Soft T-ideals of Soft BCI-algebras

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Abstarct

The notion of soft T-ideals and T-idealistic soft BCI-algebras is introduced and their basic properties are discussed. Relations between soft ideals and soft T-ideals of soft BCI-algebras are provided. Also idealistic soft BCI-algebras and T-idealistic soft BCI-algebras are being related. The intersection, union, "AND" operation and "OR" operation of soft T-ideals and T-idealistic soft BCI-algebras are established. Using soft sets, characterizations of (fuzzy) T-ideals in BCI-algebras are given. Relations between fuzzy T-ideals and T-idealistic soft BCI-algebras are discussed.

Index terms:(*T-idealistic) Soft BCI-algebra, Soft Ideal, Soft Set, Soft T-ideal*

I. INTRODUCTION

To solve complicated problems in economics, engineering, and environment, wecan't successfully use classical methods because of various uncertainties typical forthose problems. There are three theories: theory of probability, theory of fuzzysets, and the interval mathematics which we can consider as mathematical toolsfor dealing with uncertainties. But all these theories have their own difficulties.Uncertainties can't be handled using traditional mathematical tools but may bedealt with using a wide range of existing theories such as the probability theory, thetheory of (intuitionistic) fuzzy sets, the theory of vague sets, the theory of intervalmathematics, and the theory of rough sets. However, all of these theories have their own difficulties which are pointed out in [10]. Molodtsov[10] and Maji et al.[9] suggested that one reason for these difficulties may be due to the inadequacy ofthe parameterizationtool of the theory. To overcome these difficulties, Molodtsov[10] introduced the concept of soft set as a new mathematical tool for dealing withuncertainties that is free from the difficulties that have troubled the usual theoretical approaches. Molodtsov pointed out several directions for the applications ofsoft sets. At present, works on the soft set theory are progressing rapidly. Maji et al. [9] described the application of soft set theory to a decision making problem.Maji et al. [8] also studied several operations on the theory of soft sets. Chen et al. [2] presented a new definition of soft set parametrization reduction, and compared this definition to the related concept of attributes reduction in rough set theory.The algebraic structure of set theories dealing with uncertainties has been studiedby some authors. The most appropriate theory for dealing with uncertainties isthe theory of fuzzy sets developed by Zadeh [12].

In [4], Jun applied the notion of soft sets by Molodtsov to the theory of BCK/BCI algebras. He introduced the notion of soft BCK/BCI-algebras and soft sub algebras, and then derived their basic properties. In [5], Jun and Park deals with the algebraic structure of BCK/BCI-algebras by applying soft set theory. They discussed the algebraic properties of soft sets in BCK/BCI-algebras. They introduced the notion of soft ideals and idealistic soft BCK/BCI-algebras, and gave several examples. They investigated relations between soft BCK/BCI-algebras and idealistic soft BCK/BCI-algebras.

In this paper we apply the notion of soft sets by Molodtsov to T-ideals in BCI-algebras. We introduce the notion of soft T-ideals and T-idealistic soft BCI-algebras, and then derive their basic properties. Using soft sets, we give characterizations of (fuzzy) T-ideals in BCI-algebras. We provide relations between fuzzy T-ideals and T-idealistic soft BCI-algebras.

II. BASIC RESULTS ON BCI-ALGEBRAS

A BCK/BCI-algebra is an important class of logical algebras introduced by Y. Imai and K.Iseki[14]and were extensively investigated by several researchers.

An algebra $(X; *, 0)$ of type $(2, 0)$ is called a *BCI-algebra* if it satisfies the following conditions:

(a1) $(\forall x, y, z \in X)\ (((x * y) * (x * z)) * (z * y) = 0)$,

(a2) $(\forall x, y \in X)\ ((x * (x * y)) * y = 0)$,
(a3) $(\forall x \in X)\ (x * x = 0)$,
(a4) $(\forall x, y \in X)\ (x*y = 0,\ y * x = 0 \Rightarrow x = y)$.

If a BCI-algebra X satisfies the following identity:

(a5) $(\forall x \in X)\ (0 * x = 0)$,, then X is called a *BCK-algebra*.

