Signed Unidominating Functions Of Corona Product Graph $C_n \odot K_m$

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ABSTRACT *- Domination in graphs is an emerging area of research in graph theory and in recent years it has been studied extensively. An introduction and an extensive overview on domination in graphs and related topics is surveyed and detailed in the two books by Haynes et al. [5, 6]. Dominating sets have applications in diverse areas such as logistics and Communication Networks design, mobile computing, resource allocation and telecommunication etc.*

In Discrete Mathematics, Product of graphs occurs naturally as tools in combinatorial constructions. They give rise to an important classes of graphs and deep structural problems. Frucht and Harary [2] introduced a new product on two graphs G_1 and G_2, called corona product denoted by $G_1 \odot G_2$.

In this paper, we introduced a new concept signed unidominating function and studied this concept for corona product graph $C_n \odot K_m$ and determined the signed unidomination and upper signed unidomination numbers for $C_n \odot K_m$. Also the number of signed unidominating functions of minimum weight is found.

Keywords: *Signed unidominating function, signed unidomination number, minimal signed unidominating function, upper signed unidomination number.*

I. INTRODUCTION

In recent years dominating functions in domination theory is playing a key role as they have interesting applications. The concepts of dominating functions are introduced by Hedetniemi [3]. Anantha Lakshmi [1] has introduced new concepts unidomination, upper unidomination, minimal unidominating function of a graph and studied these functions for some standard graphs.

A new product on two graphs G_1 and G_2, called corona product denoted by $G_1 \odot G_2$, was introduced by Frucht and Harary [2]. The concept signed dominating function of corona product graph $C_n \odot K_m$ was studied by Siva Parvathi [4].

The authors have introduced new concepts signed unidominating function and upper signed unidomination number of a graph in this paper and this is studied for corona product graph $C_n \odot K_m$. Also the signed unidomination and upper signed unidomination number of the above said graph is found. Further, the number of signed unidominating functions of minimum weight and minimal signed unidominating functions of maximum weight for this graph are determined.

II. CORONA PRODUCT OF C_n AND K_m

The corona product of a cycle C_n with a complete graph K_m is a graph obtained by taking one copy of a n – vertex graph C_n and n copies of K_m and then joining the i^{th} - vertex of C_n to every vertex of i^{th} - copy of K_m. This is denoted by $C_n \odot K_m$.

The vertices in C_n are denoted by v_i and the vertices of complete graph K_m are denoted by u_{ij}.

III. SIGNED UNIDOMINATION NUMBER OF $C_n \odot K_m$

In this section the concepts of signed unidominating function, signed unidomination number are defined. The signed unidomination number and the number of signed unidominating functions of minimum weight of $C_n \odot K_m$ are determined.

Definition 1: Let $G(V, E)$ be a connected graph. A function $f: V \rightarrow \{-1,1\}$ is said to be a signed unidominating function if

$$\sum_{u\in N[v]} f(u) \geq 1 \quad \forall\, v \in V \; and \; f(v) = 1,$$
$$\text{and} \sum_{u\in N[v]} f(u) = 1 \quad \forall\, v \in V \;\; and \;\; f(v) = -1.$$

Definition 2: The signed unidomination number of a graph $G(V, E)$ is defined as
$\min\{f(V)/f$ is a signed unidominating function$\}$.
It is denoted by $\gamma_{su}(G)$.
Here $f(V) = \sum_{u\in V} f(u)$ is called as the weight of the signed unidominating function f.
That is the signed unidomination number of a graph $G(V, E)$ is the minimum of the weights of the signed unidominating functions of G.

Theorem 3.1: The signed unidomination number of $C_n \odot K_m$ is n when m is even.

Proof: Let $C_n \odot K_m$ be the given corona product graph.

Suppose m is even.
Define a function $f: V \to \{-1,1\}$ by
$f(v_i) = 1$ for $i = 1,2, \dots \dots, n$

$$\text{and } f(u_{ij}) = \begin{cases} -1 & for \ \frac{m}{2} \ vetices \ in \ each \ copy \ of \ K_m, \\ 1 & otherwise \end{cases}$$

for $i = 1,2, \dots\dots, n$ and j$= 1,2, \dots\dots, m$.

Now we show that f is a signed unidominating function.

