A New Approach of Multiplication with Verification

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Abstract
In this paper we prove with verification the result of multiplication of multi-digit numbers by a new method. The method is mainly based on two steps, namely step 1 and step 2, after adding these steps we find the result or product. This new method may be used in general and may be named as two step method of Multiplication.

Key words - *Two step method, prove with verification, direct vertical multiplication, cross multiplication, using block and omitting block.*

I. INTRODUCTION

The new method is a formal method of multiplication. This method is the shortest and easiest method comparing with other formal methods of multiplication. The important characteristic of this method is that we can easily verify the result (subtracting step 2) and find multiplicand and multiplier reversing the path.

The method is neither a Vedic or a Trachtenberg‟s method nor the lattice method of multiplication. This method is somewhat similar only in respect of the name of the application of technique i.e. vertical and crosswise as used in Vedic or Jakow Trachtenberg method of rapid arithmetic. New method maintain the similar rule as we apply in the lattice method for step 1 and follow an independent cross-multiplication technique for step 2.

II. PROCEDURE

Here, we discuss how the new technique applied in the method. We write the numbers one above the other, aligning place value as in the usual method of multi-digit multiplication. We put „zero‟ or „zeros‟ to equalize the vertical digits if necessary and follow the given instructions for-

Step 1: We multiply vertical digits directly one by other and write the results using two digit numbers i.e. $0 \times 0 = 00$ or $1 \times 2 = 02$. If we put a "0" (zero) to equalize digits it results only one "0" zero i.e. $3 \times 0 = 0$, instead of $3 \times 0 = 00$, because putting zero has no actual value.

Step 2: Firstly put "0" (zero) on ones place below step 1 and write down only the results of ones place after by adding various partial product of cross multiplication from tens place to the left. We keep the other digit or digits as carry number and add the carry number with the next partial product and thus in the same away as long as necessary, after final calculation, we put down all the digits in their respective place.

III. EXPLAINING WITH EXAMPLES

Now we discuss the new method in the context of several examples with verification.

Example 1-Multiply two digits by one digit Suppose, we wish to multiply 27 by 4, we write it out in this form

27
×04 put a zero to equalize digits
028 step 1: direct vertical multiplication 2 x 0 = 0 and 7 x 4 = 28
+ 080 step 2: cross multiplication 2 x 4 + 0 x 7 = 08 + 00 = 08, zero putted in ones place
108 result

Verification of the result can be done by subtracting the step 2 from the final result:

given: $27 \times 4 = 108$
Step 2: $080 = 080$
Step 1: $= 028$

Pairing from the right we have 0 and 28 for 028. Factorizing as per question (vertical digits)

$0 = 2 \times 0$ (putting "0", zero has no value)
$28 = 7 \times 4$

Setting in order we get: $108 = 27 \times 04$.

For this example 2 digits × 1 digit = 3 digits Note that final result is a 3 digit number because it is a product of two and one digit.

Example 2-Multiply two digits by two digits Suppose we multiply 77 by 77. We write it out in this form and do it

```
  77
x 77
4949   step 1: direct vertical multiplication
+ 980  step 2: cross multiplication, '0' putted on ones place
5929   result
```

Step 1: Direct vertical multiplications give $7 \times 7 = 49$ and $7 \times 7 = 49$.

Step 2: Cross multiplication and putting "0" on ones place

tens place: $7 \times 7 + 7 \times 7 = 49 + 49 = 98, (980)$
∴ step 2 = 980

Verification of the result can be done by subtracting the step 2 from the final result.

Given, $77 \times 77 = 5929$
Step 2 $= 980$
Step 1 $= 4949$

Pairing from the right we have 49 and 49 for 4949. Factorizing as per question (vertical digits)

$49 = 7 \times 7$
$49 = 7 \times 7$

Setting in order, we get: $5929 = 77 \times 77$. For this example, two digits × two digits = Four digits

Example 3-Multiply three digits by three digits Suppose we multiply 546 by 378. We write it out in this form and do it

```
  546
x 378
152848  step 1:  direct vertical multiplication
+ 53540 step 2:  cross multiplication
206388  result
```

Step 1: Direct vertical multiplications give $5 \times 3 = 15$, $4 \times 7 = 28$ and $6 \times 8 = 48$.

