Modeling the Cases of Road Traffic Crashes: A Case of Exponential Smoothing Approach

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Abstract

This study was carried out using the monthly (January 2007 to December 2017 with a total of 108 data points) reported road traffic crashes along the roads connecting the main city of Enugu State Nigeria. In this work, we gave a general overview of the three basic aspects of exponential smoothing models proposed with their applications on the cases of road traffic crashes (accident). The relationship between autoregressive integrated moving average models and exponential smoothing models was also given. Using a grid search method for the estimation of smoothing parameters, the best exponential smoothing models for each of the cases was identified based on the Mean square error (MSE), Mean absolute error (MAE) and Mean absolute percentage error (MAPE). The Holt-Winter (Triple) exponential smoothing model expounded by [1] yielded optimum value in each of the measures for each route since the series exhibit both trend and seasonality.

Keywords - *Road traffic crashes, Exponential smoothing, ARIMA models*

I. INTRODUCTION

Transportation plays a vital role in social and economic development of a nation, particularly in facilitating movement of people, goods and services from one point to another. In other words, the viability of an economy to an extent partially depends on the ease of moving people and goods from place to place. Even though modern means of transportation spurs the development pace of an economy, it has negative effects on the social life of the people. [2] succintly posited that although transportation has liberated man and makes him more mobile, his increasing reliance on vehicular movement has conferred great fatalities on him and his activities. These great fatalities as pointed out above are the results of traffic crashes along the roads.

According to [3], the causes of road traffic crashes are multi-factorial and involve the interaction of a number of pre-crash factors. These factors can be divided broadly into driver factors, vehicle factors and road way factors. Accidents can be caused by a combination of these factors. Driver factors include driver’s behaviour, visibility, decision making ability and sensitivity to speed. Drug and alcohol use while driving is an obvious predictor of road traffic crash. Human error (driver factor) contributes 64% to 95% traffic crashes in developing countries [4]. Road accidents statistics in Nigeria reveal a serious and growing problem with absolute fatality rate and casualty figure rising rapidly. To curtail the rate of accident occurence on the roads, viable programs have to be introduced by the federal road safety commission and other personels whose operations are traffic centered. To come up with such a viable program, the knowledge of the future occurence of road traffic crashes is essential. Thus, we adopt exponential smoothing model to study the accident phenomenon and come up with a reliable forecast into the future of accident cases along the major roads connecting to the city of Enugu state, Nigeria.

Exponential smoothing is a forecasting technique for smoothing time series data. This forecast technique systematically assigns weights to observations. By the procedure, the forecasts are continually revised in the light of more recent signals. One of the basic ideas of smoothing models is to construct forecasts of future values as weighted averages of past observations with the more recent observations carrying more weight in determining forecasts than observations in the more distant past. By forming forecasts based on weighted averages we are using a “smoothing” method [5]. The exponential smoothing (single exponential smoothing) procedure has a similar concept as moving averages. Whereas in moving averages the past observations are weighted equally, exponential smoothing assigns exponentially (rapidly) decreasing weights as the observation get older. That is, recent observations are assigned relatively more weight than the older observations in forecasting. In the case of moving averages, the weights assigned to the observations are the same and are equal to N^{-1}. However, in exponential smoothing, there are one or more parameters (smoothing parameters) to be determined (or estimated) and these parameters contribute in determining the weights to be assigned to the observations. Given a set of time dependent observations, $z_t; t = 1,2, \dots T$, the generation process of the series can generally be represented using an additive exponential smoothing model given as follows:

$$z_t = \mu_t + \beta_t + s_{t,m} + \varepsilon_t \quad (1)$$

Where μ_t is the time dependent mean (level) term, $\beta_t$is the time dependent slope also known as the trend term, $s_{t\,m}$ is the time dependent seasonal term for period m $(m = 1, 2, \ldots, M)$ and ε_t is the error term. Assuming that the series have no trend $(\beta_t = 0)$ and no seasonal term $(S_{t,m} = 0)$, the model reduces to single exponential smoothing. As the series are being smoothed at each time$t$, the parameters $(\mu_t, \beta_t, S_{t,m})$ are estimated. In other words, the estimates of these parameters vary based on time. Because the process is recursive,

it is necessary to determine the initial estimates first.