If $v_i \in C_n$ then

$$\sum_{u\in N[v_i]} f(u) = f(v_{i-1}) + f(v_i) + f(v_{i+1}) + f(u_{i1}) + f(u_{i2}) + \cdots + f(u_{im})$$
$$= 1 + 1 + 1 + \left[\frac{m}{2}(-1) + \frac{m}{2}(1)\right] = 3.$$

If $u_{ij} \in K_m$ then $f(u_{ij}) = 1$ or $f(u_{ij}) = -1$.

Let $u_{ij} \in K_m$ and $f(u_{ij}) = 1$. Then

$$\sum_{u\in N[u_{ij}]} f(u) = f(v_i) + f(u_{i1}) + f(u_{i2}) + \cdots + f(u_{im}) \quad = 1 + \left[\frac{m}{2}(-1) + \frac{m}{2}(1)\right] = 1.$$

Let $u_{ij} \in K_m$ and $f(u_{ij}) = -1$. Then

$$\sum_{u\in N[u_{ij}]} f(u) = f(v_i) + f(u_{i1}) + f(u_{i2}) + \cdots + f(u_{im}) \quad = 1 + \left[\frac{m}{2}(-1) + \frac{m}{2}(1)\right] = 1.$$

Hence it follows that f is a signed unidominating function.

$$\text{Now } f(V) = \sum_{u\in C_n} f(u) + \sum_{u\in K_m} f(u)$$
$$= \underbrace{(1 + 1 + 1 \dots + 1)}_{(n-times\ )} +$$

$$\underbrace{\left\{\underbrace{(1+\cdots+1)}_{(\frac{m}{2}-times\ )}+\underbrace{((-1)+\cdots+(-1))}_{(\frac{m}{2}-times\ )}\right\}+\cdots+\left\{\underbrace{(1+\cdots+1)}_{(\frac{m}{2}-times\ )}+\underbrace{((-1)+\cdots+(-1))}_{(\frac{m}{2}-times\ )}\right\}}_{(n-times\ )}$$

$$= n.$$

Thus $f(V) = n$.

Now for all other possibilities of assigning values 1 and -1 to the vertices of C_n and vertices u_{ij} in each copy of K_m, we can see that the resulting functions are not signed unidominating functions.
Therefore the function defined above is the only signed unidominating function.
Therefore $\gamma_{su}(C_n \odot K_m) = n$ when m is even.

Theorem 3.2: The signed unidomination number does not exist for $C_n \odot K_m$, if m is odd.
Proof: Let $C_n \odot K_m$ be the given corona product graph.
We show that the signed unidomination number does not exist for the graph if m is odd.
Suppose m is odd.
Define a function $f: V \to \{-1,1\}$ by

$$f(v_i) = 1 \text{ for } i = 1,2, \dots \dots, n,$$

$$\text{and } f(u_{ij}) = \begin{cases} -1 & for\ \frac{m+1}{2}\ vetices\ in\ each\ copy\ of\ K_m, \\ 1 & for\ \frac{m-1}{2}\ vetices\ in\ each\ copy\ of\ K_m \end{cases}$$

for $i = 1,2, \dots \dots, n$ and $j = 1,2, \dots \dots, m$.

If $v_i \in C_n$ then

$$\sum_{u \in N[v_i]} f(u) = f(v_{i-1}) + f(v_i) + f(v_{i+1}) + f(u_{i1}) + f(u_{i2}) + \cdots + f(u_{im})$$

$$= 1 + 1 + 1 + \left[\left(\frac{m+1}{2}\right)(-1) + \left(\frac{m-1}{2}\right)(1)\right] = 2.$$

If $u_{ij} \in K_m$ then $f(u_{ij}) = 1$ or $f(u_{ij}) = -1$.

If $u_{ij} \in K_m$ and $f(u_{ij}) = 1$ then

$$\sum_{u \in N[u_{ij}]} f(u) = f(v_i) + f(u_{i1}) + f(u_{i2}) + \cdots + f(u_{im}) = 1 + \left[\left(\frac{m+1}{2}\right)(-1) + \left(\frac{m-1}{2}\right)(1)\right] = 0.$$

That is the condition for signed unidominating function fails at the vertex u_{ij} for which $f(u_{ij}) = 1$.
Therefore f is not a signed unidominating function.
Similarly we can show that if we define a function $f: V \to \{-1,1\}$ by

$$f(v_i) = 1 \text{ for } i = 1,2, \dots \dots, n,$$

$$\text{and } f(u_{ij}) = \begin{cases} -1 & for\ \frac{m-1}{2}\ vetices\ in\ each\ copy\ of\ K_m, \\ 1 & for\ \frac{m+1}{2}\ vetices\ in\ each\ copy\ of\ K_m \end{cases}$$

for $i = 1,2, \dots \dots, n$ and $j = 1,2, \dots \dots, m$

then f is not a signed unidominating function.
For other possibilities of assigning values $1, -1$ to the vertices of K_m we can see that the resulting functions are not a signed unidominating functions.
Thus the signed unidomination number does not exist for the corona product graph $C_n \odot K_m$ when m is odd.