Step 2: Cross multiplication and putting "0" on ones place

Tens place: 6 x 7 + 8 x 4 = 42 + 32 = 74. Write 4 carry 7 (40)
Hundred place: 6 x 3 + 8 x 5 + 7 = 18 + 40 + 7 = 65, write 5 carry 6 (540)
Thousand place: 4 x 3 + 7 x 5 + 6 = 12 + 35 + 6 = 53, (53540)

∴ Step 2 = 53540

Verification of the result can be done by subtracting the step 2 from the final result.

Given, 546 × 378 = 206388
Step 2 = 53540
Step 1 = 152848

Pairing from the right we have 15, 28 and 48 for 152848. Factorizing as per question (vertical digits)

∴ = 5 × 3
28 = 4 × 7
48 = 6 × 8

Setting in order, we get: 206388 = 546 × 378. For this example, 3 digits × 3 digits = 6 digits

Example 4-Multiply four digits by four digits Suppose we multiply 2324 by 3212. We write it out in this form and do it

```
  2324
 x 3212
 06060208  step 1:  direct vertical multiplications
+1404480   step 2:  cross multiplications
 07464688  result
```

Step 1: Direct vertical multiplication give 2 x 3 = 06, 3 x 2 = 06, 2 x 1 = 01 and 4 x 2 = 08.

Step 2: Cross multiplication and putting "0" on ones places

Tens places: 4 x 1 + 2 x 2 = 04 + 04 = 08, write 8 carry 0 (80)
Hundreds place: 4 x 2 + 2 x 3 + 0 = 08 + 06 + 0 = 14, write 4 carry 1 (480)
Thousands place: 4 x 3 + 2 x 2 + 2 + 1 x 3 + 1 = 12 + 04 + 04 + 03 + 01 = 24, write 4 carry 2 (4480)

10000s place: 2 × 3 + 1 × 2 + 2 = 06 + 02 + 2 = 10, write 0 carry 1 (04480)
100000s place: 3 × 3 + 2 × 2 + 1 = 09 + 04 + 1 = 14, (1404480)
Step 2 = 1404480

Verification of the result can be done by subtracting the step 2 from the final result.

Given, 2324 × 3212 = 07464688
Step 2 = 1404480
Step 1 = 06060208

Pairing from the right we have 06, 06, 02 and 08 for 06060208. Factorizing as per question (vertical digits)

$$6 = 2 \times 3$$
$$6 = 3 \times 2$$
$$2 = 2 \times 1$$
$$8 = 4 \times 2$$

Setting in order, we get: 07464688 = 2324 × 3212. For this example, 4 digits × 4 digits = 8 digits

Example 5-Multiply ftve digits by ftve digits Suppose we multiply 34205 by 23617. We write it out in this form and do it

```
   34205
 x 23619
0612120045  step 1:   direct vertical multiplications
+ 195767850 step 2: cross multiplications
 0807887895  result
```

Step 1: Direct vertical multiplications give 3 × 2 = 06, 4 × 3 = 12,2 × 6 = 12, 0 × 1 = 00 and 5 × 9 = 45.

