On βg^* Closed Sets in Topological Ordered Spaces

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Abstract

The aim of this paper is to introduce a new class of closed sets in topological ordered spaces called increasing βg^*-closed sets, decreasing βg^*-closed sets and balance d βg^*-closed sets and obtain some of its characteristics.

Keywords - *$i\beta g^*$-closed set, $d\beta g^*$-closed set, $b\beta g^*$-closed set*

I. INTRODUCTION

Leopoldo Nachbin [6] initiated the study of topological ordered spaces. A topological ordered spaces is a triple (X, τ,≤) where 'τ' is a topology on X and '≤' is a partial order on X. For any x∈X, [x,→]={y∈X/x≤y} and [←, x]= {y∈X/y≤x}. A subset A of a topological ordered space (X,τ,≤) is said to be increasing if A= i(A) and decreasing if A = d(A) where i(A) = $\cup_{a\in A}[a,\rightarrow]$and d(A)=$\cup_{a\in A}[,\leftarrow, a]$. A subset of a topological ordered space (X,τ,≤) is said to be balanced if it is both increasing and decreasing. Veerakumar[7] introduced the study of i-closed, d-closed and b-closed sets in 2001. Levine[4] introduced the concept of generalized closed sets in topological spaces. Andrijevic[1] introduced the concept of β-open sets in 1986. The author[2] introduced βg^*-closed sets in topological spaces in 2016.[Throughout this paper *x*=i,d,b]

II. PRELIMINARIES

Throughout this paper (X, τ,≤) represent topological ordered spaces on which no separation axioms are assumed unless otherwise mentioned. For any subset A of a space(X, τ,≤), cl(A) and int(A) denote the closure of A and interior of A respectively.

Definition 2.1 A subset A of a space (X, τ) is called

1) a regular open set[3] if A=int(cl(A)) and regular-closed if A=cl(int(A)).

2) a β-open set [1]if A⊂cl(int(cl(A))) and β-closed if int(cl(int(A))) ⊂ A.

3) a semi-open set[5] if A⊂cl(int(A)) and semi-closed if int(cl(A)) ⊂ A.

Definition 2.2.A subset A of a topological space (X, τ) is called

1.Generalized closed (briefly g-closed) [4] if cl(A) ⊂ U whenever A⊂U and U is open.

Definition 2.3.[7]
A subset A of a topological space (X,τ,≤) is called
(i)an iclosed set if A is an increasing set and closed set.
(ii)a dclosed set if A is a decreasing set and closed set.
(iii)a bclosed set if A is both an increasing and decreasing set and a closed set.

III. xβG*-CLOSED SETS

***Definition 3.1*:** A subset A of (X, τ) is called βg^*-closed set if gcl(A) ⊂ U whenever A⊂U and U is β open.

Definition 3.2: A subset A of (X,τ,≤) is called $i\beta g^*$closed set if it is both increasing and βg^*closed set.
***Remark 3.3*:**∅ and X are $i\beta g^*$ closed subset of (X,τ,≤).
Theorem 3.4: Every iclosed set is an $i\beta g^*$closed set but not conversely.
Proof: Every closed set is a βg^*closed[3].Then every iclosed set is an $i\beta g^*$ closed set.

Example 3.5:Let X={a,b,c} ,τ={X,ϕ , {a}, $\leq= (a,a),(b,b),(c,c),(a,b),(b,c),(a,c)$} clearly (X,$\tau,\leq$) is a topological ordered space. iβg* closed sets are ={{c},{b,c},X, ϕ },iclosed sets are ={{b,c},X, ϕ }. Let A={c} .clearly A is an iβg* closed but not an iclosed set in X.

Theorem 3.6: Every increasing regular closed set is iβg* closed but not conversely.

Proof: Every regular closed set is βg* closed set[3].Then every ir closed is an iβg* closed set.

Example 3.7: Let X={a,b,c} ,τ={X,ϕ , {a}},$\leq=\{(a,a),(b,b),(c,c),(a,b),(b,c),(a,c)\}$ clearly (X,$\tau,\leq$) is a topological ordered space. iβg* closed sets are ={{c},{b,c},X, ϕ },irclosed sets are ={X, ϕ }. Let A={c}or{b,c}.clearly A is an iβg* closed but not an irclosed set in X.

Definition 3.8: A subset A of (X,$\tau,\leq$) is called $d\beta$g*closed set if it both decreasing and βg*closed set.

Theorem 3.9: Every d closed set is an dβg*closed set but not conversely.

Proof: Every closed set is a βg*closed. By theorem[3].Then every dclosed set is an dβg* closed set.

Example 3.10:Let X={a,b,c} ,τ={X,ϕ , {a}}, $\leq= (a,a),(b,b),(c,c),(a,b),(b,c),(a,c)$} clearly (X,$\tau,\leq$) is a topological ordered space. dβg* closed sets are ={{a},{a,b},X, ϕ },dclosed sets are ={X, ϕ }. Let A={a}or{a,b} .clearly A is an dβg* closed but not an dclosed set in X.

Theorem 3.11: Every decreasing regular closed set is dβg* closed but not conversely.

Proof: Every regular closed set is βg* closed set[3].Then every dr closed is an dβg* closed set.

Example 3.12: Let X={a,b,c} ,τ={X,ϕ , {a}},$\leq=\{(a,a),(b,b),(c,c),(a,b),(b,c),(a,c)\}$ clearly (X,$\tau,\leq$) is a topological ordered space. dβg* closed sets are ={{a},{a,b},X, ϕ },drclosed sets are ={X, ϕ }. Let A={a}or{a,b}.clearly A is an dβg* closed but not an drclosed set in X.

Definition 3.13: A subset A of (X,$\tau,\leq$) is called $b\beta$g*closed set if it both increasing and decreasing βg*closed set.

Theorem 3.14: Every bclosed set is an bβg*closed set but not conversely.

Proof: Every closed set is a βg*closed. By theorem[3].Then every bclosed set is an bβg* closed set.

Example 3.15:Let X={a,b,c} ,τ={X,ϕ , {a}}, $\leq= (a,a),(b,b),(c,c)$} clearly (X,$\tau,\leq$) is a topological ordered space. bβg* closed sets are ={{a},{b},{c},{a,b},{b,c},{a,c},X, ϕ },bclosed sets are ={{b,c},X, ϕ }. Let A={c} .clearly A is an bβg* closed but not an bclosed set in X.

Theorem 3.16: Every bregular closed set is bβg* closed but not conversely.

Proof: Every regular closed set is βg* closed set[3].Then every br closed is an bβg* closed set.

Example 3.17: Let X={a,b,c} ,τ={X,ϕ , {a}},$\leq=\{(a,a),(b,b),(c,c)\}$ clearly (X,$\tau,\leq$) is a topological ordered space. bβg* closed sets are ={{a},{b},{c},{a,b},{b,c},{a,c},X, ϕ },brclosed sets are ={X, ϕ }. Let A={a}.clearly A is an bβg* closed but not an brclosed set in X.

IV. CONCLUSION

In this paper, we have introduced increasing βg*-closed sets, decreasing βg*-closed sets and balanced βg*-closed sets and established their relationship with some of its characteristics in topological ordered spaces.

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