Any BCI-algebra X has the following properties:

(b1) $(\forall x \in X)\ (x * 0 = x)$.
(b2) $(\forall x, y, z \in X)((x * y) * z = (x * z) * y)$.
(b3) $(\forall x, y \in X)(0*(x * y) = (0*x)*(0* y)$.
(b4) $(\forall x, y \in X)(x*(x*(x* y)) = x * y)$.
(b5) $(\forall x, y, z \in X)\ (x \le y \Rightarrow x* z \le y*z,\ z*y \le z*x)$.
(b6) $(\forall x, y, z \in X)\ ((x*z)*(y*z) \le x*y)$.
(b7) $(\forall x, y, z \in X)(0*(0*((x*z) *(y*z))) = (0*y) * (0*x))$.
(b8) $(\forall x, y \in X)(0*(0*(x * y)) = (0*y)*(0*x))$.

Where $x \le y$ if and only if $x * y = 0$.

A non-empty subset S of a BCI-algebra X is called a *sub algebra* of X if $x*y \in S$ for all $x, y \in S$.

Anon-empty subset A of a BCI-algebra X is called an *ideal* of X if it satisfies the following axioms:

(c1) $0 \in A$,
(c2) $(\forall x \in X)\ (\forall y \in A)\ (x * y \in A \Rightarrow x \in A)$.

Note that every ideal A of a BCI-algebra X satisfies:

$$(\forall x \in X)\ (\forall y \in A)\ (x \le y \Rightarrow x \in A).$$

Anon-empty subset A of a BCI-algebra X is called a *T-ideal* (seeKhalid and Ahmad[13]) of X if it satisfies

(c1) and
(c3) $(\forall x, z \in X)\ (\forall y \in A)\ ((x* y)* z \in A \Rightarrow x * z \in A)$.

We know that every T-ideal of a BCI-algebra X is also an ideal of X. We refer thereader to the books [3, 11] for further study about ideals in BCK/BCI-algebras.

III. BASIC RESULTS ON SOFT SETS

Molodtsov [10] defined the soft set in the following way. Let U be an initial universeset and E be a set of parameters. Let P(U) denotes the power set of U and $A \subset E$.

Definition 3.1 **(Molodtsov[10])**:A pair (F, A) is called a *soft set* over U, where F is a mappinggiven by

$$F: A \rightarrow P(U).$$

In other words, a soft set over U is a parameterized family of subsets of the universe U. For $a \in A$,F(a) may be considered as the set of a-approximate elements of the soft set (F, A).

Definition 3.2 **(Majiet al[8]):** Let (F, A) and (G, B) be two soft sets over a common universe U. The *intersection* of (F, A) and (G, B) is defined to be the soft set (H, C)satisfying the following conditions:

(i) $C = A \cap B$,
(ii) $(\forall x \in C)$ (H (x) = F(x) or G (x), (as both are same sets)).

In this case, we write (F, A) $\tilde{\cap}$ (G, B) = (H, C).

Definition 3.3 **(Majiet al[8]):**Let (F, A) and (G, B) be two soft sets over a common universeU. The *union* of(F, A) and (G, B) is defined to be the soft set (H, C) satisfying the following conditions:

(i) C = A ∪ B,
(ii) for all x∈ C,

$$H(x) = \begin{cases} F(x), & if\ x \in A - B \\ G(x), & if\ x \in B - A \\ F(x) \cup G(x), & if\ x \in A \cap B \end{cases}$$

In this case, we write (F, A) $\tilde{\cup}$ (G,B) = (H, C).

Definition 3.4 **(Majiet al[8]):**If (F, A) and (G, B) are two soft sets over a common universe U, then "(F, A) *AND* (G, B)" denoted by (F, A) $\tilde{\wedge}$ (G, B) (see [8]) is defined by(F, A) $\tilde{\wedge}$ (G, B) = (H, A × B), whereH (x, y) = F(x)∩G (y) for all (x, y) ∈ A × B.

