The Edge Nomatic Number of Graph

N.Pratap Babu Rao
Department of mathematics S.G. degree college koppal(Karnataka)INDIA

Abstract
The line domatic number of a graph is the maximum order of a partition of the edge set into dominating sets. Analogously the edge nomadic number of a graph is the maximum order of a partition of the edge set into edge neighborhood sets. This number is determined for various graphs and some bounds are obtained .Also Nordhus Gaddum type result is established for the edge nomatic number

Graph we mean a finite undirected graph without loops or multiple edges we follow the notations and terminology given by Harary [3]

For a grap[h G, Let V(G) and E(G) respectively denote the vertex set and edge set of G. For $v\in V(G)$ the open neighborhood of v denoted by N(v) is defined by $N(v)= \{u\in V: uv\in E\}$ and the closed neighborhood of v is defined by $N[v]=N(v) \cup \{v\}$. The edge neighborhood set of graph G, is defined as $G = \cup <N[x]>$ where $N[x]=N(u) \cup N(v)$ for any x=uv in E

A subset S of E is a edge-neighborhood set (written as ln-set)of graph G if $G = \bigcup_{x\in S} <N[x]>$

Where < N[x] > is the sub graph of graph G induced by N[x].

The edge neighborhood number $\eta'_0(G)$ of graph G is the minimum cardinality of a ln-set of G.This parameter has been studied in detail by E.Sampathkumar and prabha. S. Neeralgi [5]

A set $S\subseteq V(G)$ is said to be a dominating set if each vertex not in S is adjacent to at leat one vertex in S. The domination number $\gamma(G)$ is the minimum cardinality of a dominating set of G. This parameter has been investigated by many authors {3] to [6].

A D-partition of G is a partitionof V(G) into dominating sets. The domatic number d(G) is the maximum oder of a D-partition of graph G. $\delta(G)$ ($\Delta(G)$) is the minimum (maximum)degree among the vertices of G.

The set $S\subseteq E$is saiod to be edge dominating set if each edge not in S is adjacent to at least one edge in S. The edge domination number $\gamma'(G)$ is the minimum cardinality of a dominating set of G this parameter studied by Jayram[4]

A D-partition of graph G is a partition of E(G) into dominatingsets.. The edge domatic number d'(G) is the maximum order of a D-partition of G. $\delta'(G)$ ($\Delta'(G)$) is the minimum (maximum)degree among line graph L(G).

The nomadic number of a graph G denoted by $\eta(G)$,is the maximum order of a partition of vertex set of G into ln-set of graph G this para meter is studied by jay ram[4].

The edge nomadic number of a graph G is denoted by $\eta'(G)$ is the maximum order of a partition of E(G) into ln-set of graph G.

Note $\eta'(G)= \eta(L(G))$, line nomadic number is same as nomadic number of edge graph.

Proposition A: [4] For any graph G $\eta(G) \le d(G)$

Proposition 1: For any graph G

$$\eta'(G) \le d'(G).$$

By the above it is clear that an upper bound of the edge domatic number is also an upper bound for the edge nomadic number..

Proposition B:[4] For any graph G, $\eta(G) \le \delta(G) + 1$

Th8is upper bound for the domatic number of a graph referred in Cockayne and Hedtniemi[2]

Proposition 2: For any graph G, $\eta'(G) \le \delta'(G) + 1$

A graph Gis defined to be nomatically full of $\eta(G) = \delta(G) + 1$ Analogus to this edge nomaticlly full of $\eta'(G) \le \delta'(G) + 1$

[4] have the following results Asterisks indicate that every graph in the class is full.

Proposition C:[4] For any graph G

(i) $\eta(K_p+G) = p + \eta(G)$.
(ii) $\eta(K_p) = p$ $\quad \eta(K_p)=1$
(iii) $\eta(C_{2r} + 1) = 1$, $r\ge2$ $\quad \eta(C_{2r}) = 2$
(iv) For any tree with $p \ge 2$ vertices η (T)_ = 2
(v) *If G is maximal outer planar graph then $\eta(G) =3$

As a consequence of the above results by replacing g by L(G) in proposition C, We have the following results.