II. SINGLE EXPONENTIAL SMOOTHING

Single exponential Smoothing does not fare well in a series of data which has trend. This is so because it assumes that data fluctuates around a reasonable stable mean. By the assumption of the model, the series have only level and random component thus, the general model equation reduces to

$$z_t = \mu_t + \varepsilon_t \quad (2)$$

It is used for short range forecasting and this situation exposes the shortfalls of single exponential smoothing.

III. DERIVATION OF THE SINGLE EXPONENTIAL SMOOTHING FORMULA

Let the smoothing parameter for single exponential smoothing be denoted as α. Also, let the smoothed level that estimates μ_t in (2) be denoted as L_t. The weighted expression for L_t is

$$L_t = \alpha z_t + \alpha(1-\alpha)z_{t-1} + \alpha(1-\alpha)^2 z_{t-2} + \alpha(1-\alpha)^3 z_{t-3} + \alpha(1-\alpha)^4 z_{t-4} \quad (3)$$

It follows that,

$$L_{t-1} = \alpha z_{t-1} + \alpha(1-\alpha)z_{t-2} + \alpha(1-\alpha)^2 z_{t-3} + \alpha(1-\alpha)^3 z_{t-4} \quad (4)$$

Multiplying (4) by $(1-\alpha)$, we have

$$L_{t-1}(1-\alpha) = \alpha(1-\alpha)z_{t-1} + \alpha(1-\alpha)(1-\alpha)z_{t-2} + \alpha(1-\alpha)(1-\alpha)^2 z_{t-3} + \alpha(1-\alpha)(1-\alpha)^3 z_{t-4} \quad (5)$$

Subtracting equation (5) from equation (3), implies that

$$L_t = \alpha z_t + (1-\alpha)L_{t-1} \quad (6)$$

For single (simple) exponential smoothing, (6) is used to smoothen the series and it is called the smoothing equation for (SES). The h-step ahead prediction equation therefore becomes $\hat{z}_{t+h} = L_{t-1+h};\ h = 1, 2, 3, \ldots.$

This forecast model is restricted to short term forecasting, mostly one step ahead. With the use of the simple exponential smoothing procedure, each smoothed observation is expressed as a weighted average of present observation and previous smoothed value. The smoothing parameter α can take any value between 0 and 1. If $\alpha = 0$, the new smoothed value (forecast) is expressed as the immediate past smoothed value (L_{t-1}). This implies that all the smoothed values will be constant and equal to the initial starting value. However, if $\alpha = 1$, the previous smoothed value is ignored and the new smoothed value becomes equivalent to the present (current) observation. Since when$\alpha = 1$,$L_t = Z_t$, it is assumed that smoothing is silent since there is no parameter to weigh the observations. Values of the smoothing parameter (α) close to one have less of a smoothing effect and gives heavier weight to recent changes in the data, while values of α closer to zero have a greater smoothing effect and are less responsive to recent changes. Generally, the large values of α actually reduces the level of smoothing. The single exponential smoothing model has an equivalent Box-Jenkins ARIMA model given as ARIMA (0, 1, 1). That is$(1-B)z_t = (1-\theta B)\varepsilon_t$, where z_t is the series, B is a backward shift operator, $\theta(\theta = 1-\alpha)$ is the parameter associated with the moving average part of the model, and ε_t is the white noise. This relationship will be established later in this work.

IV. DOUBLE EXPONENTIAL SMOOTHING

This procedure is specifically used for series assumed to have level, trend and noise but no seasonality. The general model equation is given as

$$z_t = \mu_t + \beta_t + \varepsilon_t \quad (7)$$

For the fact that it has two basic components: level and trend, the smoothing equation is split into two: one for the level and the other for the trend. Let L_t and T_t be the smoothed values that estimates the level and the trend respectively. Therefore,

$L_t = \alpha z_t + (1-\alpha)L_{t-1}$ (8)

$T_t = \tau(L_t - L_{t-1}) + (1-\tau)T_{t-1}$ (9)

Where α and τ (each lying between 0 and 1) are the smoothing parameters for the level and the trend respectively. After smoothing the level at time t and the trend at time t with the two smoothing equations (8 & 9) above, respectively, the h-step ahead forecast can then be made using a prediction equation

$\hat{z}_{t+h} = L_{t-1+h} + \left(\frac{\tau}{1-\alpha}\right)BL_{t+h} + \left(\frac{1-\tau}{1-\alpha}\right)T_{t-1+h};\ h = 1,2,3\ ...$ (10)

B is a backward shift operator such that $BL_t = L_t - L_{t-1}$ (11)

Also, the double exponential smoothing model has ARIMA (0, 2, 2) model equivalence. This relationship will also be established later in this work.