Definition 3.5 **(Majiet al[8]):**If (F, A) and (G, B) are two soft sets over a common universe U, then "(F, A) *OR* (G, B)" denoted by (F, A) $\tilde{\vee}$ (G, B) (see [8]) is defined by(F, A) $\tilde{\vee}$ (G, B) = (H, A × B), whereH (x, y) = F(x)∪G (y) for all (x, y) ∈ A × B.

Definition 3.6 **(Majiet al[8]):**For two soft sets (F, A) and (G, B) over a common universe U. we say that (F, A) is a *soft subset* of (G, B), denoted by (F, A) $\tilde{\subseteq}$ (G, B) if it satisfies:

(i) A ⊂ B,
(ii) For every a∈A, F(a) and G (a) are identical approximations.

IV. SOFT T-IDEALS

In what follows let X and A be a BCI-algebra and a nonempty set, respectively, and R will refer to an arbitrary binary relation between an element of A and an element ofX, that is, R is a subset of A × X without otherwise specified. A set-valued function F: A → P(X) can be defined as F(x) = {y ∈ X |xRy}for all x ∈ A. The pair (F, A) is then a soft set over X.

Definition 4.1 **(Jun and Park[5]):**Let S be a subalgebraof X. A subset I of X is called an ideal ofX related to S (briefly, S-ideal of X), denoted by I ◁ S,if it satisfies:

(i) 0 ∈ I,
(ii) (∀x∈ S) (∀y ∈ I) (x * y ∈ I⇒ x ∈ I).

Definition 4.2 **(Jun and Park[5]):**Let S bea subalgebra of X. A subset I of X is called a T-ideal ofX related to S (briefly, S-T-ideal of X), denoted by I $\lhd_T$ S, if it satisfies:

(i) 0 ∈ I,
(ii) (∀x, z ∈ S) (∀y ∈ I) ((x *y)* z∈ I ⇒ x * z ∈ I).

***Example 4.3*:**Let X = {0, a, b, c, d} be a BCK-algebra, and hence a BCI-algebra,with the following Cayley table:

*	0	a	b	c	d
0	0	0	0	0	0
A	a	0	a	a	a
B	b	b	0	b	b
C	c	c	c	0	c
D	d	d	d	d	0

Then $S=\{0,a,b\}$ is a subalgebra of X and $I=\{0,a,b,d\}$ is an S-T-ideal of X.

Note that every soft T-ideal is a soft ideal.

***Definition 4.4* (Jun[4]):**Let (F, A) be a soft set over X. Then (F, A) is called a softBCI-algebra over X if F(x) is a subalgebra of X for all $x \in A$.

***Definition 4.5* (Jun and Park[5]):**Let (F, A) be a soft BCI-algebra over X. A soft set (G, I) overX is called a soft ideal of (F,A), denoted by $(G, I)\, \tilde{\lhd}\, (F, A)$, if it satisfies:

(i) $I \subset A$,
(ii) $(\forall x \in I)\,(G(x) \lhd F(x))$.

***Definition 4.6*:**Let (F, A) be a soft BCI-algebra over X. A soft set (G, I) overX is called a soft T-idealof (F, A), denoted by $(G, I)\, \tilde{\lhd}_T\, (F, A)$, if it satisfies:

(i) $I \subset A$,
(ii) $(\forall x \in I)\,(G(x) \lhd_T F(x))$.

Let us illustrate this definition using the following examples.

***Example 4.7*:**Consider a BCI-algebra $X = \{0,a,b,c,d\}$ which is given in Example 4.3. Let (F, A) be a soft set over X,Where A = X, define a set-valued function $F: A \to P(X)$ by

$$F(x) = \{y \in X \mid y * (y * x) \in \{0, a\}\}$$

for all $x \in A$. Then F(0) = F(a) = X, F(b) = F(c) = {0, a, d}, F(d) = {0, a, b, c}. Hence (F, A) is a soft BCI-algebra over X (Jun[4]).