Theorem 3.3: If m is even then the number of signed unidominating functions of $C_n \odot K_m$ with minimum weight n is 1.

Proof: Follows by Theorem 3.1.

IV. UPPER SIGNED UNIDOMINATION NUMBER OF $C_n \odot K_m$

In this section the concepts of minimal signed unidominating function, upper signed unidomination number are defined. The upper signed unidomination number and the number of minimal signed unidominating functions of maximum weight of $C_n \odot K_m$ are determined. Further the results obtained are illustrated.

Definition 1: Let f and g be functions from V to $\{-1,1\}$. We say that $g < f$ if $g(u) \leq f(u)\ \forall\, u \in V$, with strict inequality for at least one vertex u.

Definition 2: A signed unidominating function $f: V \to \{-1,1\}$ is called a minimal signed unidominating function if for all $g < f, g$ is not a signed unidominating function.

Definition 3: The upper signed unidomination number of a graph $G(V, E)$ is defined as $\max\{f(V)/f \text{ is a minimal signed unidominating function}\}$.
It is denoted by $\Gamma_{su}(G)$.

Theorem 4.1: The upper signed unidomination number of $C_n \odot K_m$ is n when m is even.
Proof: Let $C_n \odot K_m$ be the given graph.
Define a function $f: V \to \{-1,1\}$ by

$$f(v_i) = 1 \text{ for } i = 1,2, \dots \dots, n,$$

$$\text{and } f(u_{ij}) = \begin{cases} -1 & for\ \frac{m}{2}\ vetices\ in\ each\ copy\ of\ K_m, \\ 1 & otherwise \end{cases}$$

for $i = 1,2, \dots \dots, n$ and $j = 1,2, \dots \dots, m$.

This function is same as the function defined in Theorem 3.1 and it is shown that f is a signed unidominating function.

Now we check for the minimality of f.
Define a function $g: V \to \{-1,1\}$ by

$$g(v_i) = \begin{cases} -1 & for\ v_i = v_k \in C_n\ for\ some\ k, \\ 1 & otherwise \end{cases}$$

$$\text{and } g(u_{ij}) = \begin{cases} -1 & for\ \frac{m}{2}\ vetices\ in\ each\ copy\ of\ K_m, \\ 1 & otherwise \end{cases}$$

for $i = 1,2, \dots \dots, n$ and j= $1,2, \dots \dots, m$.

Then by the definition of g it is obvious that $g < f$.
Suppose $i = k$.

Then $g(v_k) = -1$.

If $u_{kj} \in K_m$ then $g(u_{kj}) = 1$ or $g(u_{kj}) = -1$.

Let $u_{kj} \in K_m$ and $g(u_{kj}) = 1$. Then

$$\sum_{u \in N[u_{kj}]} g(u) = g(v_k) + g(u_{k1}) + g(u_{k2}) + \dots + g(u_{km}) = (-1) + \left[\left(\frac{m}{2}\right)(-1) + \left(\frac{m}{2}\right)(1)\right] = -1 \neq 1.$$

That is the condition for unidominating function fails at the vertex $u_{kj} \in K_m$ for which $g(v_k) = -1$.

So $\sum_{u\in N[u_{kj}]} g(u) \neq 1$, as $g(v_k) = -1$.
Thus g is not a unidominating function.

Since g is defined arbitrarily, it follows that there exists no $g < f$ such that g is a signed unidominating function.

For all other possibilities of defining a function $g < f$, we can see that g is not a signed unidominating function.

Hence f is a minimal signed unidominating function.