V. HOLT-WINTER (TRIPLE) EXPONENTIAL SMOOTHING

This is third order exponential smoothing which is the recursive application of an exponential filters three times. The smoothing method is used when the series shows level, trend and seasonality. To handle seasonality, we have to add a third parameter γ and introduce a third equation to take care of seasonality with seasonal length, m=12 (since we are considering monthly series). The general model equation is given as

$Z_t = \mu_t + \beta_t + S_{t,m} + \varepsilon_t.$ (12)

Let m be the length of the season. The seasonal indices are defined such that they sum to zero. That is $\sum_{t=1}^{m} S_t = 0$.

The model has three smoothing equations which are defined as follows;

$L_t = \alpha(z_t - S_{t-m}) + (1-\alpha)(L_{t-1} + T_{t-1})$ (13)

$T_t = \tau(L_t - L_{t-1}) + (1-\tau)T_{t-1}$ (14)

$S_t = \gamma(z_t - L_t) + (1-\gamma)S_{t-m}$ (15)

The h-step prediction equation is $\hat{z}_{t+h} = \left(\frac{\tau-\gamma}{1-\alpha-\gamma}\right)L_{t+h} + \left(\frac{1-\alpha-\tau}{1-\alpha-\gamma}\right)L_{t-1+h} + \left(\frac{2-\alpha-\tau}{1-\alpha-\gamma}\right)T_{t-1+h} - \left(\frac{\alpha+\gamma+1}{1-\alpha-\gamma}\right)S_{t-m+h}$, h=1,2,… (16)

that is forecasting z h-steps ahead by using the last available estimated level state and incrementing it by h times using the last available trend while at the same time adding the last available smoothed seasonal factor, S_{t-m+h} that matches the month of the forecast horizon.

VI. DETERMINATION OR CHOOSING OF INITIAL/ STARTING VALUES

In order to implement these methods mentioned above, the user must provide starting values for the level L_t, trend T_t and seasonal indices S_t at the beginning of the series in order to initiate the updating procedure. There are many different ways of choosing these initial values.

For simple exponential smoothing, the initial value can be determined by taking the average of observations in the first year or simply setting L_o=z_1(the first observation).

In double exponential smoothing, we set L_o=z_1 and $T_0 = z_2$-z_1

For Holt-Winter, we set L_o equal to the average observation in the first year. That is

$$L_0 = \frac{1}{m}\sum_{i=1}^{m} z_t$$

where m is the number of seasons in the year. The starting value for the trend

$$T_0 = \frac{1}{m}\sum_{i=1}^{m}(z_{m+i} - z_i)$$

Finally, the seasonal index starting value (S_0) can be calculated as follows

$S_0 = Z_m - l_0,$

$S_{-1} = Z_{m-1} - l_0, \ldots, S_{-m+1} = Z_1 - l_0$

VII. CHOOSING THE BEST VALUE FOR THE SMOOTHING CONSTANT

The accuracy of the forecasting depends on the smoothing constant. The user must also provide values for the three smoothing parameters(α, τ, β). There are two general ways of selecting the parameters. The first is to estimate them by minimizing sum function of the forecast errors of the historical data. The second is simply guestimate (an estimate that combines reasoning with guessing). Selecting a smoothing constant is basically a matter of judgment or trial and error, using forecast errors to guide the decision. The goal is to select a smoothing constant that balances the benefits of smoothing random variations with the benefits of responding to real changes if and when they occur. The smoothing constant serves as the weighting factor. When α is close to 1, the new forecast will include a substantial adjustment for any error that occurred in the preceding forecast. When α is close to 0, the new forecast is very similar to the old forecast. The smoothing constant is not an arbitrary choice. Low values of α gives less weight to recent data while higher values of α permit the more recent data to have a greater influence on the predictions. In practice, the smoothing constant is chosen by a grid search within the parameter space. That is, different solutions for α are tried starting, for example, with α = 0.1 to α = 0.9, with increment of 0.1. The value of α with the smallest MAE, MSE or MAPE is chosen for use in producing the future forecasts.