(1)Let (G, I) be a soft set over X, where I = {a, b, c} and $G: I \to P(X)$ is a set-valuedfunction defined by

$$G(x) = \{y \in X \mid y * (y * x) \in \{0, d\}$$

for all $x \in I$. Then G (a) = {0, b, c, d} $\lhd_T$ X = F(a), G (b) = {0, a, c, d} $\lhd_T$ {0, a, c, d} = F(b) and G(c) = {0, a, b, d} $\lhd_T$ {0, a, b, d} = F(c). This means that$(G, I)\, \tilde{\lhd}_T\, (F, A)$, and hence $(G, I)\, \tilde{\lhd}\, (F, A)$.

(2) For I = {a, b, c}, let $H: I \to P(X)$ be a set-valued function defined by

$$H(x) = \{0\} \cup \{y \in X \mid x \leq y\}$$

for all $x \in I$. Then H (a) = {0, a} $\lhd_T$ X = F(a), H (b) = {0, b} $\lhd_T$ {0, a, c, d} =F(b) andH (c) = {0, c} $\lhd_T$ {0, a, b, d} = F(c), which implies that $(H, I)\, \tilde{\lhd}_T\, (F, A)$and $(H, I)\, \tilde{\lhd}\, (F, A)$.

Note that every soft T-ideal is a soft ideal. But, the converse is not true as seen in the following example.

***Example 4.8*:**Let $X = \{0, a, b, c\}$ be a BCI-algebra with the followingCayley table:

*	0	a	b	c
0	0	c	b	a
a	a	0	c	b
b	b	a	0	c
c	c	b	a	0

Let (F, A) be a soft set over X, where A= X and $F: A \to P(X)$is a set-valued function defined by

$$F(x) = \{0\} \cup \{y \in X \mid y * (y * x) \in \{0, a\}$$

for all $x \in A$. Then $F(0) = F(a) = X$ and $F(b) = F(c) = \{0\}$, which are subalgebras of X.
Hence (F, A) is a soft BCI-algebra over X. Now, consider $I = \{0,a\} \subset A$ and define a set-valued function $G: I \to P(X)$ by

$$G(x) = \{0\} \cup \{y \in X \mid x \leq y\}$$

for all $x \in I$. Then $G(0) = \{0\} \lhd X = F(0)$ and $G(a) = \{0,a\} \lhd X = F(a)$.
Hence (G, I) is a soft ideal of (F, A), but G (0) is not a F(0)-T-ideal of X (see Example 4.3(2)).Because $G(a)$ is not an $F(a)$-h-ideal of X.Since $(b*0)*c = b*c = c \notin G(a)$ but $b*c = c \notin G(a)$.

Example 4.9:Let X = {0, 1, a, b, c} be a BCI-algebra with the following Cayley table.

*	0	1	a	b	c
0	0	0	a	b	c
1	1	0	a	b	c
a	a	a	0	c	b
b	b	b	c	0	a
c	c	c	b	a	0

For $A = \{0, 1\} \subset X$, let $F: A \to P(X)$ be a set-valued function defined by

$$F(x) = \{y \in X \mid y * x = y\}$$

for all $x \in A$. Then $F(0) = X$ and $F(1) = \{0, a, b, c\}$, which are subalgebras of X, and hence (F, A) is a soft BCI-algebra over X. If we take I = A and define a set-valued function $G: I \to P(X)$ by

$$G(x) = \{y \in X \mid x * (x * y) \in \{0, b\}\}$$

for all $x \in I$, then we obtain that

$$G(0) = \{0, 1, b\} \lhd_T F(0) \text{ and } G(1) = \{0, 1, b\} \lhd_T F(1).$$

This means that $(G, I) \tilde{\lhd}_T (F, A)$. Now, consider J = {0} which is not disjoint with I, and let $H: J \to P(X)$ be a set-valued function defined by

$$H(x) = \{y \in X \mid x * (x * y) \in \{0, c\}\}$$

for all $x \in J$. Then $H(0) = \{0, 1, c\} \lhd_T F(0)$, and so $(H, J) \tilde{\lhd}_T (F, A)$. But if
$(K, U) := (G, I) \tilde{\cup} (H, J)$, then $K(0) = G(0) \cup H(0) = \{0, 1, b, c\}$, which is not a T-ideal of X related to F(0) since $(a * b) * 0 = c \in K(0)$ and $a * 0 = a \notin K(0)$.
Hence $(K, U) = (G, I) \tilde{\cup} (H, J)$ is not a soft T-ideal of (F, A).