Further f is the only one minimal signed unidominating function because any other possible assignment of

values $-1, 1$ to the vertices of C_n and K_m does not make f no more a signed unidominating function.

$$\text{Now } f(V) = \sum_{u\in C_n} f(u) + \sum_{u\in K_m} f(u)$$

$$= \underbrace{(1+1+1\ldots+1)}_{(n-times)} +$$

$$\underbrace{\left\{\underbrace{(1+\cdots+1)}_{(\frac{m}{2}-times)} + \underbrace{((-1)+\cdots+(-1))}_{(\frac{m}{2}-times)}\right\} + \cdots + \left\{\underbrace{(1+\cdots+1)}_{(\frac{m}{2}-times)} + \underbrace{((-1)+\cdots+(-1))}_{(\frac{m}{2}-times)}\right\}}_{(n-times)}$$

$$= n + \left(\frac{m}{2}\right)(n) + \left(\frac{m}{2}\right)(-n) = n.$$

Thus $f(V) = n$.

Now $\max\{f(V)/f$ is a minimal signed unidominating function$\} = n$, because f is the only one minimal signed unidominating function.
Therefore $\Gamma_{su}(C_n \odot K_m) = n$.

Theorem 4.2: The upper signed unidomination number does not exist for the corona product graph $C_n \odot K_m$ if m is odd.

Proof: By Theorem 3.2, we know that the signed unidominating function does not exist for

$C_n \odot K_m$ when m is odd.

Therefore the upper signed unidomination number does not exist for the corona product

graph $C_n \odot K_m$ when m is odd.

Theorem 4.3: If m is even then the number of minimal signed unidominating functions of $C_n \odot K_m$ with maximum weight n is 1.

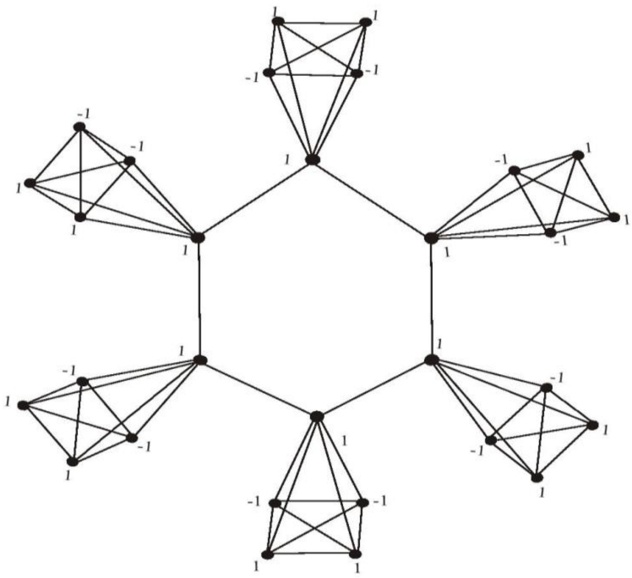
Proof: Follows by Theorem 4.1.

V. ILLUSTRATIONS

Signed unidomination number

Theorem 3.1:

The functional values are given at each vertex of the graph $C_6 \odot K_4$.



$\gamma_{su}(C_6 \odot K_4) = 6$

VI. CONCLUSION

It is interesting to study various graph theoretic properties and domination parameters of corona product graph of a cycle with a complete graph. Signed unidominating functions and upper signed unidomination number of this graph are studied by the authors. Study of these graphs enhances further research and throws light on future developments.

REFERENCES

[1] Anantha Lakshmi, V. A Study on Unidominating and Total Unidominating functions of Some Standard Graphs, Ph.D. thesis, Sri Padmavati Mahila Visvavidyalayam, Tirupati, Andhra Pradesh, India, (2015).

[2] Frucht, R. Harary, F On the corona of Two Graphs. AequationesMathematicae, 1970,Volume 4, Issue 3, pp. 322-325

[3] Hedetniemi S .M, Hedetniemi, S.T. and Wimer, T. V. - Linear time resource allocation algorithms for trees. Technical report URI – 014, Department of Mathematics, Clemson University, 1987.

[4] Siva Parvathi, M. Some Studies on Dominating functions of Corona Product Graphs, Ph. D. Thesis, Sri Padmavathi Mahila Visvavidyalayam, tirupathi, Andhra Pradesh, India, 2013

[5] T.W. Haynes, T. Hedetniemi, and P.J. Slater, Fundamentals of Domination in Graphs, Marcel Dekker, New York, 1998.

[6] T.W. Haynes, S.T. Hedetniemi, and P.J. Slater, Domination in Graphs: Advanced Topics, Marcel Dekker, New York, 199