VIII. DISCUSSIONS ON THE RESULTS OF THE EXPONENTIAL SMOOTHING MODELS

In previous section, we established that there is seasonality in the series of each of the routes. This informs the use of Holt-Winter (Triple) exponential smoothing model which will inculcate the seasonality that is visible in the series. Here, the analysis was run using the non-adaptive technique for additive triple exponential smoothing model. The non-adaptive technique uses the data set to build the model and establish smoothing factors. Since it is non-adaptive, once the optimum smoothing factors α, τ and γ are established, they are not modified again. In this research work, a grid search was adopted to identify the optimum smoothing factors (that is the smoothing factors that gives minimum MAE, MSE, SSE and MAPE). However, we still considered single and double exponential smoothing models assuming that there is no seasonality. The essence of doing this is to know whether the model (single or double) will perform better if seasonal component is ignored. We are applying the models on the cases of road traffic crashes along three routes: Enugu-Abakeliki(ENU-ABK), Nsukka-9thmiles (NSK-9MILE) and Enugu-Onitsha (ENU-ONITSHA), connecting Enugu State, Nigeria. The results are given in the table below:

IX. TABLE 1: SINGLE EXPONENTIAL SMOOTHING MODEL FOR THE ROUTES

ROUTES	SMOOTHING FACTORS	PERFORMANCE MEASURES			
	∝	MAE	MSE	SSE	MAPE
ENU-ABK	0.3	2.9189	16.3905	1770.17	∞
NSK-9MILE	0.4	2.7829	15.3718	1660.16	∞
ENU-ONITSHA	0.2	3.2404	19.3095	2085.43	47.4046

X. TABLE 2: DOUBLE EXPONENTIAL SMOOTHING MODEL FOR THE ROUTES

ROUTES	SMOOTHING FACTORS		PERFORMANCE MEASURES			
	∝	T	MAE	MSE	SSE	MAPE
ENU-ABK	0.3	0.1	2.9531	17.2853	1866.81	∞
NSK-9MILE	0.4	0.1	2.8496	16.4398	1775.50	∞
ENU-ONITSHA	0.2	0.1	3.3121	20.9090	2258.17	45.2986

XI. TABLE 3: HOLT-WINTER (TRIPLE) EXPONENTIAL SMOOTHING MODEL FOR THE ROUTES

ROUTES	SMOOTHING FACTORS			PERFORMANCE MEASURES			
	∝	T	Γ	MAE	MSE	SSE	MAPE
ENU-ABK	0.3	0.1	0.1	2.8879	14.9525	1614.87	∞
NSK-9MILE	0.5	0.1	0.1	2.5364	12.5325	1353.51	∞
ENU-ONITSHA	0.2	0.1	0.1	3.0174	14.8241	1601.00	42.5465

The results of the exponential smoothing models put in the table above shows that the Holt-winter exponential smoothing model forecasts each of the series better as it yields minimum values of the model performance measures considered in the work. As stated earlier, H-W exponential smoothing model acknowledges the presence of seasonal effects in the series. Therefore, it gives better results since we have been able to establish in previous section that the rate of accident occurrence is generally, seasonal.

XII. ESTABLISHMENT OF THE RELATIONSHIPS BETWEEN ARIMA AND EXPONENTIAL SMOOTHING MODELS

Exponential smoothing and ARIMA models are the two most widely used approaches to time series forecasting. They provide complementary approaches to a problem. While exponential smoothing models were based on a description of trend and seasonality in the data, ARIMA models aim to describe the autocorrelations in the data. ARIMA model has been pioneered by [6]. These models are intended for the forecasting of traffic flow data and have since been successfully used.
As stated earlier, the simple/single exponential smoothing is optimal for an ARIMA (0,1,1) model while double exponential smoothing(two parameter model) model is optimal for an ARIMA (0,2,2) model. The 3-parameter Holt-Winters method with additive seasonality is so complicated that it would never be identified in practice [7]. The multiplicative Holt-Winter does not have an ARIMA equivalent at all. These relationships (that is ARIMA equivalence to exponential smoothing) can be proved using their innovation state space model. Each model consists of a measurement equation that describes the observed data and some transition equations that describe how the unobserved components or states (level, trend, seasonal) change over time.