Theorem 4.10:Let (F, A) be a soft BCI-algebra over X. For any soft sets $(G_1, I_1)$and (G_2, I_2) over X where$I_1 \cap I_2 \neq \phi$, we have$(G_1, I_1) \tilde{\lhd}_T (F, A), (G_2, I_2) \tilde{\lhd}_T (F, A) \Rightarrow (G_1, I_1) \tilde{\cap} (G_2, I_2) \tilde{\lhd}_q (F, A)$.

Proof. Using Definition 3.2, we can write

$$(G_1, I_1) \tilde{\cap} (G_2, I_2) = (G, I),$$

where $I = I_1 \cap I_2$ and $G(x) = G_1(x)$ or $G_2(x)$ for all $x \in I$. Obviously, $I \subset A$ and $G: I \to P(X)$ is a mapping. Hence (G, I) is a soft set over X. Since$(G_1, I_1) \tilde{\lhd}_T (F, A)$ and $(G_2, I_2) \tilde{\lhd}_T (F, A)$,weknow that $G(x) = G_1(x) \lhd_T F(x)$ or$G(x) = G_2(x) \lhd_T F(x)$ for all $x \in I$. Hence

$$(G_1, I_1) \tilde{\cap} (G_2, I_2) = (G, I) \tilde{\cap} (F, A).$$

This completes the proof.

Theorem 4.11:Let (F, A) be a soft BCI-algebra over X. For any soft sets (G, I)and (H, J) over X in which I and J are disjoint, we have(G, I) $\tilde{\triangleleft}_T$ (F, A), (H, J) $\tilde{\triangleleft}_T$ (F, A)$\Rightarrow$ (G, I) $\tilde{\cup}$ (H, J) $\tilde{\triangleleft}_T$ (F, A).

Proof. Assume that (G, I) $\tilde{\triangleleft}_T$ (F, A) and (H, J) $\tilde{\triangleleft}_T$ (F, A). By means of Definition3.3, we can write (G, I) $\tilde{\cup}$ (H, J) = (K, U) where U = I $\cup$ J and for every x $\in$ U,

$$K(x) = \begin{cases} G(x), & if\ x \in I - J \\ H(x), & if\ x \in J - I \\ G(x) \cup H(x), & if\ x \in I \cap J \end{cases}$$

Since I $\cup$ J = ϕ, either x $\in$ I - J or x $\in$ J - I for all x $\in$ U. If x $\in$ I - J,then K (x) = G (x) $\triangleleft_T$ F(x) since (G, I) $\tilde{\triangleleft}_T$ (F, A). If x $\in$ J - I, then K (x) = H (x) $\triangleleft_T$ F(x) since (H, J) $\tilde{\triangleleft}_T$ (F, A).

Thus K(x) $\triangleleft_T$ F(x) for all x $\in$ U, and so (G, I) $\tilde{\cup}$ (H, J) = (K,U) $\tilde{\triangleleft}_T$ (F, A).

If I and J are not disjoint in Theorem 4.11, then Theorem 4.11 is not true.

V. T-IDEALISTIC SOFT BCI-ALGEBRAS

Definition 5.1**(Jun and Park[5]):** Let (F, A) be a soft set over X. Then (F, A) is called an idealisticsoft BCI-algebra over X if F(x) is an ideal of X for all x $\in$ A.

Definition 5.2:Let (F, A) be a soft set over X. Then (F, A) is called a T-idealisticsoft BCI-algebra over X if F(x) is a T-ideal of X for all x $\in$ A.

Theorem 5.3**:**Let (F, A) and (G, B) be two T-idealistic soft BCI-algebras over X. If A $\cap$B $\neq$Ø, then the intersection (F, A) $\tilde{\cap}$ (G, B) is a T-idealistic soft BCI-algebra over X.