Step 2: Cross multiplication and putting "0" on ones place

Tens place: 5 × 1 + 9 × 0 = 05 + 00 = 05, write 5 carry 0 (50)

Hundreds place: 5 × 6 + 9 × 2 + 0 = 30 + 18 + 0 = 48, write 8 carry 4 (850)

Thousands place: 5 × 3 + 9 × 4 + 0 × 6 + 1 × 2 + 4 = 15 + 36 + 00 + 02 + 4 = 57, write 7 carry 5 (7850)

10000s place: 5 × 2 + 9 × 3 + 0 × 3 + 1 × 4 + 5 = 10 + 27 + 00 + 04 + 5 = 46 , write 6 carry 4 (67850)

100000s place: 0 × 2 + 1 × 3 + 2 × 3 + 6 × 4 + 4 = 00 + 03 + 06 + 12 + 4 = 37, write 7 carry 3 (767850)

1000000s place: 2 × 2 + 6 × 3 + 3 = 04 + 18 + 3 = 25, write 5 carry 2 (5767850)

10000000s place: 4 × 2 + 3 × 3 + 2 = 08 + 09 + 2 = 19, (195767850)

∴ Step 2 = 195767850

For this example, 5 digits × 5 digits = 10 digits

Example 6-Multiply six digits by six digits Suppose we multiply 534672 by 482935. We write it out in this form and do it

```
   534672
 x 482935
202108542110  step 1:   direct vertical multiplications
+ 55803280210 step 2:   cross multiplications
 258211822320  result
```

Step 1: Direct vertical multiplications give 5 × 4 = 20, 3 × 8 = 24, 4 × 2 = 08, 6 × 9 = 54, 7 × 3 = 21 and 2 × 5 = 10.

Step 2: Cross multiplication and putting "0" on ones place

Tens place: $2 \times 3 + 5 \times 7 = 06 + 35 = 41$, write 1 carry 4 (10)

Hundreds place: $2 \times 9 + 5 \times 6 + 4 = 18 + 30 + 4 = 52$, write 2 carry 5 (210)

Thousands place: $2 \times 2 + 5 \times 4 + 7 \times 9 + 3 \times 6 + 5 = 04 + 20 + 63 + 18 + 5 = 110$
,write 0 carry 11 (0210)

10000s place: 2 x 8 + 5 x 3 + 7 x 2 + 2 + 3 x 4 + 11 = 16 + 15 + 14 + 12 + 11 = 110
write 8 carry 6 (80210)

100000s place: $2 \times 4 + 5 \times 5 + 7 \times 8 + 3 \times 3 + 6 \times 2 + 9 \times 4 + 6 = 152$,
write 2 carry 15 (280210)

1000000s place: $7 \times 4 + 3 \times 5 + 6 \times 8 + 9 \times 3 + 15 = 133$, write 3, carry 13 (3280210)

10000000s place: $6 \times 4 + 9 \times 5 + 4 \times 8 + 2 \times 3 + 13 = 24 + 45 + 32 + 06 + 13 = 120$,
write 0, carry 12 (03280210)

100000000s place: $4 \times 4 + 2 \times 5 + 12 = 38$, write 8, carry 3 (803280210)

1000000000s place: $3 \times 4 + 8 \times 5 + 03 = 12 + 40 + 03 = 55$, (55803280210)

∴ Step 2 = 55803280210

For this example, 6 digits × 6 digits = 12 digits

Example 7-Multiply seven digits by seven digits Suppose we multiply 3456789 by 5234210. We write it out in this form and do it

```
   3456789
 × 5234210
15081524140800 step 1:  direct vertical multiplications
+ 3012035410830 step 2:  cross multiplications
 18093559551690 result
```

Step 1: Direct vertical multiplications give $3 \times 5 = 15$, $2 \times 4 = 08$, $5 \times 3 = 15$, $6 \times 4 = 24$, $7 \times 2 = 14$, $8 \times 1 = 08$ and $9 \times 0 = 00$.