- ***For simple exponential smoothing,***

We have that The ARIMA model equivalence to simple exponential smoothing is ARIMA (0, 1, 1) written as $(1-B)Z_t = (1-\theta B)\varepsilon_t$, where θ =1-α. (17)

Here we try to prove the relationship using the innovation state space model and the transition equation.

$z_t = \mu_{t-1} + \varepsilon_t$ this is the innovation state space model and (1-B)$\mu_t = \alpha\varepsilon_t$ (Transition equation) . Apply the difference operator (1-B) to both sides of innovation state space model

$(1-B)z_t = (1-B)\mu_{t-1} + (1-B)\varepsilon_t$ (18)

$(1-B)z_t = \alpha\varepsilon_{t-1} + \varepsilon_t - \varepsilon_{t-1}$ (19)

$(1-B)z_t = \alpha\varepsilon_{t-1} - \varepsilon_{t-1} + \varepsilon_t = \varepsilon_t - (1-\alpha)\,\varepsilon_{t-1}$ (20)

Let $(1-\alpha) = \theta$

$(1-B)z_t = \varepsilon_t - \theta\varepsilon_{t-1}$ (21)

$(1-B)z_t = (1-\theta B)\varepsilon_t$ (22)

- ***For double exponential smoothing:***

The ARIMA equivalence to double exponential smoothing is the ARIMA (0, 2, 2) model written as

$(1-B)^2Z_t = (1-\theta B)^2\varepsilon_t$, where θ =1-α (23)

Given $z_t = \mu_{t-1} + \beta_{t-1} + \varepsilon_t$ then the innovation state space model for double exponential smoothing is

$(1-B)\mu_t = \beta_{t-1} + \alpha\varepsilon_t$ (24)

$(1-B)\beta_t = \tau\varepsilon_t$ (25)

- To create the relationship, multiply both sides of ISSM by $(1-B)^2$ and apply the transition equations appropriately.

$(1-B)^2 z_t = (1-B)^2\mu_{t-1} + (1-B)^2\beta_{t-1} + (1-B)^2\varepsilon_t$

If $(1-B)\mu_t = \beta_{t-1} + \alpha\varepsilon_t$ then $(1-B)\mu_{t-1} = \beta_{t-2} + \alpha\varepsilon_{t-1}$

$(1-B)^2\mu_{t-1} = (1-B)\,\beta_{t-2} + \alpha(1-B)\varepsilon_{t-1}$

For $(1-B)^2\beta_{t-1}$, we know that $(1-B)\beta_t = \tau\varepsilon_t$

Therefore $(1-B)\beta_{t-1} = \tau\varepsilon_{t-1} \Longrightarrow (1-B)^2\beta_{t-1} = \tau(1-B)\varepsilon_{t-1}$

$(1-B)^2 z_t = (1-B)\beta_{t-2} + \alpha(1-B)\varepsilon_{t-1} + \tau(1-B)\varepsilon_{t-1} + \varepsilon_t - 2\varepsilon_{t-1} + \varepsilon_{t-2}$

Since $(1-B)\beta_t = \tau\varepsilon_t$

$(1-B)\beta_{t-2} = \tau\varepsilon_{t-2}$

$$\begin{aligned}(1-B)^2 z_t &= \tau\varepsilon_{t-2} + \alpha(\varepsilon_{t-1} - \varepsilon_{t-2}) + \tau(\varepsilon_{t-1} - \varepsilon_{t-2}) + \varepsilon_t - 2\varepsilon_{t-1} + \varepsilon_{t-2} \\ &= \tau\varepsilon_{t-2} + \alpha\varepsilon_{t-1} - \alpha\varepsilon_{t-2} + \tau\varepsilon_{t-1} - \tau\varepsilon_{t-2} + \varepsilon_t - 2\varepsilon_{t-1} + \varepsilon_{t-2} \\ &= \varepsilon_t + (\alpha + \tau - 2)\varepsilon_{t-1} + (\tau - \alpha - \tau + 1)\varepsilon_{t-2}\end{aligned}$$

$= \varepsilon_t - (2 - \alpha - \tau)\varepsilon_{t-1} + (1 - \alpha)\varepsilon_{t-2}$

$= \varepsilon_t - (2 - \alpha - \tau)\varepsilon_{t-1} - (\alpha - 1)\varepsilon_{t-2}$

$if\ \theta_1 = 2 - \alpha - \tau, \theta_2 = \alpha - 1$

Therefore $(1 - B)^2 z_t = \varepsilon_t - \theta_1\varepsilon_{t-1} - \theta_2\varepsilon_{t-2} \Longrightarrow (1-\theta_1 B - \theta_2 B^2)\varepsilon_t$ (26)

XIII. FOR HOLT-WINTER EXPONENTIAL SMOOTHING

The 3-parameter Holt-Winters method with additive seasonality is so complicated that it would never be identified in practice [7]. The multiplicative Holt-Winter does not have an ARIMA equivalent at all.