Proof. Using Definition 3.2, we can write (F, A) $\tilde{\cap}$ (G, B) = (H, C), where C =A $\cap$ B and H (x) = F(x) or G (x) for all x $\in$ C. Note that H: C $\rightarrow$ P(X) is a mapping, and therefore (H, C) is a soft set over X. Since (F, A) and (G, B) are T-idealistic soft BCI-algebras over X, it follows that H (x) = F(x) is a T-ideal of X, or H (x) = G (x) is a T-ideal of X for all x $\in$ C.

Hence (H, C) = (F, A) $\tilde{\cap}$ (G, B) is a T-idealistic soft BCI-algebra over X.

Theorem 5.4**:**Let (F, A) and (G, B) be two T-idealistic soft BCI-algebras over X. If A and B are disjoint, then the union (F, A) $\tilde{\cup}$ (G, B) is a T-idealistic soft BCI-algebra over X.

Proof. Using Definition 3.3, we can write (F, A) $\tilde{\cup}$ (G, B) = (H, C), where C =A $\cup$ B and for every x $\in$ C,

$$H(x) = \begin{cases} F(x), & if\ x \in A - B \\ G(x), & if\ x \in B - A \\ F(x) \cup G(x), & if\ x \in A \cap B \end{cases}$$

Since A $\cap$ B = ϕ, either x $\in$ A - B or x $\in$ B - A for all x $\in$ C. If x $\in$ A - B, then H (x) = F(x) is a T-ideal of X since (F, A) is a T-idealistic soft BCI-algebra over X. If x $\in$ B - A, then H (x) = G(x) is a T-ideal of X since (G, B) is a T-idealistic soft BCI-algebra over X. Hence (H, C) = (F, A) $\tilde{\cup}$ (G, B) is a T-idealistic soft BCI-algebra over X.

Corollary 5.5**:**Let (F, A) and (G, A) be two T-idealistic soft BCI-algebras over X. Then their intersection (F, A) $\triangleleft_q$ (G, A) is a T-idealistic soft BCI-algebra over X:

Proof. Straightforward.

***Theorem 5.6*:** If (F, A) and (G, B) are T-idealistic soft BCI-algebras over X,then(F, A) $\tilde{\wedge}$ (G,B) is a T-idealistic soft BCI-algebra over X.

Proof. By means of Definition 3.4, we know that (F, A) $\tilde{\wedge}$ (G, B) = (H, A × B),where H (x, y) = F(x) ∩ G (y) for all (x, y) ∈ A × B. Since F(x) and G (y) areT-ideals of X, the intersection F(x)∩ G (y) is also a T-ideal of X. Hence H (x, y) is a T-ideal of X for all (x, y) ∈ A× B, and therefore (F, A) $\tilde{\wedge}$ (G, B) = (H, A × B)is a T-idealistic soft BCI-algebra over X.

***Example 5.7*:**Let X = {0, 1, 2, a, b} be a BCI-algebra with the following Cayley table:

*	0	1	2	a	b
0	0	0	0	a	a
1	1	0	1	b	a
2	2	2	0	a	a
a	a	a	a	0	0
b	b	a	b	1	0

Let (F, A) be a soft set over X. where A = {0, a} and F: A → P(X) isa set-valued function defined by

F(x) = {y ∈ X | x * y = x}

for all x ∈ A. Then F(0) = F(1) = {0, 1, 2} $\lhd_T$ X, and so (F, A) is a T-idealisticsoft BCI-algebra over X.

Obviously, every T-idealistic soft BCI-algebra over X is an idealistic soft BCI-algebra over X,but the converse is not true in general as seen in the followingexample.

***Example 5.8*:**Let X = {0, a, b, c, d, e, f,g} and consider the following Cayleytable:

*	0	a	b	c	d	e	f	g
0	0	0	0	0	d	d	d	d
a	a	0	0	0	e	d	d	d
b	b	b	0	0	f	f	d	d
c	c	b	a	0	g	f	e	d
d	d	d	d	d	0	0	0	0
e	e	d	d	d	a	0	0	0
f	f	f	d	d	b	b	0	0
g	g	f	e	d	c	b	a	0

Then (X; *, 0) is a BCI-algebra (see [1]). Let (F, A) be a soft set over X, where A = {a, b, c}⊂ X and F: A → P(X) is a set-valued function defined as follows:

F(x) = {y ∈ X | o(x) = o(y)}

for all x ∈ A. Then F(a) = F(b) = F(c) = {0, a, b, c} $\lhd_T$ X. But F(d) = {d, e, f, g} is a T-ideal of X. Hence (F, A) is a T-idealistic soft BCI-algebra over X.
Now, if we take B = {a, b, f, g}⊆ Xand defined a set-valued function by

$$G(x)=\{0\}\cup\{y\in X|o(x)=o(y)\}$$

for all $x\in B$, then (G, B) is not an T-idealistic soft BCI-algebra over X, since $G(f)=\{0,d,e,f,g\}$ is not an T-ideal of X because $(g*f)*d=$ a*d=$e\in G(f)$ and $e \in G(f)$ but $g*d=c\notin G(f)$.

***Definition 5.9*:(Khalid and Ahmad[13]):**A fuzzy set μ in X is a fuzzy T-ideal of X (see [6]) if it satisfiesthe following assertions:

(i) $(\forall x\in X)\ (\mu(0) \geq \mu(x))$,
(ii) $(\forall x, y, z \in X)\ (\mu(x * z) \geq \min\{\mu(x * (y * z)), \mu(y)\})$.

***Lemma 5.10*:(Khalid and Ahmad[13]):**A fuzzy set μin X is a fuzzy T-ideal of X if and only if it satisfies:

$$(\forall t\in [0, 1])(U(\mu; t) \neq \emptyset \Rightarrow U(\mu; t) \text{ is a T-ideal of X}).$$

***Theorem 5.11*:** For every fuzzy T-idealμ of X, there exists a T-idealistic soft BCI-algebra (F, A) over X.

Proof. Let μ be a fuzzy T-ideal of X: Then $U(\mu; t) := \{x \in X \mid \mu(x) \geq t\}$ is a T-ideal of X, for all$t\in Im(\mu)$. If we take $A = Im(\mu)$ and consider a set-valued function $F : A \to P(X)$ given by$F(t) = U(\mu; t)$ for all $t \in A$, then (F, A) is a T-idealistic soft BCI-algebra over X.

Conversely, the following theorem is straightforward.

***Theorem 5.12*:**Let μ be a fuzzy set in X and let (F, A) be a soft set over X in which A = [0, 1] and $F: A \to P(X)$ is given by (5.1). Then the following assertions are equivalent:

1. μ is a fuzzy T-ideal of X,
2. for every $t \in A$ with $F(t) \neq \emptyset$, F(t) is a T-ideal of X.

Proof. Assume that $\mu$is a fuzzy T-ideal of X. Let $t \in A$ be such that $F(t) \neq \emptyset$. If we select $x\in F(t)$, then $\mu(0) + t \geq \mu(x) + t > 1$, and so $0\in F(t)$. Let $t \in A$ and $x, y, z \in X$ be such that $y \in F(t)$ and $(x * y)* z \in F(t)$. Then $\mu(y) + t > 1$ and$\mu((x* y)* z) + t>1$. Since μ is a fuzzy T-ideal of X, it follows that

$$\mu(x * z) + t \geq \min \{\mu((x * y)* z), \mu(y)\} + t$$

$$= \min \{\mu((x * y)* z) + t, \mu(y) + t\}$$

$$>1,$$

so that $x * z \in F(t)$. Hence F(t) is a T-ideal of X for all $t \in A$ with $F(t) \neq \emptyset$.

Conversely, suppose that the second assertion is valid. If there exists $a\in X$ such that $\mu(0)<\mu(a)$, then we can select $t_a\in A$ such that $\mu(0)+t_a\leq 1 <\mu(a)+t_a$. Itfollows that $a \in F(t_a)$ and $0 \notin F(t_a)$, a contradiction. Hence $\mu(0) \geq \mu(x)$ for all$x\in X$. Now, assume that

$$\mu(a* c) < \min \{\mu((a * b)* c), \mu(b)\}$$

for some $a, b, c \in X$. Then

$$\mu(a * c) + s_0 \leq 1 < \min \{\mu((a * b)* c), \mu(b)\} + s_0,$$

for some $s_0\in A$,which implies that $(a * b)* c\in F(s_0)$ and $b \in F(s_0)$, but $a * c \notin F(s_0)$. This is a contradiction. Therefore

$$\mu(x * z) \geq \min \{\mu((x * y)* z), \mu(y)\}$$

for all $x, y, z \in X$, and thus μ is a fuzzy T-ideal of X.

