A Review of Vedic Mathematical Techniques for Solving Algebraic Equations

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Abstract:
In terms of Efficiency and Simplicity, Vedic Methods are performing exceptionally well. It actually mirroring traditions of versatile and varied methods in mathematics, as a result there are several tactics to simplify equations. Arithmetic computations cannot be obtained faster by any other known method. This paper describes the different techniques used in ancient Vedic mathematics for solving algebraic equations.

Keywords - *Vedic Mathematics, Sutras.*

I. INTRODUCTION

Mathematics is practical science helps us to understand the mysteries of universe. It have been seen that mathematical study comes under influence of two directions, termed as Outer and Inner. The outer direction moves us to applying number, order and mathematical relationships in world around us. It is practical, useful & beneficial. Next one is Inner direction, it puts back to the credentials unto which entire mathematics stands. Ultimately, it reminds us of our origin, the unity, supreme self, which is the basis of entire creation. So, these are two directions but they are not exclusive. The understanding of outer direction has been enhanced by inner direction. It is the Vedic system, that enables these two directions to be studied in mutual harmony and this may be accomplished through correct appreciation of the sutras.

Vedic mathematics has mainly 16 Sutras. These Sutras are enlisted as follows.

1	Anurupye - Shunyamanyat	If one variable is in ratio, the other variable is zero.
2	Chalana-Kalanabyham	Differences and Similarities.
3	Ek-Adhikina Purvena	By one more than the previous One
4	Ek-Anyunena Purvena	By one less than the previous one
5	Gunaka-Samuchyah	The factors of the sum is equal to the sum of the factors
6	Gunita-Samuchyah	The product of the sum is equal to the sum of the product
7	Nikhilam Navatash -Charamam Dashatah	All from 9 and last from 10
8	ParaavartyaYojayet	Transpose and adjust
9	Puranapuranabyham	By the completion or non completion
10	Sankalana-Vyavakalanabhyam	By addition and by subtraction.
11	Shes-Anyankena Charamena	The remainders by the last digit.
12	Shunyam Saamya-Samuccaye	When the sum is the same that sum is zero.
13	Sopaantyadvayamantyam	The ultimate and twice the penultimate
14	Urdhva-tiryabhyam	Vertically and crosswise.
15	Vyashtisamanstih	Part and Whole.
16	Yaavadunam	Whatever the extent of its deficiency.

II. IMPLEMENTATION OF VEDIC MATHEMATICAL METHODS IN ALGEBRAIC EQUATIONS

A. Vilokanam:

The Sutra 'Vilokanam' means 'Observation'. Generally some problems can be solved by mere observation.

Example 1: $t + \frac{1}{t} = \frac{5}{2}$

can be viewed as $t + \frac{1}{t} = 2 + \frac{1}{2}$ gives t = 2 or ½

Example 2: $\frac{t}{t+2} + \frac{t+2}{t} = \frac{34}{15}$

Vilokanam gives $\frac{34}{15} = \frac{9+25}{5\times3} = \frac{3}{5} + \frac{5}{3} = \frac{t}{t+2} + \frac{t+2}{t}$

Gives $\frac{3}{5} = \frac{t}{t+2}$ or $\frac{5}{3} = \frac{t}{t+2}$

5t = 3t + 6 or 3t = 5t + 10

2t = 6 or -2t = 10

t = 3 or t = -5

Example 3: m + n = 9 and mn = 14.

by Vilokanam, mn = 14 gives m = 2, n = 7 or m = 7, n = 2 and these two sets satisfy m + n = 9 since 2 + 7 = 9 or 7 + 2 = 9. Hence the solution.

B. ***Paravartya:***

It means 'transpose and apply'. The rule relating to transposition enjoins invariable change of sign with every change of side. Such that + becomes - and × becomes ÷ and conversely.

Type (i) : Consider the Algebraic equation of the type (at + b) = (ct + d)

By Paravartya technique, solution of this equation is given by $t = \frac{d-b}{a-c}$

Example 1. Consider the problem (7t – 5) = (5t + 1)

Here a = 7, b = - 5, c = 5, d = 1

Hence $t = \frac{d-b}{a-c} = \frac{1+5}{7-5} = \frac{6}{2} = 3$

Type (ii) : Consider the equation of the type (t + p) (t + q) = (t + r) (t + s).

By paravartya technique , we get solution as $t = \frac{rs-pq}{(p+q)-(r+s)}$

Example 2. Consider the problem (t + 7) (t – 6) = (t +3) (t – 4).

Here p = 7, q = - 6, r = 3, s = -4

Hence $t = \frac{\mathrm{rs-pq}}{\mathrm{(p+q)-(r+s)}} = \frac{3(-4)-7(-6)}{(7-6)-(3-4)} = 15$

Type (iii) : Consider the equation of the type $\frac{\mathrm{at+b}}{\mathrm{ct+d}} = \frac{\mathrm{p}}{\mathrm{q}}$

paravartya gives pd - qb, qa - pc and $\mathrm{t} = \frac{\mathrm{pd-qb}}{\mathrm{qa-pc}}$

Example 3. Consider the problem $\frac{3t+1}{4t+3} = \frac{13}{19}$

paravartya gives $\mathrm{t} = \frac{\mathrm{pd-qb}}{\mathrm{qa-pc}} = \frac{13(3)-19(1)}{19(3)-13(4)} = \frac{39-19}{57-52} = \frac{20}{5} = 4$