Proposition 3: For any graph G,

(i) $\eta'(K_p+G) = p + \eta$ '(G).

(ii) $\eta'(K_p) = q$ $\eta'(K_p)=1$

(iii) $\eta'(C_{2r}+1) = 1$, $r\geq 2$ $\eta'(C_{2r}) = 2$

(iv) For any tree with $p \geq 2$ vertices $\eta'(T)\_ = 2$

(v) *If G is maximal outer planar graph then $\eta'(G) = 3$

Proposition D:[4] For any graph G, $\eta(G) + \eta(\bar{G}) \leq p + 1$ and the equality holds if and only if $G = K_p$ or $\bar{K}_p$

Proposition 4: For any graph G, $\eta'(G) + \eta'(\bar{G}) \leq q + 1$ and the equality holds if and only if $G = K_p$ or $\bar{K}_p$\

Proof: By proposition 2 $\eta'(G) \leq \delta'(G) + 1$. Hence for $\bar{G}$ the complement of G.

$\eta'(G) \leq \delta'(G) + 1 \leq \Delta(G) + 1$ and hence

$\eta'(G) + \eta'(\bar{G}) \leq \delta'(G) + 1 \leq \delta'(G) + \Delta(\bar{G}) + 2$

$\leq q - 1 + 2 \leq q + 1.$

Since $\eta_0'(K_p) = \gamma'(K_p) = 1$ and $\eta_0'(\bar{K}_p) = \gamma'(\bar{K}_p) = q$ the rest of the proof analogously as given by Cockayne and Hedetneimi [2]

Theorem :[5] For any graph G

$P - q + q_0 \leq \eta_0(G) \leq p - \Delta(G)$ here $q_0 = \min\{q, (<D>)\}$ and D is the minimum dominating set of graph G.

Theorem 5: For any graph G

$P - q + p_0 \leq \eta_0'(G) \leq q - \Delta'(G)$ here $p_0 = \min\{p, (<D>)\}$ and D is the minimum edge dominating set of graph G.

Proposition 6: or any graph G $\eta_0'(G) + \eta'(G) \leq q + 1$ equality holds if and only if $G = K_p$ or $\bar{K}_p$.

Proof:: By proposition 2 we have

$\eta_0'(G) \leq q - \Delta'(G) \leq q - \delta'(G)$ and we have $\eta'(G) \leq \delta'(G) + 1$ Hence,

$\eta_0'(G) + \eta'(G) \leq q + 1$.

Now let G be a graph, $G \neq K_p$ or $\bar{K}_p$. such that

$\eta_0'(G) + \eta'(G) \leq q + 1$

It is clear that one of the inequalities $\eta_0'(G) \leq q - \delta'(G)$ or $\eta'(G) \leq \delta'(G) + 1$ is strict.

For otherwise $\eta_0'(G) = q - \delta'$

$= q - \eta'(G) + 1.$

Since $\eta_0'(G) \leq q - \Delta'(G)$ we have $q - \Delta'(G) \geq \eta_0'(G) = q - \eta'(G) + 1$.

There fore $\eta'(G) \geq \Delta' + 1$. Contradiction to above result.

Hence one of the inequalities is strict and this implies that $\eta_0'(G) + \eta'(G) \leq q + 1$ which contradict the hypothesis.

Hence $G = K_p$ or $\bar{K}_p$.

CONCLUSION

We have investigated some results corresponding to the concept of edge nomatic number. Ww have taken up the issue to obtain the edge nomatic number. Also Nordhus Gaddum type result is established for the edge nomatic number

REFERENCES

[1] C .Berge, Graphs and Hyper graph, North Holland, Amsterdam.

[2] E.J. Cockayne and S.T. Hedetneimi, towards a theory of Domination in Graphs Networks (1977).

[3] F.Harary, Graph theory Addison Wesley Reading mass (1972)

[4] S.R .Jay ram, The nomatic number of a graph Nat, Acad. Sci -letters Vol 10 No.1 91987)

[5] E. Sampathkumar and P.S.Neerlagi, line neighborhood number of graph Indian J. pure.app;.math 17(2) 142-149.

[6] Development in the theory of Domination in graph. M.R.I.Lecture notes No.1 Mehta research institute Allahabad.