Step 2: Cross multiplication and putting "0" on ones place

Tens place: $1 \times 9 + 8 \times 0 = 09$, write 9 carry 0 (90)

Hundreds place: $9 \times 2 + 0 \times 7 + 0 = 18$, write 8 carry 1 (890)

Thousands place: $9 \times 4 + 0 \times 6 + 8 \times 2 + 1 \times 7 + 01 = 60$, write 0 carry 6 (0890)

10000s place: $9 \times 3 + 0 \times 5 + 8 \times 4 + 1 \times 6 + 6 = 71$, write 1 carry 7 (10890)

100000s place: $9 \times 2 + 0 \times 4 + 8 \times 3 + 1 \times 5 + 6 \times 2 + 4 \times 7 + 7 = 94$,
write 4 carry 9 (410890)

1000000s place: $9 \times 5 + 0 \times 3 + 8 \times 2 + 1 \times 4 + 7 \times 3 + 2 \times 5 + 9 = 105$,
write 5, carry 10 (5410890)

10000000s place: $3 \times 1 + 5 \times 8 + 4 \times 2 + 2 \times 7 + 5 \times 4 + 3 \times 6 + 10 = 113$,
write 3, carry 11 (35410890)

100000000s place: $3 \times 2 + 5 \times 7 + 4 \times 4 + 2 \times 6 + 11 = 0$,
write 0, carry 8 (035410890)

1000000000s place: $3 \times 4 + 5 \times 6 + 4 \times 3 + 2 \times 5 + 8 = 72$, write 2, carry 7(2035410890)

10000000000s place: $3 \times 3 + 5 \times 5 + 7 = 41$, write 1, carry 4 (12035410890)

100000000000s place: $3 \times 2 + 5 \times 4 + 4 = 30$, (3012035410890)

∴ Step 2 = 3012035410890

For this example, 7 digits × 7 digits = 14 digits

IV. CONCLUSION

From the above examples it is clear that when we use 1 digit ×1 digit = 2 digit without neglecting "0" before the result or product in the case of multiplication then we find the sum of the digits of the multiplicand and multiplier are equal to the total digits of the product. If we use a_1, b_1, c_1, d_1, e_1 etc are unknown digits of multiplicand and a_2, b_2, c_2, d_2, e_2 etc are unknown digits of multiplier then we find the following formula in this form where we find place value of step 1 are in 2 digits using blocks. We must keep the place value of step 2 are in 1 digit in each block forwarding other digit or digits as carry number from the block of tens place to the second left most block respectively and before final calculation we omit the block symbols. We then add these two steps to find final product or result.

1. $(a_1b_1)\,(a_2b_2) = a_1.a_2|b_1.b_2 + (a_1.b_2 + a_2.b_1)\,|0$

2. $(a_1b_1c_1)\,(a_2b_2c_2) = a_1.a_2|b_1.b_2|c_1.c_2 + (a_1.b_2 + a_2.b_1)\,|\,(b_1.c_2 + b_2.c_1)\,|0$

3. $(a_1b_1c_1d_1)\,(a_2b_2c_2d_2) = a_1.a_2|b_1.b_2|c_1.c_2|d_1.d_2 + (a_1.b_2 + a_2.b_1)\;|(a_1.c_2 + a_2.c_1)\,|\,(a_1.d_2 + a_2.d_1) + (b_1.c_2 + b_2.c_1)\,|\,(b_1.d_2 + b_2.d_1)\,|\;(c_1.d_2 + c_2.d_1)\,|0$

4. $(a_1b_1c_1d_1e_1)\,(a_2b_2c_2d_2e_2) = a_1.a_2|b_1.b_2|c_1.c_2|d_1.d_2|e_1.e_2 + (a_1.b_2 + a_2.b_1)\,|\,(a_1.c_2 + a_2.c_1)\,|\,(a_1.d_2 + a_2.d_1) + (b_1.c_2 + b_2.c_1)\,|\,(a_1.e_2 + a_2.e_1) + (b_1.d_2 + b_2.d_1)\,|\,(b_1.e_2 + b_2.e_1) + (b_1.c_2 + b_2.c_1)\,|\,(c_1.e_2 + c_2.e_1)\,|\,(d_1.e_2 + d_2.e_1)\,|0$

and so on.

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