Type (iv): Consider the equation of the type $\frac{p}{t+a} + \frac{q}{t+b} = 0$

paravartya gives $t = \frac{-pb-qa}{p+q}$

Example 4. Consider the problem $\frac{3}{t+4} + \frac{4}{t-6} = 0$

Here a = 4, b = - 6, p = 3, q = 4

Hence $t = \frac{-pb-qa}{p+q} = \frac{-3(-6)-4(4)}{3+4} = \frac{18-16}{7} = \frac{2}{7}$

C. *Shunyam Saamya-Samuccaye:*

It says when the 'Samuccaya is the same term, that Samuccaya is zero.' i.e, it should be equated to zero.

Type (i) 'Samuccaya' is a term which occurs as a common factor in all the terms concerned and proceed as follows.

Example 1: The equation 7t + 3t = 4t + 5t has the same factor ‘ t ‘ in all its terms.
Hence by the sutra it is zero, i.e; t = 0.

Example 2: 4(t+1) = 9(t+1) has the common factor (t+1)
Now Samuccaya is (t + 1), i.e t + 1 = 0 gives t = -1

Type (ii) let 'Samuccaya' as product of independent terms in expressions like (t + p) (t + q) = (t + r) (t + s)

Example 3: (t + 9) (t + 4) = (t – 12) (t – 3)
Here Samuccaya is 9 x 4 = 36 = -12 x -3
Since it is same , we derive t = 0

Type (iii) We interpret ' Samuccaya 'as the sum of the denominators of two fractions.
It is applicable for all problems of the type $\frac{m}{\text{at}+\text{b}}+\frac{m}{\text{ct}+\text{d}}=0$

Samuccaya is (at+b)+(ct+d) and solution is ($m \neq 0$)

$$t=\frac{-(\text{b}+\text{d})}{(\text{a}+\text{c})}$$

Example 4: $\frac{1}{3t-2}+\frac{1}{2t-1}=0$

we directly put the Samuccaya i.e., sum of the denominators as
3t – 2 +2 t - 1 = 5t - 3 = 0
Gives 5t = 3 or t = 3 / 5

Type (iv) We now interpret 'Samuccaya' as combination.
If the addition of the numerators and the addition of the denominators be the equal, then that sum = 0.

Consider examples of type $\frac{\text{at}+\text{b}}{\text{at}+\text{c}}=\frac{\text{at}+\text{c}}{\text{at}+\text{b}}$

As per Samuccaya (at+b) + (at+c) = 0
2at+b+c = 0
2at = -b-c

$$t=\frac{-(\text{b}+\text{c})}{2\text{a}}$$

Example 5: $\frac{3t+4}{3t+5}=\frac{3t+5}{3t+4}$
Since $N_1 + N_2$ = 3t + 4 + 3t + 5 = 6t + 9 ,
And $D_1 + D_2$ = 3t + 4 + 3t + 5 = 6t + 9
We have $N_1 + N_2 = D_1 + D_2$ = 6t + 9
Hence from Sunya Samuccaya we get 6t + 9 = 0
6t = -9

$$t=\frac{-3}{2}$$

D. *Anurupye-Sunyamanyat:*

'If one variable is in ratio, the other variable is zero'. This Sutra is used in solving a special type of simultaneous equations in which the coefficients of 'one' variable are in the same ratio to each other as

the independent terms are to each other. According to the Sutra the 'other' variable is zero, from which we get two equations in the first variable (already considered) and get the same value for the variable.

Example : $3m + 7n = 2$
$4m + 21n = 6$

We observe that the n-coefficients are in the ratio 7 : 21 i.e., 1 : 3, which is same as the ratio of independent terms i.e., 2 : 6 i.e., 1 : 3.

Hence the other variable $m = 0$ and $7n = 2$ or $21n = 6$ gives $n = 2 / 7$

E. ***Sankalana –Vyavakalanabhyam*** **:**

This Sutra means 'by sum and by subtraction'. It can be used in solving a special type of simultaneous equations where coefficients of x and coefficients of y are found interchanged.

Example : $45m – 23n = 113$
$23m – 45n = 91$

From Sankalana – Vyavakalanabhyam

Add them,

i.e., $(45m – 23n) + (23m – 45n) = 113 + 91$

i.e., $68m – 68n = 204$

$m – n = 3$

Subtract one from other,

i.e., $(45m – 23n) – (23m – 45n) = 113 – 91$

i.e., $22m + 22n = 22$

$m + n = 1$

and repeat the same sutra, we get $m = 2$ and $n = - 1$

Simple addition and subtraction are sufficient for solving equation by this method.

III. CONCLUSION

Vedic Mathematical methods are derived from ancient systems of computations, now available in present form by Jagad Guru Swami Sri Bharati Krishna Tirthaji Maharaja, who published book on Vedic mathematics in 1965.

In this paper we use different methods of Vedic Ganit like Viloknam, Paravyatya, Sunyam Samya Samuccaya, Anurupye , Sankalan and Vyavkalanabhyam to solve algebraic equations and we observe that the whole mathematics is governed by 16 Sutras and sub sutras and by these methods we get results very quickly and accurately. these methods are computationally faster and easy to perform.

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