All exponential smoothing methods need some estimation of smoothing parameters which is either α, β, τ. The minimization of the mean square error is the common method of estimating these parameters and this is normally done through the grid search method.

Emperically, the relationship established above can be pictured in the performance measures of the two models (Exponential smoothing and ARIMA model)

XIV. TABLE 4: COMPARISON TABLE FOR SINGLE EXPONENTIAL SMOOTHING MODEL AND ARIMA (0, 1, 1)

ROUTES	Exponential Smoothing Model			ARIMA Model		
	MAE	RMSE	MAPE	MAE	RMSE	MAPE
ENU-ABK	2.9189	4.048	∞	2.87	4.107	64.143
NSK-9MILE	2.7829	3.9206	∞	2.839	3.916	67.966
ENU-ONITSHA	3.2404	4.3943	47.4046	3.274	4.515	47.172

XV. TABLE 5: COMPARISON TABLE FOR DOUBLE EXPONENTIAL SMOOTHING MODEL AND ARIMA (0, 2, 2)

ROUTE	Exponential Smoothing Model			ARIMA Model		
	MAE	RMSE	MAPE	MAE	RMSE	MAPE
ENU-ABK	2.9531	4.158	∞	3.177	4.555	63.378
NSK-9MILE	2.8496	4.055	∞	2.945	4.322	66.611
ENU-ONITSHA	3.3121	4.573	45.298	3.577	5.377	46.163

XVI. SUMMARY

Holt-Winter (Triple) exponential smoothing model expounded by [1] is also used when the series under study exhibit both trend and seasonality. These informed the use of the two approaches, in addition to the other two forms of exponential smoothing models to study cases of Road traffic crashes (RTC) in some selected routes in Enugu state, Nigeria.

On the other hand, the series were smoothed (remove irregularities from the series) using three different exponential smoothing methods. The first one (single exponential smoothing method) assumed that the series has no trend, the second one (double exponential smoothing method) assumed that the series has trend while the third one (Holt-winter (Triple) exponential smoothing method) has the assumption that the series has both trend and seasonality. From the result of the exponential smoothing modeling, the acknowledgment of the presence of seasonality improves the performance of the model. This is obvious in the result of the Holt- Winters exponential smoothing model.

Even as the smoothing models are reliable for forecasting, it has some inherent shortfalls. As a modeling technique, exponential smoothing methods have a significant shortfall emanating from not having an objective statistical identification and diagnostic checking system for evaluating the "goodness" of competing exponential smoothing models. Because of these shortfalls, exponential smoothing models are statistically regarded as ad hoc models.

REFERENCES

[1]Peter, W. R. (1960). Forecasting sales by exponentially weighted moving average. Management Science, 6. 324-342.

[2] Sumaila, A. F. (2013). Road crashes trends and safety management in Nigeria. Journal of Geography and Regional Planning, 6 (3), 53-62.

[3] Salako A. J., Adeoke, B. O & Akanmu, T. A. (2014). Time series analysis for modeling and detecting seasonality pattern of auto-crash cases recorded at Federal Road Safety Commission, Osun State. International Journal of Engineering and Advanced Technology Studies, 2 (4), 25

[4] Atubi, A. O. (2012). Epidemiology of Injuries from Road Traffic Accidents in Lagos State, Nigeria. International Journal of Science and Technology, 1(2), 56-75.

[5] Fomby, T. B. (2008). Exponential smoothing models in tax: Department of Economics southern Methodist University Dallas.
[6] Box & Jenkins (1970). Time series analysis, Forecasting and control. Holden day, San Francisco, C.A. New York.
[7] Chatfield, C. & Yar, M. (1988). Holt-Winters forecasting: some practical issues. Journal of the Royal statistical society, 37(2), 129-140