***Theorem 5.13*:**Let μ be a fuzzy set in X and let (F, A) be a soft set over X in which A = (0.5, 1] and $F: A \to P(X)$ is defined by $(\forall t\in A)\ (F(t) = U(\mu; t))$.Then F(t) is a T-ideal of X, for all $t \in A$ with $F(t) \neq 0$, if and only if the following assertions are valid:

1. $(\forall x\in X)\ (\max \{\mu(0), 0.5\} \geq \mu(x))$,

2. $(\forall x, y, z \in X)$ (max $\{\mu(x * z), 0.5\} \geq$ min $\{\mu((x * y) * z), \mu(y)\}$).

Proof. Assume that F(t) is a T-ideal of X for all $t \in A$ with $F(t) \neq \emptyset$. If there exists $x_0 \in X$ such that $\max\{\mu(0), 0.5\} < \mu(x_0)$, then we can select $t_0 \in A$ such thatmax $\{\mu(0), 0.5\} < t_0 < \mu(x_0)$. It follows that $\mu(0) < t_0$ so that $x_0 \in F(t_0)$ and $0 \notin F(t_0)$. This is a contradiction, and so the first assertion is valid. Suppose that there exist a, b, c $\in$ X such that

$$\max\{\mu(a * c), 0.5\} < \min\{\mu((a * b) * c), \mu(b)\}.$$

Then

$$\max\{\mu(a * c), 0.5\} < u_0 \leq \min\{\mu((a * b) * c), \mu(b)\}$$

for some $u_0 \in A$. Thus $(a * b) * c \in F(u_0)$ and $b \in F(u_0)$, but $a * c \notin F(u_0)$.
This is a contradiction, and so the second assertion is valid.
Conversely, suppose that conditions (1) and (2) are valid. Let $t \in A$ with $F(t) \neq \emptyset$. For anyx $\in F(t)$, we have

$$\max\{\mu(0), 0.5\} \geq \mu(x) \geq t > 0.5$$

and so $\mu(0) \geq t$, i.e., $0 \in F(t)$. Let x, y, z $\in$ X be such that $y \in F(t)$ and $x * (y * z) \in F(t)$. Then $\mu(y) \geq t$ and$\mu((x * y) * z) \geq t$. It follows from the second condition that

$$\max\{\mu(x * z), 0.5\} \geq \min\{\mu((x * y) * z), \mu(y)\} \geq t > 0.5$$

so that $\mu(x * z) \geq t$, i.e., $x * z \in F(t)$. Therefore F(t) is a T-ideal of X for all $t \in A$ with $F(t) \neq \emptyset$.

VI. CONCLUSION

The concept of soft set, which is introduced by Molodtsov[10], is a new mathematical tool for dealing with uncertainties that is free from the difficulties that have troubled the usual theoretical approaches. Soft sets are deeply related to fuzzy sets and rough sets. We introduced the notion of soft T-ideals and T-idealistic soft BCI-algebras and discussed related properties. We established the intersection, union, "AND" operation and "OR" operation of soft T-ideals and T-idealistic soft BCI-algebras. From above discussion it can be observed that fuzzy T-ideals can be characterized using the concept of soft sets. For a soft set (F, A) over X, a fuzzy set μ in X is a fuzzy T-ideal of Xif and only if for every $t \in A$ with $F(t) = \{x \in X | \mu(x) + t > 1\} \neq \emptyset$, $F(t)$ is an T-ideal of X. Finally we have discussed the relations between fuzzy T-ideals and T-idealistic soft BCI-algebras.

ACKNWLEDEMENT

The authors are highly graceful to the references for their valuable comments and suggestions for improving the paper.

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