A. Then$r1_{\widetilde{R}[\alpha]}(x \wedge y) \geq r1_{\widetilde{R}[\alpha]}(x) \wedge r1_{\widetilde{R}[\alpha]}(y)$, and $r2_{\widetilde{R}[\alpha]}(x \wedge y) \leq r2_{\widetilde{R}[\alpha]}(x) \vee r2_{\widetilde{R}[\alpha]}(y)$. Now $1- r2_{\widetilde{R}[\alpha]}(x \wedge y) \geq 1 - \{r2_{\widetilde{R}[\alpha]}(x) \vee r2_{\widetilde{R}[\alpha]}(y)\} \Rightarrow r^C2_{\widetilde{R}[\alpha]}(x) \geq \{1 - r2_{\widetilde{R}[\alpha]}(x)\} \wedge \{1 - r2_{\widetilde{R}[\alpha]}(y)\} \Rightarrow r^C2_{\widetilde{R}[\alpha]}(x) \geq r^C2_{\widetilde{R}[\alpha]}(x) \wedge r^C2_{\widetilde{R}[\alpha]}(y)$. Hence $r1_{\widetilde{R}[\alpha]}(x)$ and $r^C2_{\widetilde{R}[\alpha]}(x)$ are anti-fuzzy prime ideals of X. The converse is easy, we omit the proof.

***4.6 Theorem*:**Let $(\widetilde{R}, A)$ be a reluctant IAFSIDof X. Then $\triangle(\widetilde{R}, A)$ is a reluctant IAFSPID of X, where $r^C1_{\widetilde{R}[\alpha]}(x) = 1- r1_{\widetilde{R}[\alpha]}(x)$.

Proof. Let $(\widetilde{R}, A)$ be a reluctant IAFSPID of X. Since by Theorem 4.4, $(\widetilde{R}, A)$is a reluctant IAFSID of X and so $\triangle(\widetilde{R}, A)$ is a reluctant IAFSID of X. For any x, y in X and αin A, $r1_{\widetilde{R}[\alpha]}(x \wedge y) \geq r1_{\widetilde{R}[\alpha]}(x) \wedge r1_{\widetilde{R}[\alpha]}(y) \Rightarrow 1 - r1_{\widetilde{R}[\alpha]}(x \wedge y) \leq 1 - \{r1_{\widetilde{R}[\alpha]}(x) \wedge r1_{\widetilde{R}[\alpha]}(y)\} \Rightarrow r^C1_{\widetilde{R}[\alpha]}(x \wedge y) \leq \{1 - r1_{\widetilde{R}[\alpha]}(x)\} \vee \{1 - r1_{\widetilde{R}[\alpha]}(y)\} \Rightarrow r^C1_{\widetilde{R}[\alpha]}(x\wedge y) \leq r^C1_{\widetilde{R}[\alpha]}(x) \vee r^C1_{\widetilde{R}[\alpha]}(y)$. Hence $\triangle(\widetilde{R}, A)$ is a reluctant IAFSPID of X.

***4.7 Theorem*:**Let $(\widetilde{R}, A)$ be a reluctant IAFSPID of X. Then $\Diamond(\widetilde{R}, A)$ is a reluctant IAFSPID of X, where $r^C2_{\widetilde{R}[\alpha]}(x) = 1- r2_{\widetilde{R}[\alpha]}(x)$.

Proof. Let $(\widetilde{R}, A)$ be a reluctant IAFSPID of X. Since by Theorem 4.4, $(\widetilde{R}, A)$is a reluctant IAFSID of X and so $\Diamond(\widetilde{R}, A)$ is a reluctant IAFSID of X. For any x, y in X and αin A, $r2_{\widetilde{R}[\alpha]}(x \wedge y) \leq r2_{\widetilde{R}[\alpha]}(x) \vee r2_{\widetilde{R}[\alpha]}(y) \Rightarrow 1 - r2_{\widetilde{R}[\alpha]}(x \wedge y) \geq 1 - \{r2_{\widetilde{R}[\alpha]}(x) \vee r2_{\widetilde{R}[\alpha]}(y)\} \Rightarrow r^C2_{\widetilde{R}[\alpha]}(x \wedge y) \geq \{1 - r2_{\widetilde{R}[\alpha]}(x)\} \wedge \{1 - r2_{\widetilde{R}[\alpha]}(y)\} \Rightarrow r^C2_{\widetilde{R}[\alpha]}(x\wedge y) \geq r^C2_{\widetilde{R}[\alpha]}(x) \wedge r^C2_{\widetilde{R}[\alpha]}(y)$. Hence$\Diamond(\widetilde{R}, A)$ is a reluctant IAFSPID of X.

V. CONCLUSION

In this paper, we have introduced reluctant intuitionistic anti- fuzzy soft commutative ideals with example and proved that every reluctant intuitionistic anti-fuzzy soft commutative ideal is a reluctant intuitionistic anti-fuzzy soft ideal with converse also. Also, we defined reluctant intuitionistic anti-fuzzy soft prime ideal and proved that reluctant intuitionistic anti- fuzzy soft prime ideal is a reluctant intuitionistic anti-fuzzy soft ideal of commutative BCK-algebra. Finally, necessity and possibility operator of reluctant intuitionistic anti-fuzzy soft prime ideals is a reluctant intuitionistic anti- fuzzy soft prime ideal.For future, we can work some other reluctant intuitionistic anti-fuzzy soft ideals in B-algebras

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